Laser Wake Field Acceleration

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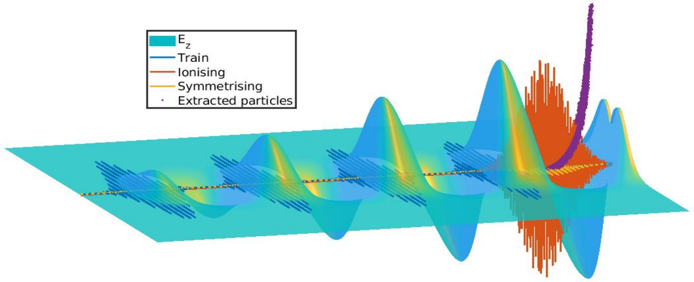

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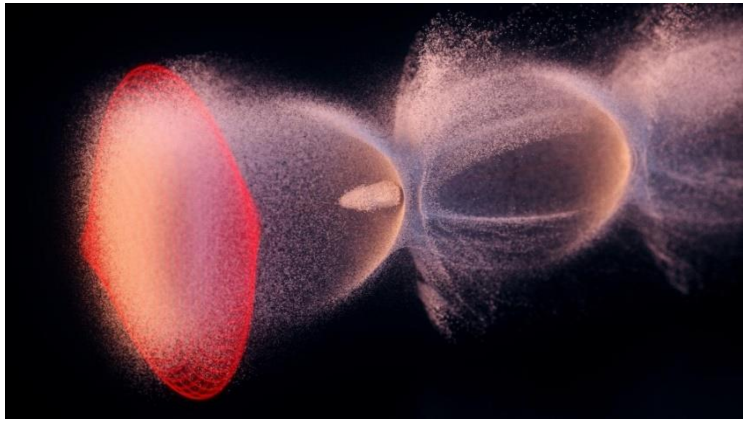

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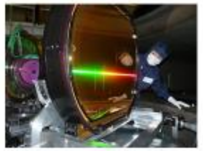
THE FULL MASTER/PhD COURSE

The lessons for the full master/Ph.D. course (40h) on LWFA are available upon request

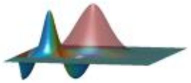
The course in modules

MODULE 1
Cold plasmas, Vlasov equations and plasma waves in underdense plasmas

MODULE 2
Ultra-intense laser pulses, radiation pressure and ponderomotive force.

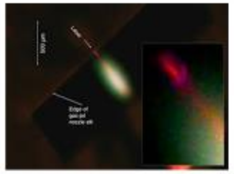


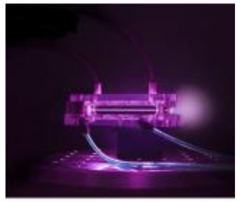
MODULE 3
Plasma waves excitation, laser-plasma accelerators. Linear and fully nonlinear regimes.



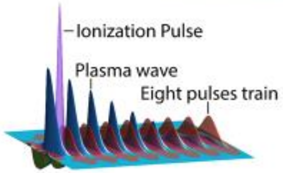
The course

MODULE 4
Propagation of laser pulses into plasmas. Self-focusing and laser guiding





MODULE 5
Particle's injection into the plasma waves: bubble, downramp. Ionization injection and ReMPI



The course

MODULE 6
The simulation codes: fluid, hybrid and PIC

MODULE 7
Design of a laser-plasma accelerator. Scale laws

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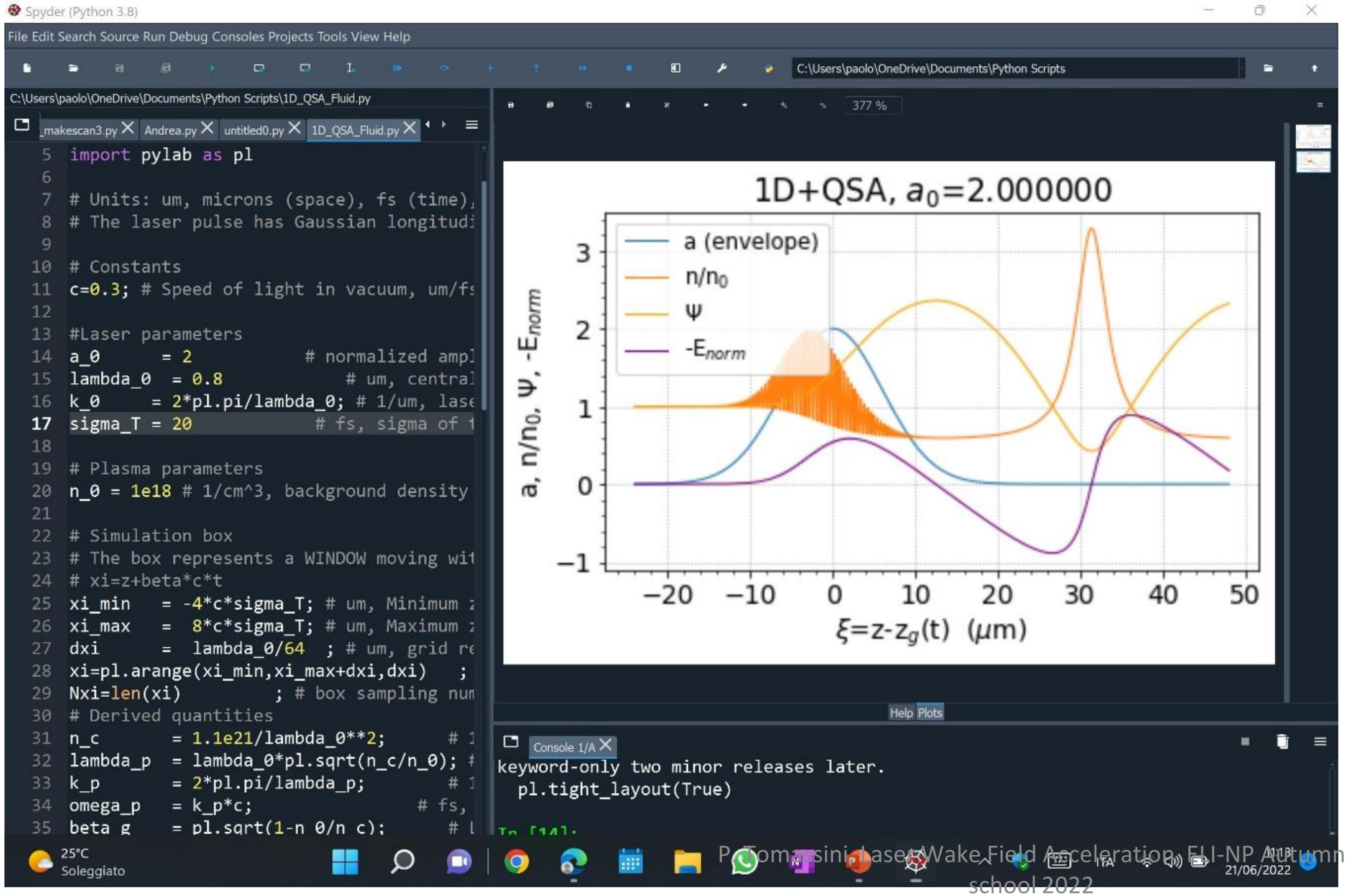
1D self consistent code for wakefield excitation and acceleration studies

nuclear physics

Python code available for the attendees (very simple and fast)
Suitable to take contact with LWFA and for preparatory 1D simulations

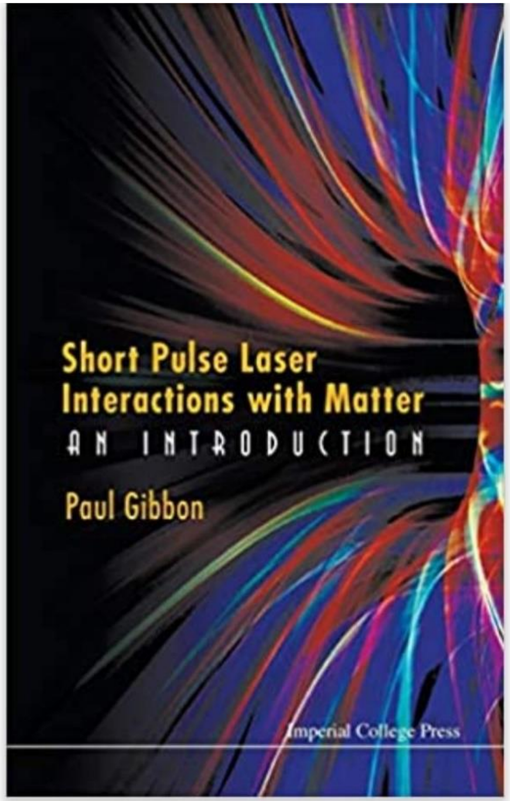


Note: Full simulations are usually performed with **2D or 3D PIC**/Fluid/Hybrid codes

See the lecture of D'Humieres on tomorrow

Basic and less basic references to start with

Paul Gibbon, Short Pulse Laser Interactions with Matter: An Introduction
ICP, ISBN-13: 978-1860941351



Eric Esarey et al., Physics of laser-driven plasma-based electron accelerators,
REVIEWS OF MODERN PHYSICS, 81, (2009)

REVIEWS OF MODERN PHYSICS, VOLUME 81, JULY–SEPTEMBER 2009

Physics of laser-driven plasma-based electron accelerators

E. Esarey, C. B. Schroeder, and W. P. Leemans
Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA

(Published 27 August 2009)

Laser-driven plasma-based accelerators, which are capable of supporting fields in excess of 100 GV/m, are reviewed. This includes the laser wakefield accelerator, the plasma beat wave accelerator, the self-modulated laser wakefield accelerator, plasma waves driven by multiple laser pulses, and highly nonlinear regimes. The properties of linear and nonlinear plasma waves are discussed, as well as electron acceleration in plasma waves. Methods for injecting and trapping plasma electrons in plasma waves are also discussed. Limits to the electron energy gain are summarized, including laser pulse diffraction, electron dephasing, laser pulse energy depletion, and beam loading limitations. The basic physics of laser pulse evolution in underdense plasmas is also reviewed. This includes the propagation, self-focusing, and guiding of laser pulses in uniform plasmas and with preformed density channels. Instabilities relevant to intense short-pulse laser-plasma interactions, such as Raman, self-modulation, and hose instabilities, are discussed. Experiments demonstrating key physics, such as the production of high-quality electron bunches at energies of 0.1–1 GeV, are summarized.

DOI: 10.1103/RevModPhys.81.1229 PACS number(s): 52.38.Kd, 41.75.Lx, 52.40.Mj

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I. INTRODUCTION

OUTLOOK

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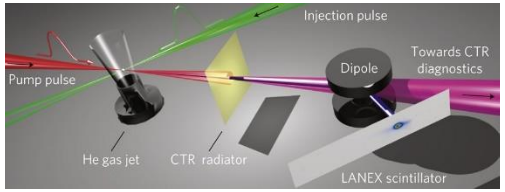
4. Limiting factors to high energy gain accelerators
5. 3D effects on
6. Downramp (or shock) injection
7. Two-Color injection
8. The Resonant Multi-Pulse Ionisation Injection
9. High-Brilliance e-bunches



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The original idea of Laser driven acceleration

VOLUME 43, NUMBER 4 PHYSICAL REVIEW LETTERS 23 JULY 1979

43 years!

Laser Electron Accelerator

T. Tajima and J. M. Dawson
Department of Physics, University of California, Los Angeles, California 90024
(Received 9 March 1979)

An intense electromagnetic pulse can create a weak of plasma oscillations through the action of the nonlinear ponderomotive force. Electrons trapped in the wake can be accelerated to high energy. Existing glass lasers of power density 10^{18}W/cm^2 shone on plasmas of densities 10^{18} cm^{-3} can yield gigaelectronvolts of electron energy per centimeter of acceleration distance. This acceleration mechanism is demonstrated through computer simulation. Applications to accelerators and pulsers are examined.

Collective plasma accelerators have recently received considerable theoretical and experimental investigation. Earlier Fermi[1] and McMillan[2] considered cosmic-ray particle acceleration by moving magnetic fields[1] or electromag-

the wavelength of the plasma waves in the wake:

$$L_t = \lambda_w/2 = \pi c/\omega_p . \quad (2)$$

An alternative way of exciting the plasmon is to inject two laser beams with slightly different

LWFA in a nut shell

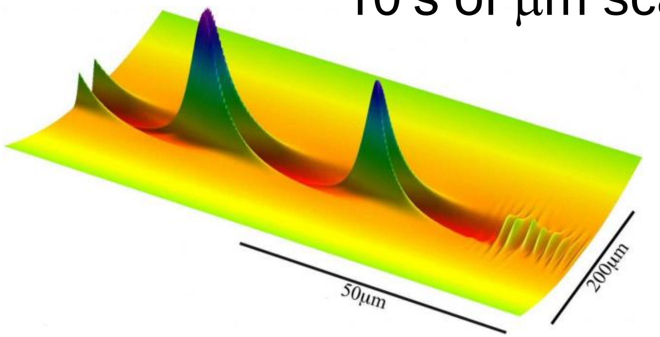
2+2 ingredients for a LWFA process

1. Ultraintense and ultrashort laser pulse

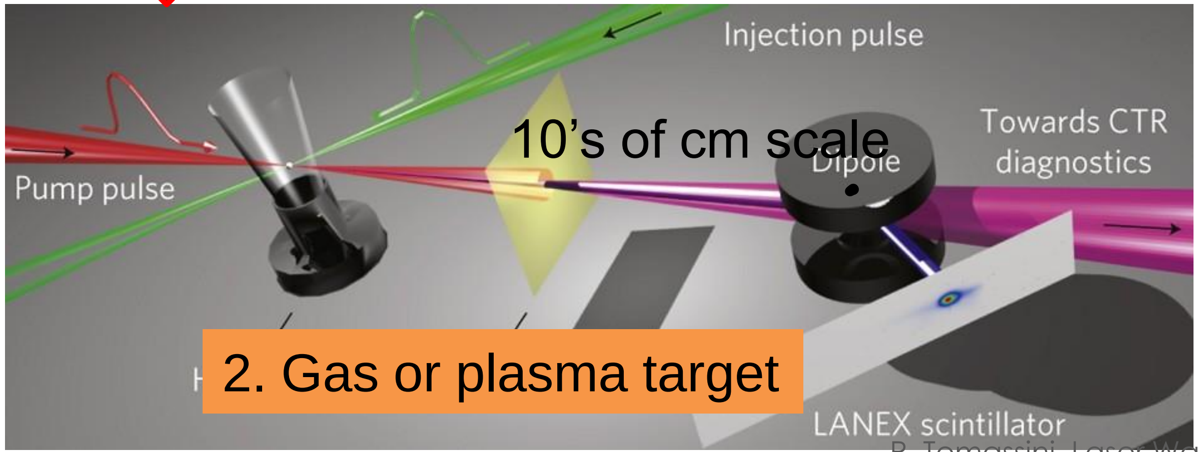
10's of m scale

3. Excitation of the plasma wave

10's of μm scale

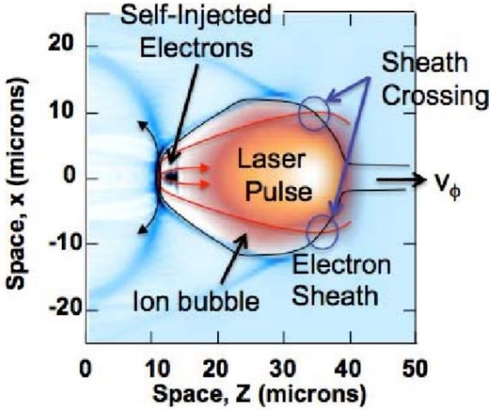


4. Trapping and energy boosting of the particles



10's of cm scale

2. Gas or plasma target

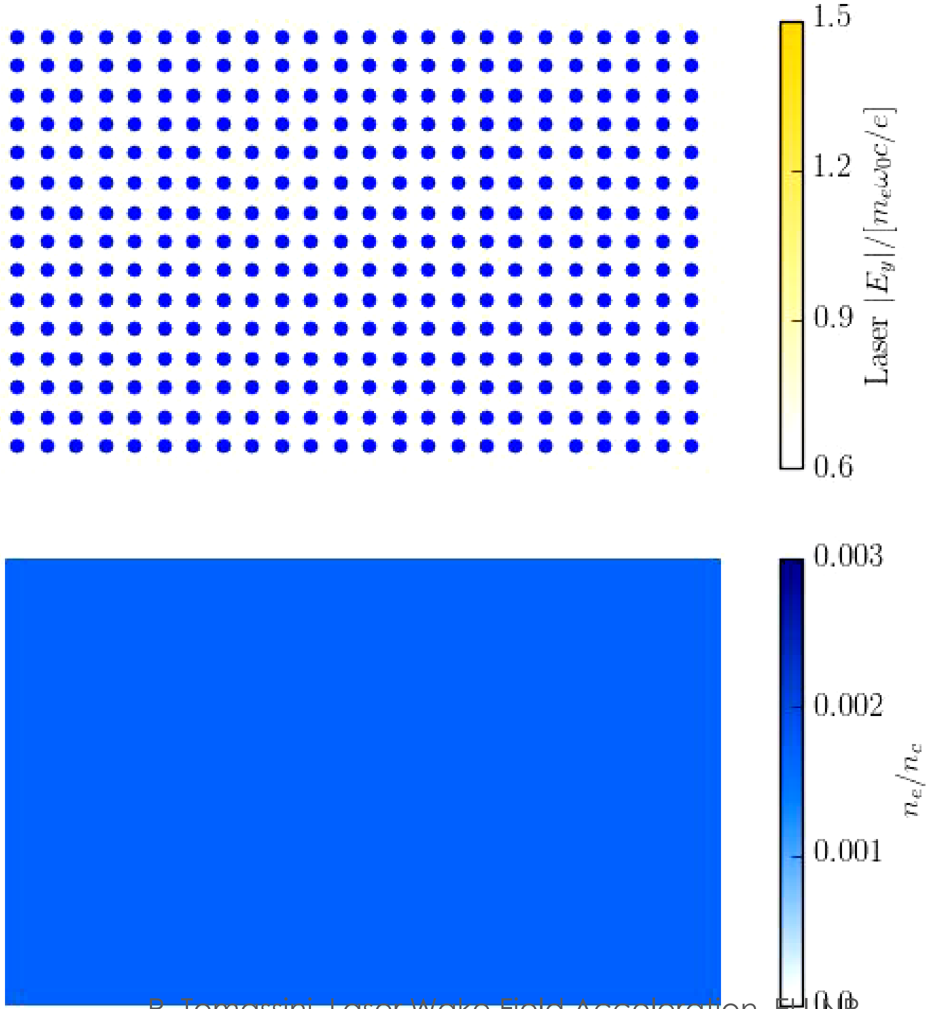


Before to start

Effect of the laser pulse ponderomotive force

- Sampled e^-
- Sampled ions +

Ions are (almost) immobile: we can focus on e^- dynamics



Electron density

We often need an High Beam Quality

- **Compactness** in phase-space (low energy spread, divergence…)
- High charge or high current
- **Linear correlation** in the transverse x-u_x and y-u_y planes (low emittance)

Preserves the focusability

$$u = p/mc$$

Normalized emittance

$$\epsilon_n^2 = \langle x^2\rangle\langle u_x^2\rangle - (\langle x \cdot u_x\rangle)^2$$

Normalized Brighness (5D)

$$B_{n,5D} = \frac{2}{\pi^2}\frac{I}{\epsilon_{n,x}\epsilon_{n,y}} \; A/(m\times rad)^2$$

Footprint

Normalized Brighness (6D)

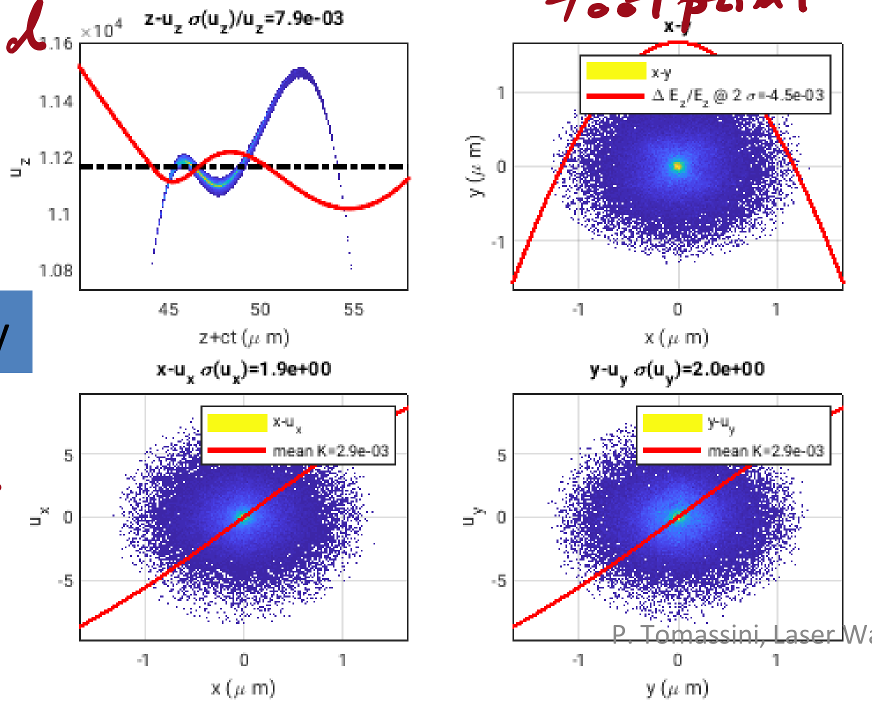
$$B_{n,6D} = \frac{B_{n,5D}}{\delta E/E/0.1\%}$$

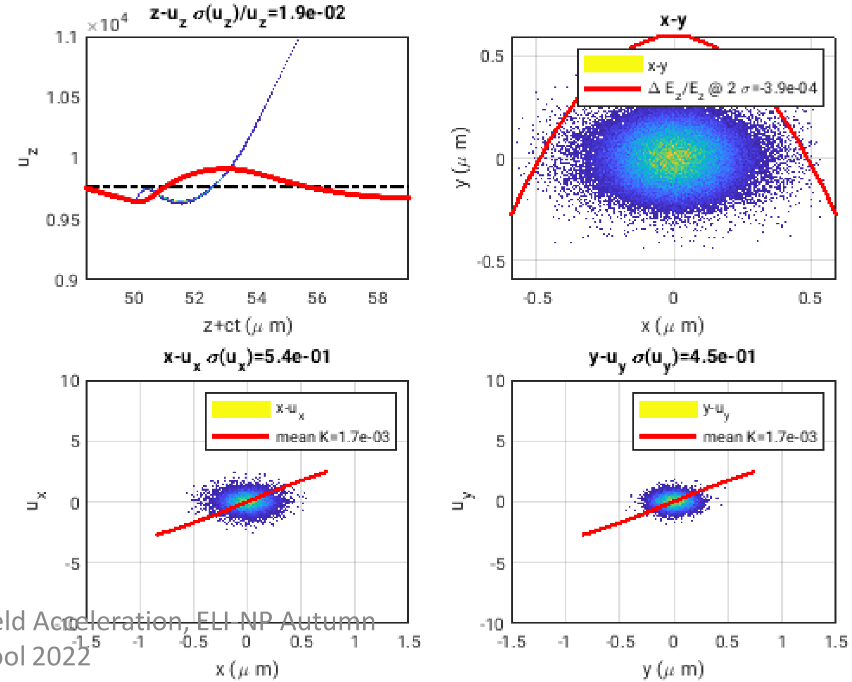
Energy spread

Longitudinal

Transverse

Good Quality





Excellent Quality

But for some application (e.g VHEE) the focus is on beam charge

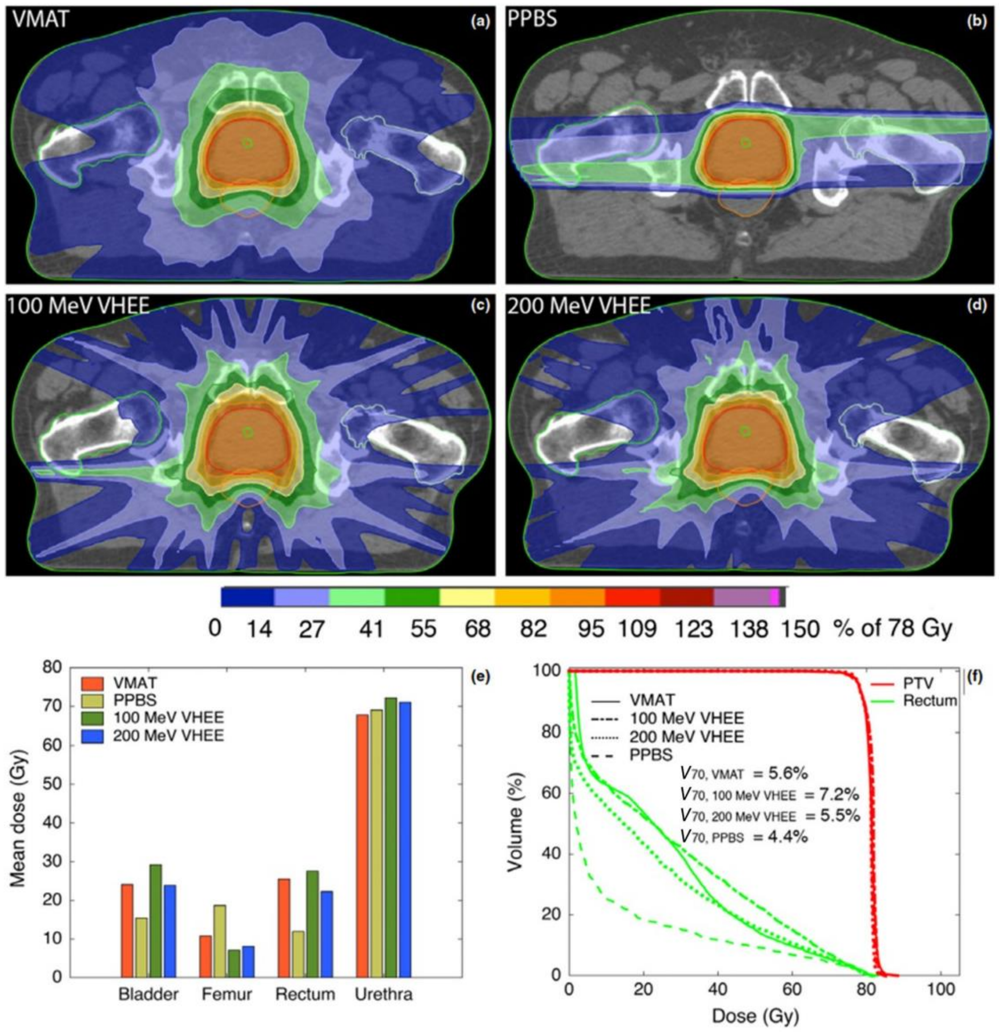
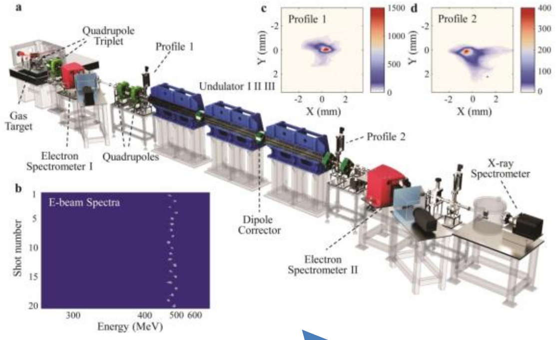


FIGURE 17 Comparison of treatment plans for prostate cancer. Treatment planning comparison between VMAT, PPBS, 100 MeV VHEE, and 200 MeV VHEE plans (a–d, respectively); (e) mean doses to the bladder, femurs, rectum, and urethra; (f) dose volume histogram for the planning target volume and rectum, together with the reported percent of prescription dose to 70% of the rectal volume (V70). Adapted with permission from Schüler et al.[102]

Ultra-high dose rate radiation production and delivery systems intended for FLASH, Jonathan Farr et al. Medical Physics 2021
DOI: 10.1002/mp.15659

More on the lectures from V. Patera on Friday

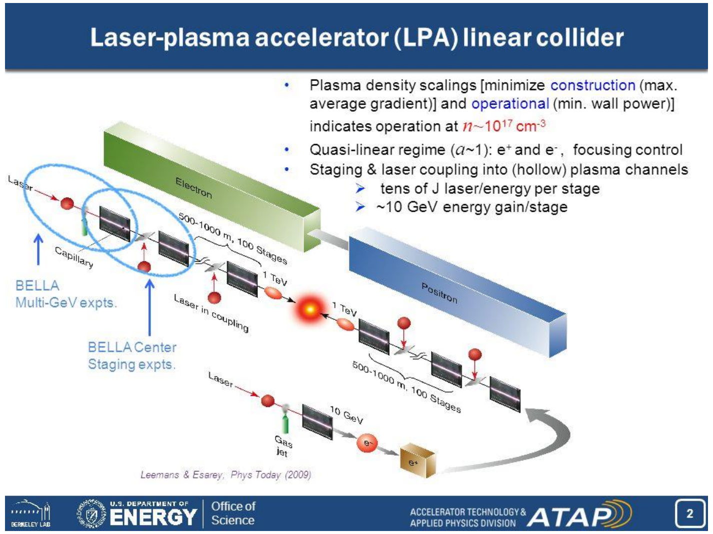
Small scale 0.5-10’s GeV accelerator for X-ray FEL or Compton gamma radiation



Device schematic of table-top free ele[...]aser based on LWFA.

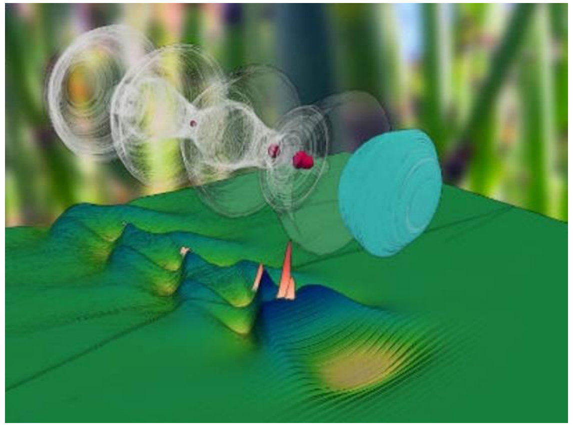
W. T. Wang, K. Feng, *et al.*, *Nature*, **595**, 561 (2021).

Small scale 10’s GeV modules for TeV scale multistage acceleration



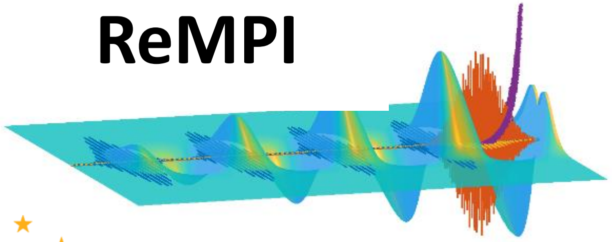
What can we do with LWFA?

Table-Top 100’s MeV accelerator For medical applications (VHEE, imaging)



High-Brigthness accelerators and attosecond bunches generation

ReMPI



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And more…

OUTLOOK

1. *Introduction to LWFA*
2. ***Understanding the excitation and the structure of the plasma waves***
3. The wide spot (1D and QSA) limiting case
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Maxwell equations and particle dynamics chart (recap from CD and ED courses)

MKS	$\mu_0\epsilon_0 = \frac{1}{c^2}$	CGS
$\nabla\cdot\vec{E} = \rho/\epsilon_0$	$\epsilon_0 \to \frac{1}{4\pi}$	$\nabla\cdot\vec{E} = 4\pi\rho$
$\nabla\cdot\vec{B} = 0$		$\nabla\cdot\vec{B} = 0$
$\nabla\times\vec{E} = -\partial_t\vec{B}$	$B \to B/c$	$\nabla\times\vec{E} = -\frac{1}{c}\partial_t\vec{B}$
$\nabla\times\vec{B} = \mu_0\left(\vec{J} + \epsilon_0\partial_t\vec{E}\right)$	$\mu_0 \to 4\pi/c^2$	$\nabla\times\vec{B} = 4\pi\left(\frac{1}{c}\vec{J} + \frac{1}{4\pi}\frac{1}{c}\partial_t\vec{E}\right)$
$d_t\vec{p} = q\left(\vec{E} + \vec{v}\times\vec{B}\right)$	$B \to B/c$	$d_t\vec{p} = q\left(\vec{E} + \frac{\vec{v}}{c}\times\vec{B}\right)$

Maxwell Equations

Dynamics

Normalised potentials, momenta and dynamics

$$\vec{a} \equiv \frac{e}{mc^2}\vec{A} \qquad \varphi \equiv \frac{e}{mc^2}\phi$$

$$\vec{u} \equiv \vec{p}/mc = \gamma\vec{v}/c = \gamma\vec{\beta}$$

Hamiltonian of an e^-

$$\mathcal{H} = \sqrt{1 + (\pi + a)^2} - \varphi \qquad \frac{\partial}{\partial ct}\vec{\pi} = -\frac{\partial\mathcal{H}}{\partial\vec{x}}$$

$$\vec{\pi} \equiv \frac{\partial\mathcal{L}}{\partial\vec{\beta}} = \vec{u} - \vec{a} \qquad \frac{\partial}{\partial ct}\vec{\beta} = \frac{\partial\mathcal{H}}{\partial\vec{\pi}}$$

→ Conjugate momentum

Very useful description of e^- dynamics

$$\frac{1}{c}d_t\left(\vec{u} - \vec{a}\right) = \vec{\nabla}\left(\varphi - \vec{\beta}\cdot\vec{a}\right) \;!$$

Understanding the Ponderomotive Force (even in the relativistic regime)

Useful description of the plane-wave laser pulseg

For an **EM** pulse (ϕ=0) If the pulse is propagating towards z+

$$\vec{E}(z,t) = \hat{x} E_0 f(z - v_g t) \sin\left[k_0 (z - v_\phi t)\right]$$

$$\vec{E} = -\tfrac{1}{c}\partial_t \vec{A} \qquad k_0 v_\phi = \omega_0$$

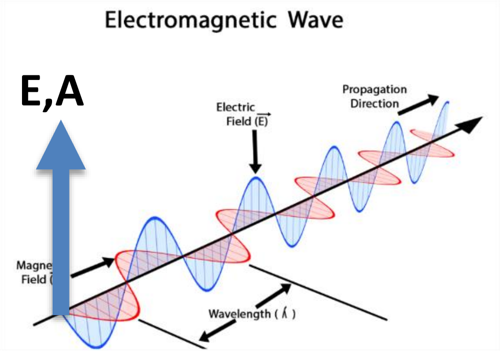
$$T \gg \tau_0 = \lambda_0 / c$$ **Many laser cycles/ pulse**

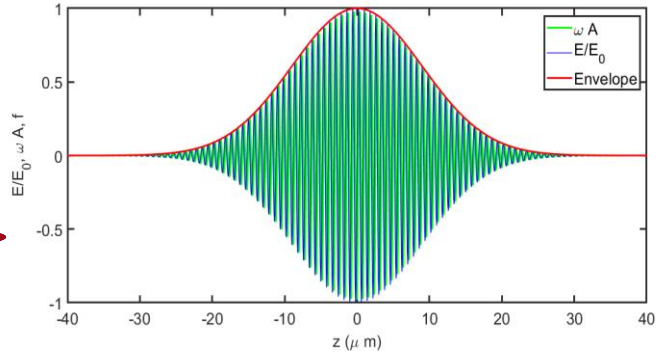
Envelope — Oscillating term

$$\vec{A}(z,t) \simeq \hat{x}\tfrac{1}{\omega_0} E_0 f(z - v_g t) \cos\left[k_0 (z - v_\phi t)\right]$$

$$\vec{a}(z,r,t) \simeq \hat{x} a_0 f(z - v_g t, r) \cos\left[k_0 (z - v_\phi t + \phi(z,r))\right]$$

$$a_0 = \frac{e}{mc^2}\max(A) = \frac{e}{mc^2\omega_0}\max(E) \simeq 8.5\cdot 10^{-10}\sqrt{I[W/cm^2]\lambda[\mu m]^2}$$





A obtained by exact integration of **E**

! Key parameter in laser-electron interaction, *we'll give a physical interpretation in a few slides*

Understanding the Ponderomotive Force (even in the relativistic regime)

IN THE VACUUM(ϕ=0) for pulse polarized along x

$$\vec{a} = \vec{x} a_0 f(z+ct) g(x,y)$$

LONGITUDINAL — TRANSVERSE

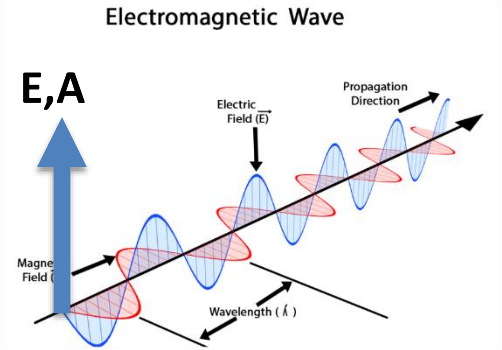
Propagation trough z

$$\frac{1}{c} d_t (\vec{u} - \vec{a}) = \vec{\nabla}\left(\varphi - \vec{\beta}\cdot\vec{a}\right) \quad \begin{cases} \frac{1}{c} d_t (\vec{u}_\perp - \vec{a}_\perp) = -\vec{\beta}\cdot\vec{\nabla}_\perp \vec{a} \\ \frac{1}{c} d_t (\vec{u}_\parallel - \vec{a}_\parallel) = -\vec{\beta}\cdot\vec{\nabla}_\parallel \vec{a} \end{cases}$$

negligible

$$\nabla_\parallel = \partial_z$$

$$\frac{\partial \mathcal{H}}{\partial t} = \frac{d\mathcal{H}}{dt} \qquad \frac{1}{c} d_t \gamma = \frac{1}{\gamma} u_x \, \frac{1}{c} d_t a$$



→ Quasistatic approximation

If the pulse depends on (x,y,z+ct) solely (i.e. with a ***rigid motion*** within the selected time interval), by using $\frac{1}{c}\partial_t = \partial_z$ we get :

$$\left.\begin{array}{l} \frac{1}{c} d_t u_z = -\frac{1}{\gamma} u_x \partial_z a \\ \frac{1}{c} d_t \gamma = \frac{1}{\gamma} u_x \, \frac{1}{c} d_t a \end{array}\right\} \rightarrow$$

Longitudinal

$$d_t(\gamma + u_z) = 0 \rightarrow \gamma + u_z \equiv h_0 = constant$$

$$\frac{1}{c} d_t (\vec{u}_x - \vec{a}_x) = -\beta_x \cdot \partial_x a$$

$$\frac{1}{c} d_t (\vec{u}_y - 0) = -\beta_x \cdot \partial_y a$$

Transverse

Understanding the Ponderomotive Force (even in the relativistic regime)

$$d_t(\gamma + u_z) = 0 \rightarrow \gamma + u_z \equiv h_0 \quad \frac{1}{c} d_t\left(\vec{u}_x - \vec{a}_x\right) = -\beta_x \cdot \partial_x a \quad \frac{1}{c} d_t \vec{u}_y = -\beta_x \cdot \partial_y a$$

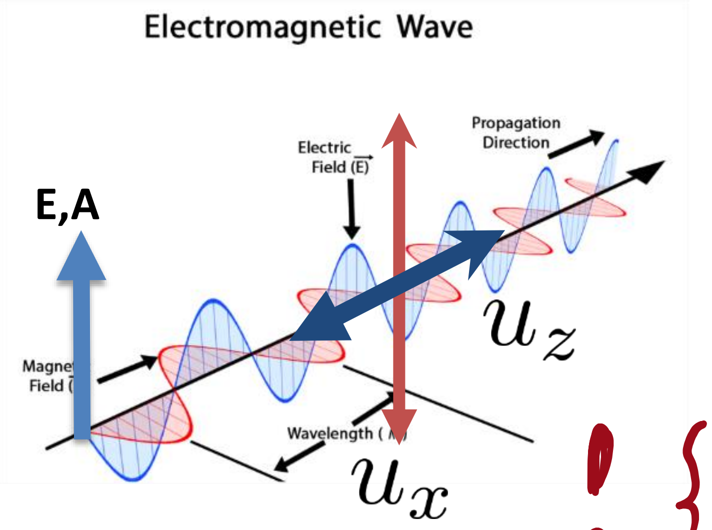
The wide pulse case

If the radial **gradients** of a can be neglected (large waist) we get :

$$\dot{u}_x - \dot{a} \simeq 0 \rightarrow u_x - a \simeq constant$$
$$\dot{u}_y \simeq 0 \rightarrow u_y \simeq constant$$
$$\gamma + u_z \equiv h_0 = constant$$

$$u_x = u_{x;0} + (a - a_{;0})$$
$$u_y = u_{y,0}$$
$$u_z = \frac{1}{2h_0}\left[h_0^2 - (1 + u_x^2 + u_y^2)\right]$$
$$\gamma = \frac{1}{2h_0}\left[h_0^2 + (1 + u_x^2 + u_y^2)\right]$$

Analytical solution of the dynamics for the momenta



Finally we got: the oscillation of the transverse momentum in a laser cycle ***is the same of the normalized potential***. Therefore a_0 gives the measure of the maximum transverse momentum acquired during one cycle.

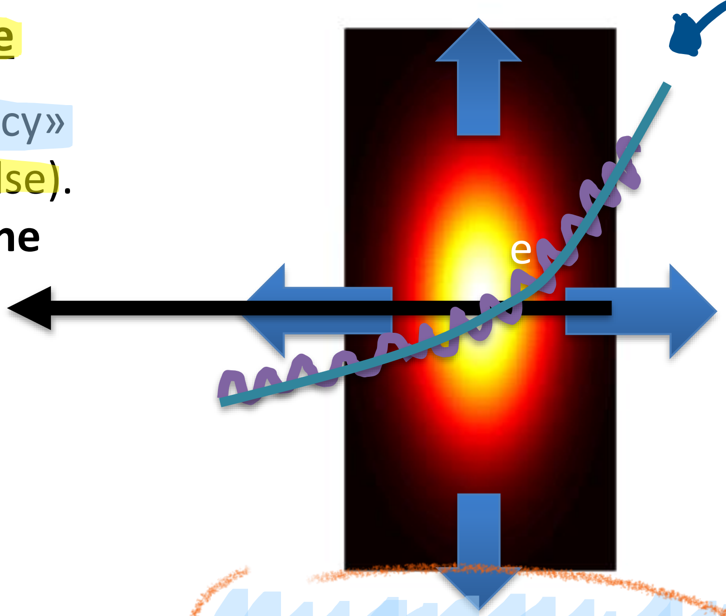
If $a_0 \ll 1$ the quivering is nonrelativistic and u_z is almost constant

If $a_0 \sim 1$ the quivering is relativistic and u_z quivers (with a drift)

The Ponderomotive Force (LWFA)

The effective Hamiltonian for the low-frequency phase-space

The electromagnetic field is supposed to have a «low frequency» component and a «high-frequency» component (the laser pulse). From now on we **retain the low-frequency parts, leaving to the <a^2> term the role of ponderomotive action**



Low frequency u_s

The average kinetic energy ***including quivering***

$$< \gamma >= \gamma_s = \sqrt{1 + u_s^2 + < a^2 >}$$

The average total energy ***including the potential***

$$E_s = \gamma_s - \phi = \sqrt{1 + u_s^2 + < a^2 >} - \phi_s$$

The canonical momentum and the EFFECTIVE HAMILTONIAN are

$$\pi \equiv u_s - a_s \Rightarrow u_s = \pi + a_s$$

$$\mathcal{H}_s = \sqrt{1 + (\pi + a)^2 + < a^2 >} - \phi_S$$

$$\frac{1}{c}\dot{\vec{u}}_s = -\frac{1}{2\gamma_s}\vec{\nabla} < a^2 >$$

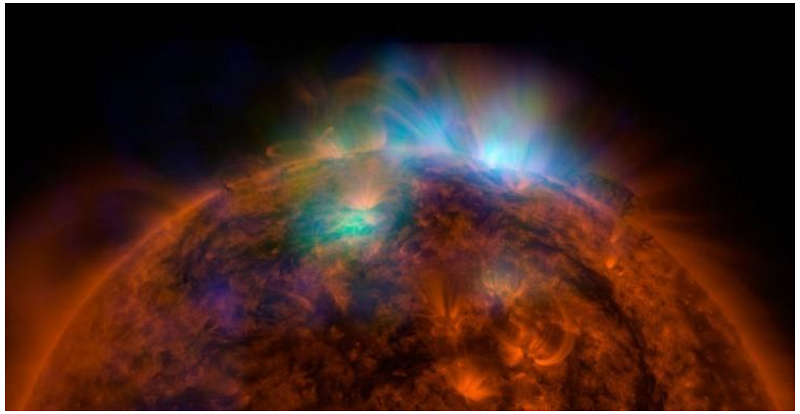
PONDEROMOTIVE FORCE

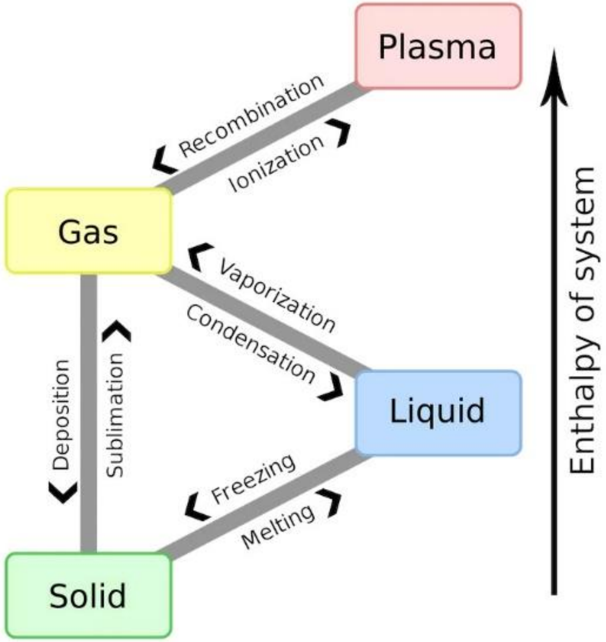
Acts as a pressure term!

$$\frac{1}{c} d_t \vec{u}_s = -\frac{\partial \mathcal{H}}{\partial \vec{x}} = -\left(\vec{E} + \vec{\beta} \times \vec{B}\right)\Big|_{s;norm} - \frac{1}{2\gamma_s}\vec{\nabla}\langle a^2 \rangle$$

Full dynamics for the cycle averaged (secular or low-frequency) trajectories

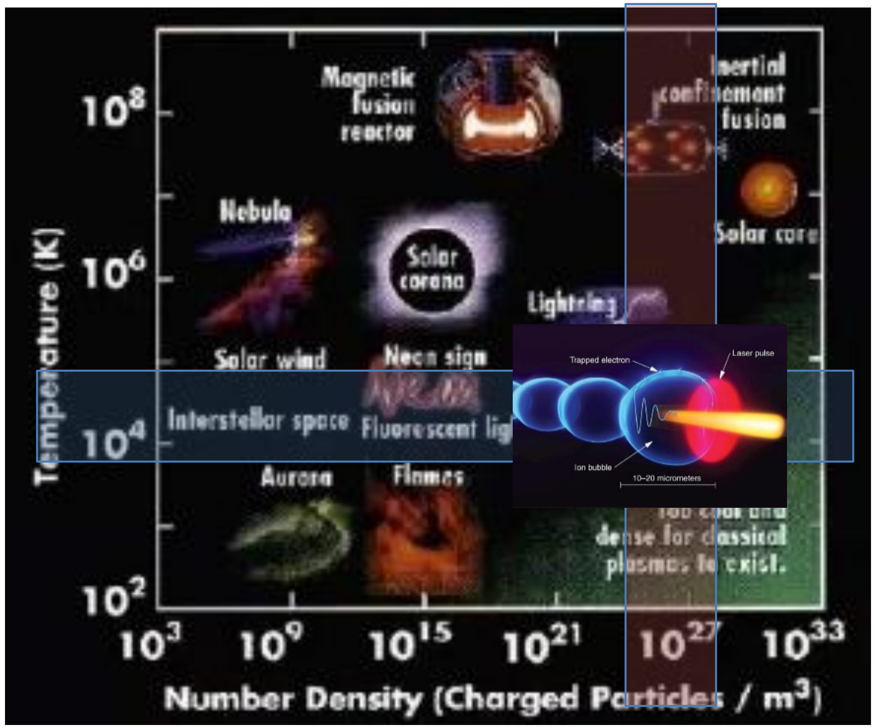
The cold, unmagnetized plasma side





«**COLD**» , Kin. En. In the range 1-10eV

$E_C = 3/2\ K_B\ T$ $T = 2/3\ E_C/K_B = 10^4 - 10^5 K$



Rigorous definition of plasma:

plasma is an ionized gas…

globally neutral…

*that displays **collective** phenomena.*

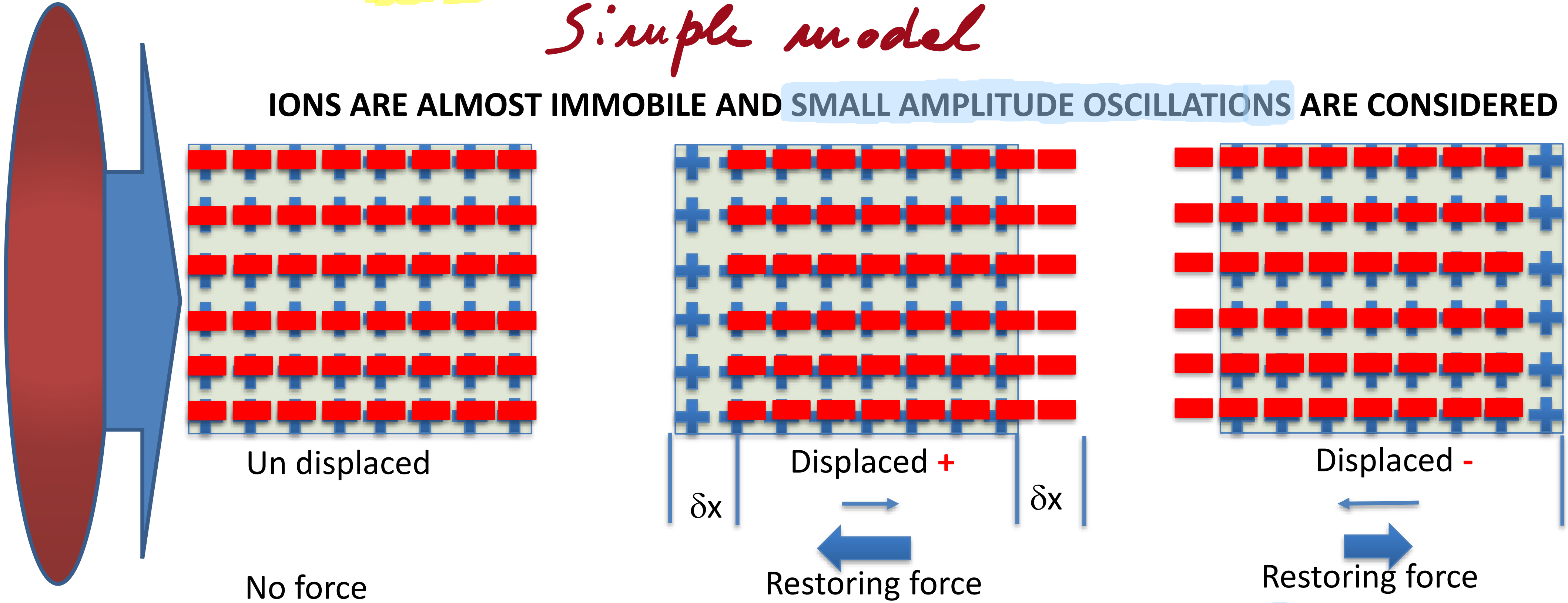
«**DENSE**» (it's confusing….it's called «**UNDERDENSE**» because light must propagate into it)

$n_e = 10^{22} - 10^{26}$ electrons/m^3 i.e. $10^{16} - 10^{20}$ electrons/cm^3

How does plasma responds to any attempt to displace positive (**heavy)** ions from negative (**light**) electrons?

Simple model

IONS ARE ALMOST IMMOBILE AND SMALL AMPLITUDE OSCILLATIONS ARE CONSIDERED



$$E = \frac{\sigma}{\epsilon_0} = \frac{e n_0 \delta x}{\epsilon_0} \rightarrow m_e \frac{d^2 \delta x}{dt^2} = -eE = -\frac{e^2 n_0}{\epsilon_0}\delta x \rightarrow \frac{d^2 \delta x}{dt^2} + \omega_p^2 \delta x = 0$$

capacitor model

$$\omega_p = \sqrt{\frac{n_0 e^2}{m_e \epsilon_0}}$$

$$\omega_p \propto \sqrt{n_0}$$

Understanding the plasma wave in the linear (small amplitude perturbations) regime

For a deeper understanding:

FLUID equations

Fluid momentum conservation

$$n_e \partial_t \vec{v} + n_e \vec{v} \cdot \nabla \vec{v} = \frac{q n_e}{m} \left[\vec{E} + \vec{v} \times \vec{B} \right] - \frac{1}{m} \nabla P$$

NEEDS TO BE MODELED with a closure (equation of state) term.

$$n_e \frac{d\vec{v}}{dt} = \frac{q n_e}{m} \left[\vec{E} + \vec{v} \times \vec{B} \right] - \frac{1}{m} \nabla P$$

Pressure term

$$\frac{d\vec{v}}{dt} \equiv \partial_t \vec{v} + \vec{v} \cdot \nabla \vec{v}$$

Total (convective) derivative

ORDER 2 CLOSURE (perfect gas)

$$P = \mathcal{I} p \quad P_{ij} = p \delta_{ij}$$

$$p = n_e k_B T$$

$$k_B T = m v_{th}^2$$

$$p = n_e m v_{th}^2$$

Compare with the Ponderomotive Force

$$F_p \sim -\vec{\nabla} \langle a^2 \rangle \sim -\vec{\nabla} I$$

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Understanding the plasma wave in the linear (small amplitude perturbations) regime

We are now going to get the dispersion relations for the interesting plasma waves (longitudinal [electrostatic] and transverse [electromagnetic] electron plasma waves.

Wake field   Laser pulse

Small-amplitude (linear regime) perturbations

$$n_e = n_0 + \delta n \quad \vec{v} = \delta\vec{v} \quad p = p_0 + \delta p \quad \vec{E} = \delta\vec{E} \quad \vec{B} = \delta\vec{B}$$

= first order perturbations

$$n_e \partial_t \delta\vec{v} + n_e \delta\vec{v} \cdot \nabla\delta\vec{v} = \frac{q n_e}{m}\left[\delta\vec{E} + \delta\vec{v} \times \delta\vec{B}\right] - \frac{1}{m}\nabla\delta P$$

= second order

$$\nabla p_0 = 0$$

$$\partial_t \delta\vec{v} = \frac{q}{m}\delta\vec{E} - \frac{1}{m n_0}\nabla\delta P$$

CLOSURE (perfect gas)

$$pV^\gamma = const. \rightarrow \delta p/p_0 = -\gamma\delta V/V = \gamma\delta n/n_0$$

$\gamma = 3$ **1D perturbation**

ELI-NP Autumn School

Understanding the plasma wave in the linear (small amplitude perturbations) regime

Small-amplitude (linear regime) perturbations
MAXWELL EQUATIONS

Exact

$$\nabla \cdot \vec{E} = q(n_e - n_0)/\epsilon_0 \quad \nabla \cdot \vec{B} = 0 \quad \nabla \times \vec{E} = -\partial_t \vec{B} \quad \nabla \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \partial_t \vec{E} \right)$$

1st order

$$\nabla \cdot \delta\vec{E} = q\delta n/\epsilon_0 \quad \nabla \cdot \delta\vec{B} = 0 \quad \nabla \times \delta\vec{E} = -\partial_t \delta\vec{B} \quad \nabla \times \delta\vec{B} = \mu_0 \left(q n_0 \delta\vec{v} + \epsilon_0 \partial_t \delta\vec{E} \right)$$

+

$$\partial_t \delta n + n_0 \nabla \cdot \delta\vec{v} = 0 \quad \partial_t \delta\vec{v} = \frac{q}{m}\delta\vec{E} - \frac{1}{m n_0}\nabla \delta P \quad \delta p/p_0 = \gamma \delta n/n_0$$

$$(p_0 = n_0 m v_{th}^2)$$

Complete set of coupled linear terms

FOURIER DECOMPOSITION

$$f(\vec{x},t) \simeq f(\vec{k},\omega)e^{i(\vec{k}\cdot\vec{x}-\omega t)} \qquad \nabla \to i\vec{k} \quad \partial_t \to -i\omega$$

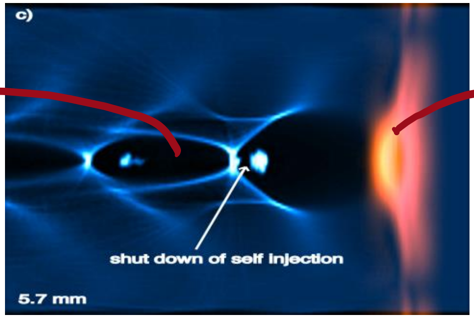
Wave propagation

$$\nabla^2 \delta\vec{E} - \frac{1}{c^2}\partial_t^2 \delta\vec{E} = \vec{S}$$

Source term due to the plasma response

$$\vec{S} = \frac{\omega_p^2}{c^2}\delta\vec{E} - \left(1 - \gamma\frac{v_{th}^2}{c^2}\right)\vec{k}(\vec{k}\cdot\delta\vec{E})$$

Understanding the plasma wave in the linear (small amplitude perturbations) regime



LONGITUDINAL WAVES (wakefield)

$\delta B \sim 0$ Electrostatic

$$\delta\vec{v} \parallel \delta\vec{E} \qquad \delta\vec{k} \parallel \delta\vec{E}$$

$$\vec{S} = \frac{\omega_p^2}{c^2}\delta\vec{E} - \left(1 - \gamma\frac{v_{th}^2}{c^2}\right)k^2\delta\vec{E}$$

$$\left(-k^2 + \frac{\omega^2}{c^2}\right)\delta\vec{E} = \frac{\omega_p^2}{c^2}\delta\vec{E} - \left(1 - \gamma\frac{v_{th}^2}{c^2}\right)k^2\delta\vec{E}$$

$$\left(\omega^2 - \omega_p^2 - \gamma v_{th}^2 k^2\right)\delta\vec{E} = 0$$

Has nontrivial solution $\delta E \neq 0$ **if**

$$\omega^2 = \omega_p^2 + \gamma v_{th}^2 k^2$$

Relation dispersion for the longitudinal plasma wave

TRANSVERSE WAVES (laser)

$\delta B \neq 0$ Electromagnetic

$$\delta\vec{v} \parallel \delta\vec{E} \qquad \delta\vec{k} \perp \delta\vec{E}$$

$$\vec{S} = \frac{\omega_p^2}{c^2}\delta\vec{E}$$

$$\left(-k^2 + \frac{\omega^2}{c^2}\right)\delta\vec{E} = \frac{\omega_p^2}{c^2}\delta\vec{E}$$

$$\left(\omega^2 - \omega_p^2 - k^2c^2\right)\delta\vec{E} = 0$$

Has nontrivial solution $\delta E \neq 0$ **if**

$$\omega^2 = \omega_p^2 + k^2c^2$$

Relation dispersion for the transverse plasma wave

Remind from EM courses

Phase velocity and group velocity

Phase velocity: wavefront velocity for each monochromatic component

$$f(z - v_\phi t) = f_0 \exp\left[i(kz - \omega t)\right] = f_0 \exp\left[ik\left(z - \frac{\omega}{k} t\right)\right]$$

$$v_\phi = \frac{\omega}{k}$$

How fast the phase fronts move

Group velocity: Envelope center-of-mass-velocity

$$v_g = \frac{d\omega}{dk}$$

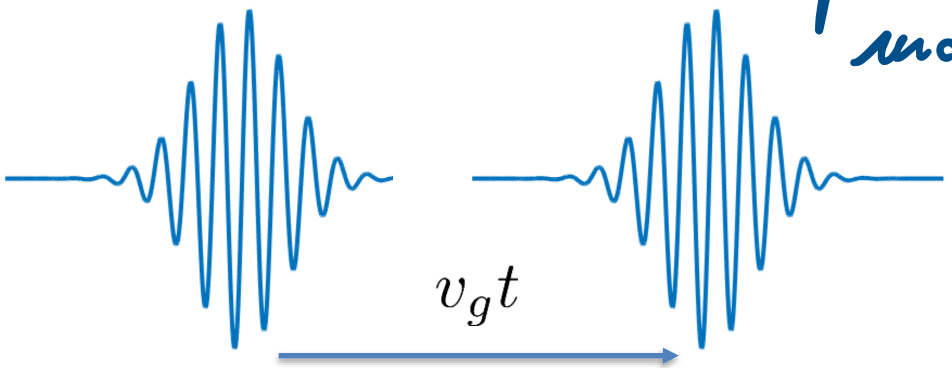
How fast the envelope (packet) moves

Wave packet $f(z,t)$

$$f(z,t) = \int \frac{dk}{2\pi}\frac{d\omega}{2\pi} f(k,\omega) e^{i(kz-\omega t)}$$

$\omega = \omega(k)$ Dispersion Relation

$$f(z,t) \to \int dk f(k,\omega(k)) e^{i(kz - \omega(k) t)}$$



If f(k) is ***almost monochromatic***

$$\omega(k) \simeq \omega_0 + \frac{d\omega(k)}{dk}(k - k_0)$$

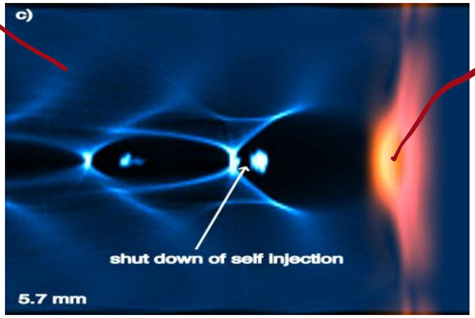
$$f(z,t) \to \int dk f(k,\omega(k)) e^{i(kz-\omega(k)t)} \simeq e^{i(k_0 z - \omega_0(k) t)} \int dk f(k,\omega(k)) e^{i(k-k_0)\left(z - \frac{d\omega}{dk} t\right)}$$

Carrier speed $v_\phi = \frac{\omega}{k}$

Envelope speed $v_g = \frac{d\omega}{dk}$

Understanding the plasma wave in the linear (small amplitude perturbations) regime

Small-amplitude PLASMA WAVES



LONGITUDINAL WAVES (wakefield)

$$\omega^2 = \omega_p^2 + \gamma v_{th}^2 k^2$$

$$v_\phi = \frac{\omega}{k} \simeq \frac{\omega_p}{k} + \frac{3}{2} v_{th}^2 \frac{k}{\omega_p} \approx \frac{\omega_p}{k}$$

Very small

$$v_g = \frac{d\omega}{dk} \simeq 3 v_{th}^2 \frac{k}{2\omega_p} = \frac{3 v_{th}^2}{2 v_\phi} \sim 0$$

The phase velocity of the plasma wave **is the same of the source** moving into the plasma.

The group velocity is usually neglibible
(longitudinal waves don't contribute to energy transfer)

TRANSVERSE EM WAVES

$$\omega^2 = \omega_p^2 + c^2 k^2$$

$$v_\phi = \frac{\omega}{k} = \frac{c}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} > c$$

$$v_g \simeq \frac{d\omega}{dk} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}} < c$$

The phase velocity of the transverse EM wave is superluminal

The group velocity <u>can be very close to c</u>, provided that

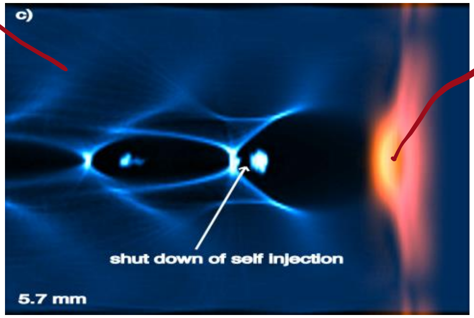
$$\omega_p / \omega \ll 1$$

<u>The higher is the frequency, the faster is the wave.</u>

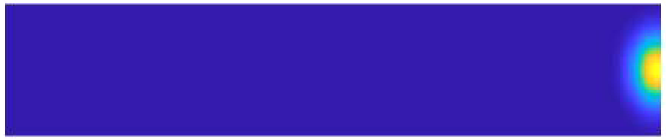
Understanding the plasma wave in the linear (small amplitude perturbations) regime

Small-amplitude PLASMA WAVES

LONGITUDINAL WAVES (wakefield)



TRANSVERSE EM WAVES



$$v_\phi = \frac{\omega}{k} \simeq \frac{\omega_p}{k} + \frac{3}{2} v_{th}^2 \frac{k}{\omega_p} \approx \frac{\omega_p}{k} = v_{DRIVER}$$

$$v_g \simeq 0$$

$$v_\phi = \frac{\omega}{k} = \frac{c}{\sqrt{1 - \frac{\omega_p^2}{\omega^2}}} > c$$

$$v_g = \frac{d\omega}{dk} \simeq c\sqrt{1 - \frac{\omega_p^2}{\omega^2}} < c$$

Understanding the plasma wave in the linear (small amplitude perturbations) regime

What we learned about waves in cold, unmagnetized plasmas

Remember:

$$v_g \simeq \frac{d\omega}{dk} = c\sqrt{1 - \frac{\omega_p^2}{\omega^2}}$$

Transverse plasma waves (Laser pulses for Laser WakeField Accelerarion) can propagate in plasmas provided that .

$$\beta_g > 0 \rightarrow \omega_p/\omega_0 < 1$$

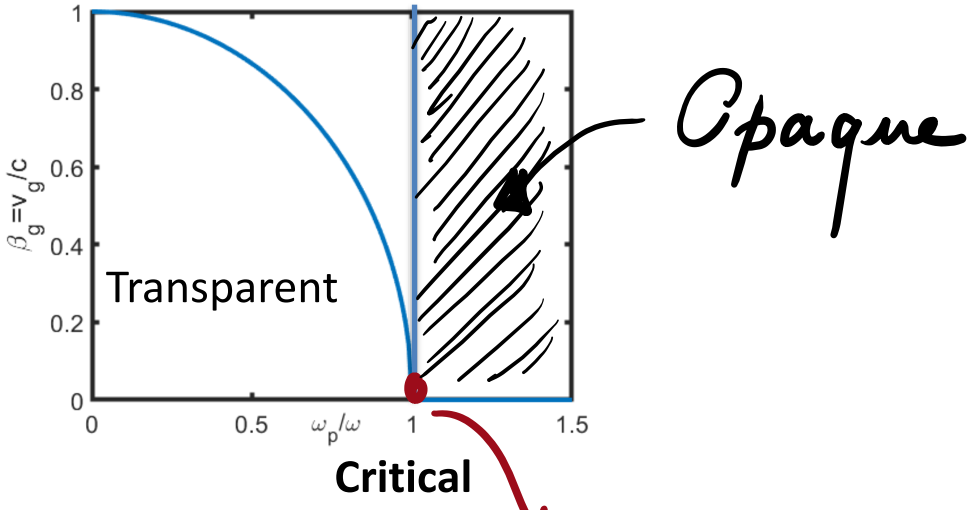
The **critical density** of a plasma depends on the pulsation of the EM pulse propagating into it and **sets the boundary between transparent and opaque plasmas**

$$v_g = 0 \rightarrow \omega_p^2 = \omega_0^2 \rightarrow \frac{n_0 e^2}{\epsilon_0 m} = \omega_0^2$$

$$n_c[cm^{-3}] = \frac{\epsilon_0 m \omega_0^2}{e^2} \simeq 1.1 \cdot 10^{21}/\lambda_0[\mu m]^2$$

$$n_c = \frac{\epsilon_0 m \omega_0^2}{e^2} \qquad n_c = \frac{m\omega_0^2}{4\pi e^2}$$

MKS CGS



$n_e = n_c$

$\omega_0 = \omega_p$

Understanding the plasma wave in the linear (small amplitude perturbations) regime

What we learned about waves in cold, unmagnetized plasmas

REMIND

NUMBERS

$$n_0 = 10^{16} cm^{-3} = 10^{22} m^{-3}$$
$$\omega_p = 5.6 \times 10^{12} rad/s$$

Infrared

$$n_0 = 10^{18} cm^{-3} = 10^{24} m^{-3}$$
$$\omega_p = 5.6 \times 10^{13} rad/s$$

$$n_c[cm^{-3}] = \frac{\epsilon_0 m \omega_0^2}{e^2} \simeq 1.1 \cdot 10^{21}/\lambda_0[\mu m]^2$$

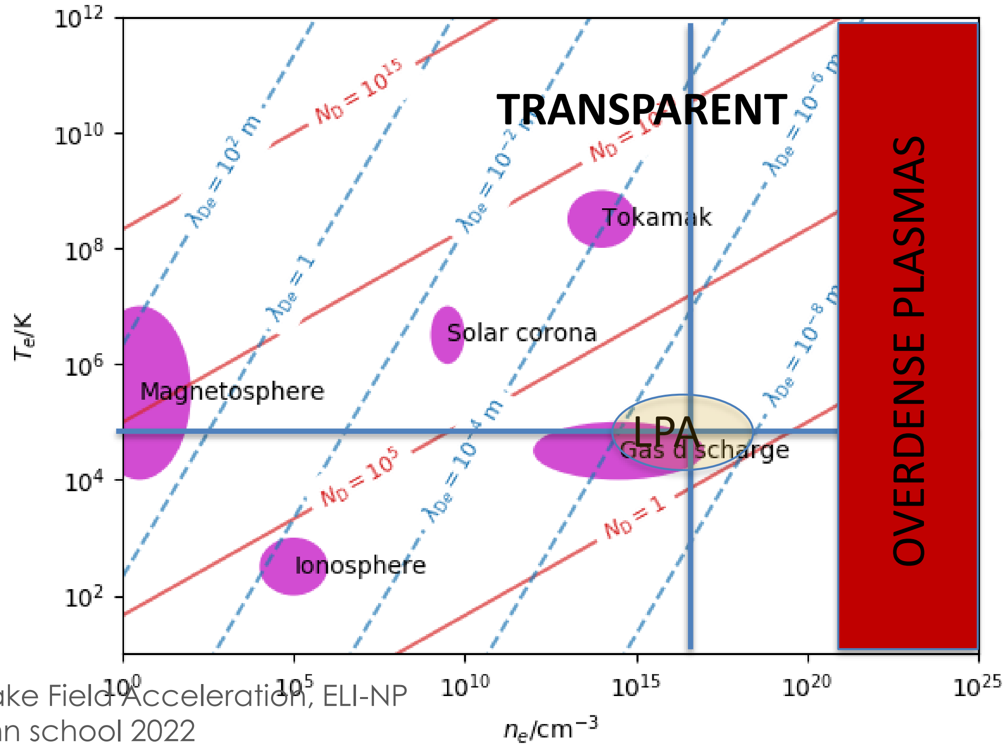
Generally used carrier wavelength

$$\lambda_0 \simeq 0.8 \mu m$$
$$\omega_0 = 2\pi c/\lambda_0$$
$$\omega_0 = 2.4 \times 10^{15} rad/s$$

$$n_c[cm^{-3}] \simeq 1.6 \cdot 10^{21}$$

$$10^{-4} < n_0/n_c < 10^{-2}$$

Our plasmas are well underdense!



Understanding the plasma wave in the linear (small amplitude perturbations) regime

What we learned about waves in cold, unmagnetized plasmas

Therefore in our range of densities

$$\left(\frac{\omega_p}{\omega_0}\right)^2 = \frac{n_0}{n_c} \approx 10^{-4} - 10^{-2}$$

$$v_g = \frac{d\omega}{dk} \simeq c\sqrt{1-\frac{\omega_p^2}{\omega^2}} < c$$

How close are we to c?

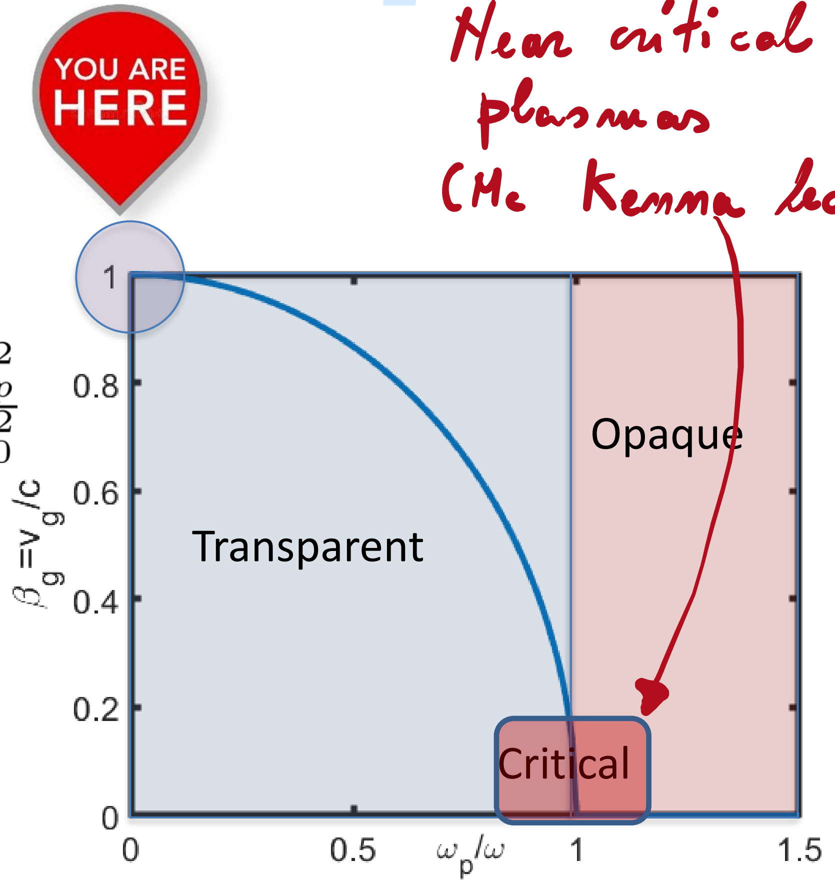
$$\beta_g = v_g/c = \frac{d\omega_0}{dk} \simeq \sqrt{1-\frac{\omega_p^2}{\omega^2}} \simeq 1 - \frac{1}{2}\frac{\omega_p^2}{\omega_0^2}$$

We can define the Lorentz γ of the pulse as

$$\gamma_g = \frac{1}{\sqrt{1-\beta_g^2}}$$

$$\gamma_g = \frac{\omega_0}{\omega_p} = \sqrt{\frac{n_c}{n_0}}$$

Very important wakefield Parameter



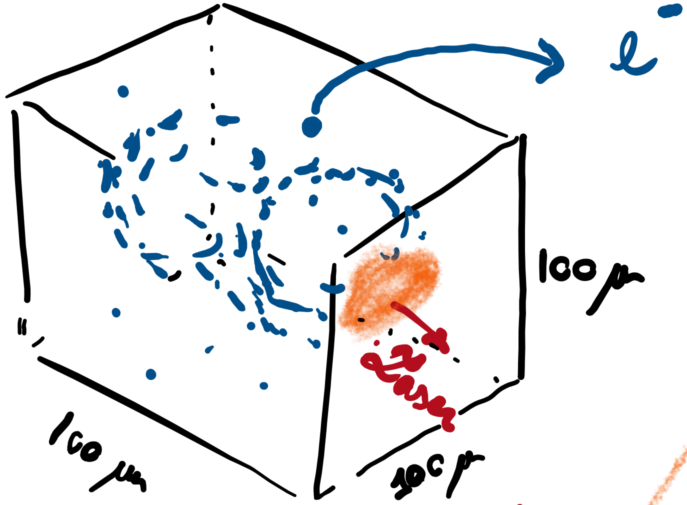
Near critical plasmas (Mc Kenna lecture)

In our standard configuration $10 < \gamma_g < 100$ **(not so large!!! We will come back on this later on)**

Approximations map

It's impossible to fully model the dynamics of the plasma electrons in the dynamic volume of the interaction

Laser PLASMA



BOLTZMANN EQUATION (KINETIC DESCRIPTION)

We should model the interaction of

$N \approx 10^{18} \frac{e^-}{cm^3} \cdot (100 \cdot 10^{-4})^3\, cm^3$

! $N \approx 10^{12}\, e^-$!

We describe the DISTRIBUTION of the particles in the POSITION and Momentum space

Phase-Space

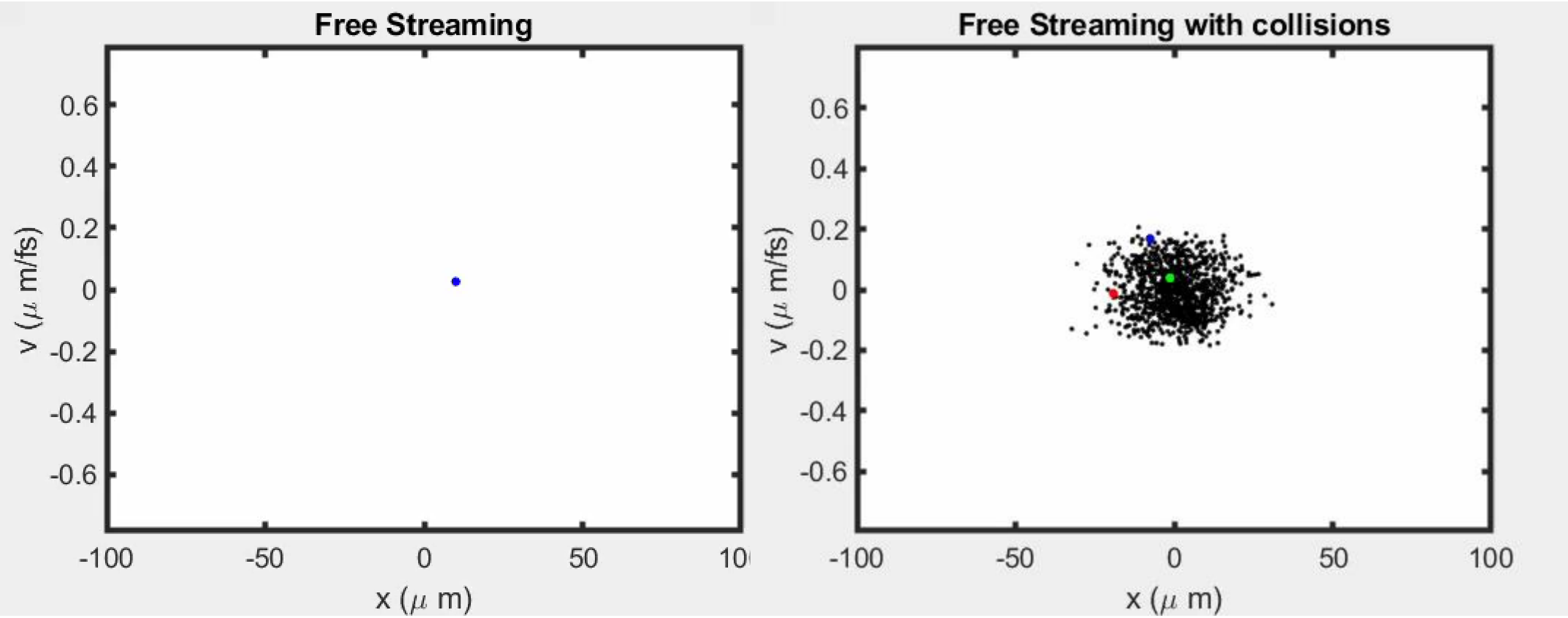
Approximations map

The Vlasov / Boltzman equations

Binary interactions

$$\frac{\partial}{\partial t}f + v\frac{\partial}{\partial q}f + F\frac{\partial}{\partial p}f = 0$$

$$\frac{\partial}{\partial t}f + v\frac{\partial}{\partial q}f + F\frac{\partial}{\partial p}f = \partial_t f|_{collisions}$$

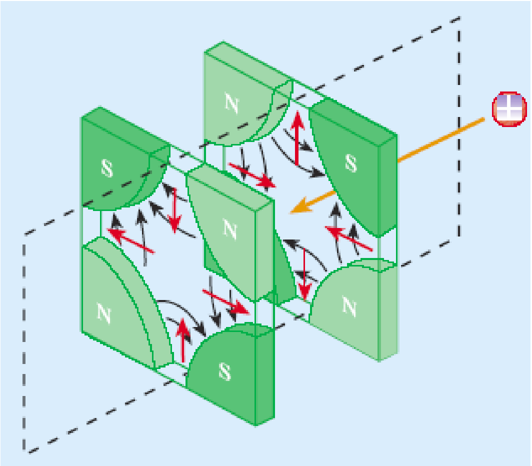


Vlasov **Boltzmann**

Approximations map

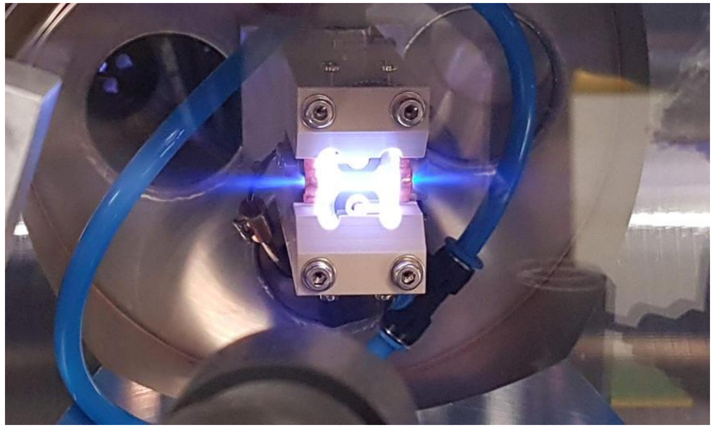
EXAMPLE Linear restoring force **(F=-kx)**

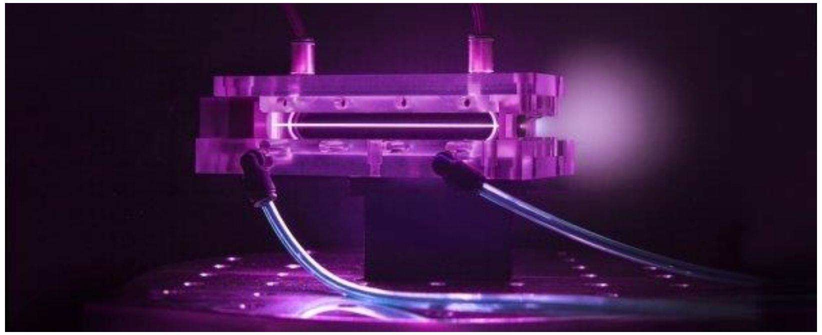
Standard magnetic quadrupoles



meter(s) scale

Plasma lenses





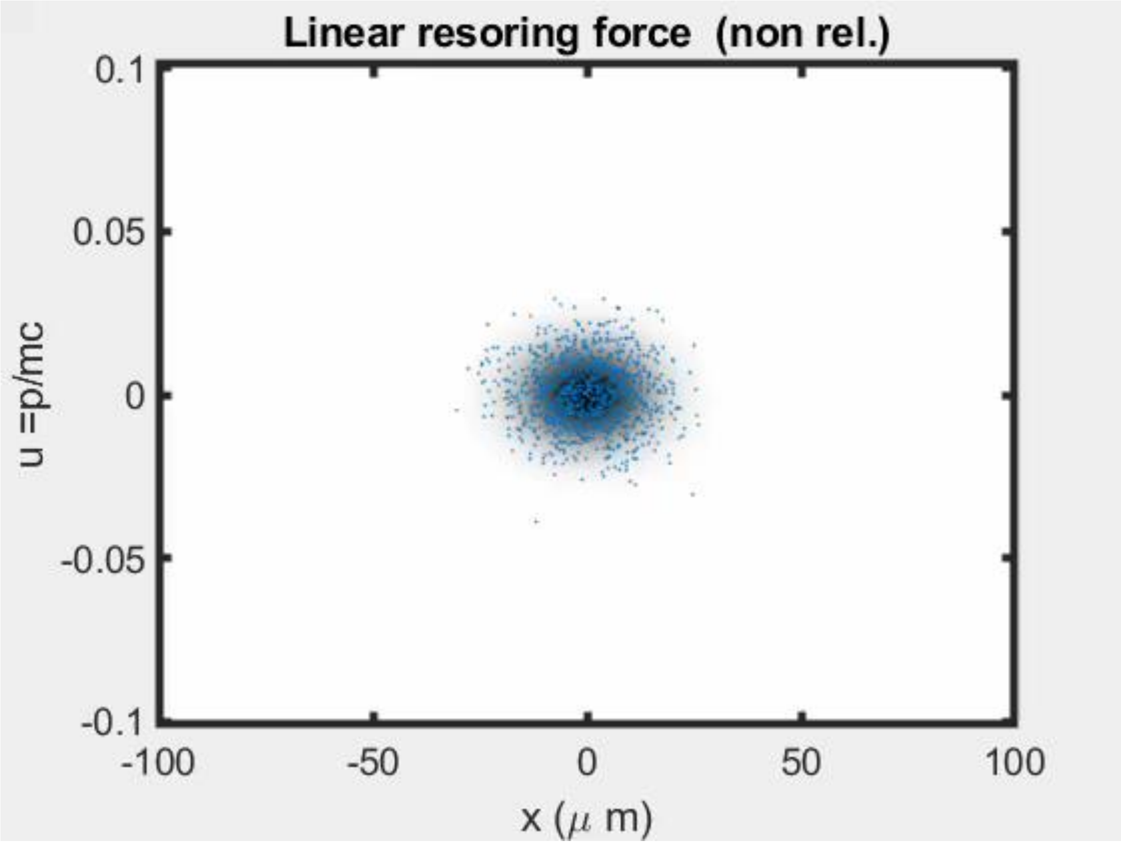
cm(s) scale

Approximations map

Solving the Vlasov equation with the «method of the characteristics»

EXAMPLE Linear restoring force **(F=-kx)**

$$f(x,p,t) = f\left(x\cos(\omega t) - \frac{v}{\omega}\sin(\omega t), v\cos(\omega t) + \omega x \sin(\omega t), 0\right)$$

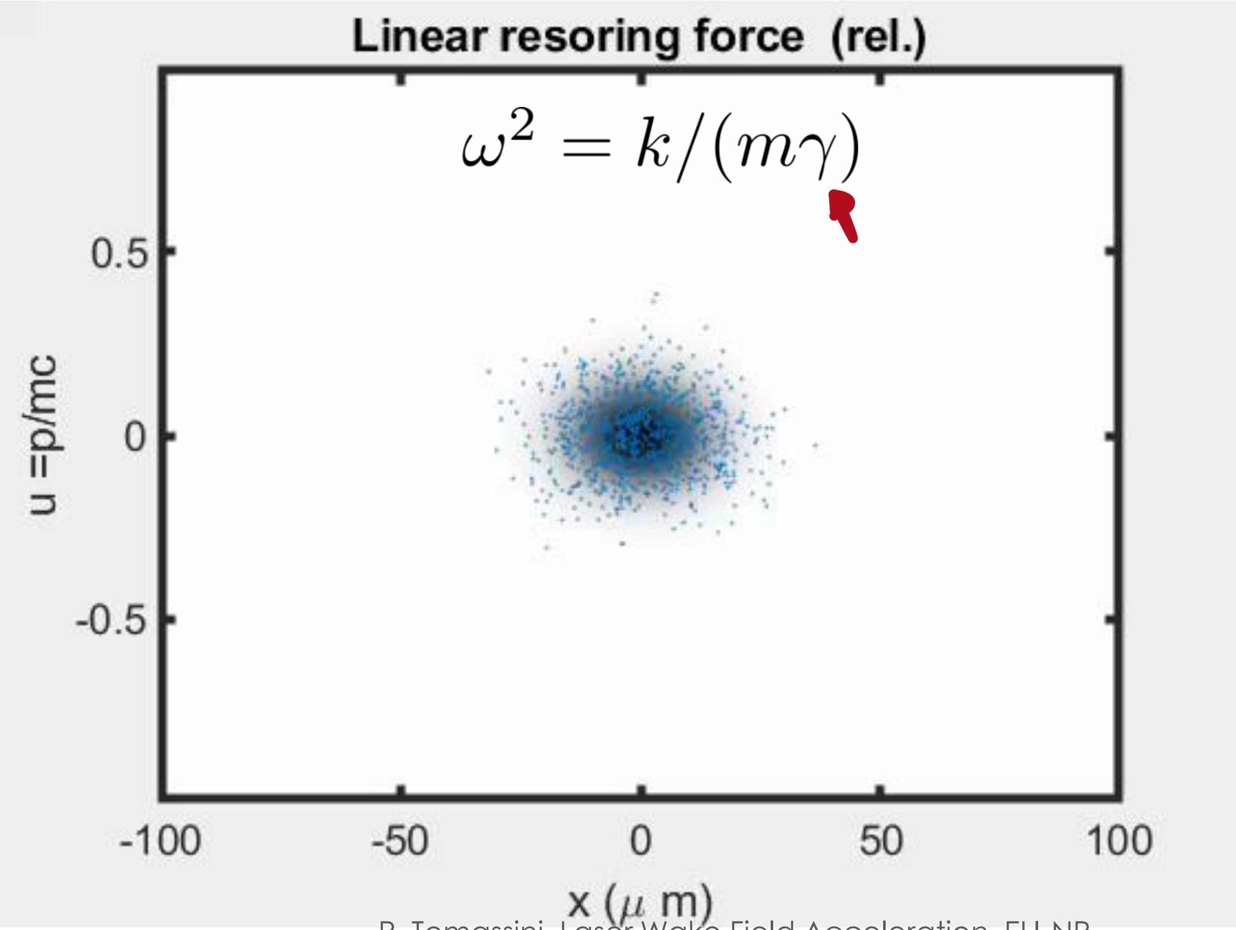


Approximations map

Solving the Vlasov equation with the «method of the characteristics»

EXAMPLE Linear restoring force **(F=-kx)**

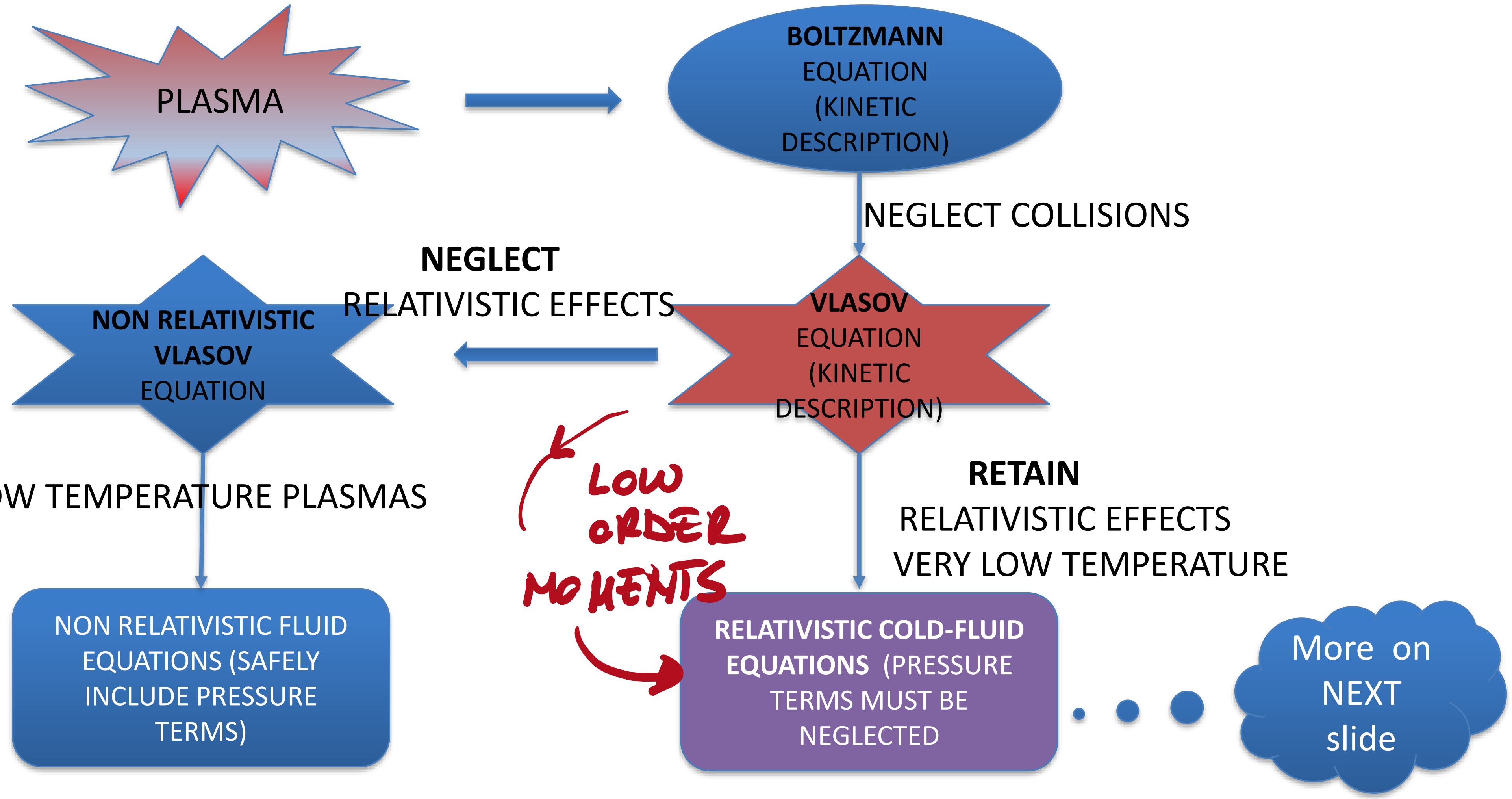
RELATIVISTIC EFFECTS ON



This phenomenon can happen in LWFA and it can degrade beam quality

Final Approximations map (1/2)

We start from a PLASMA STATE (we already know what this means)



Final Approximations map (2/2)

PLASMA

NEGLECT COLLISIONS →

VLASOV EQUATION (KINETIC DESCRIPTION)

RETAINS RELATIVISTIC EFFECTS

VERY LOW TEMPERATURE ↓

RELATIVISTIC COLD-FLUID EQUATIONS (PRESSURE TERMS MUST BE NEGLECTED

← RETAIN ALL THE SCALES

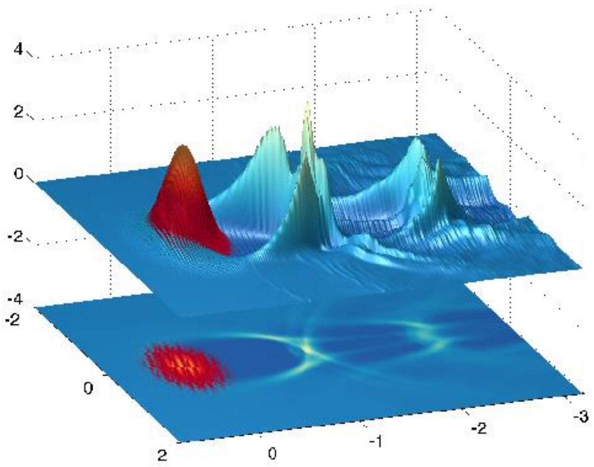
IRROTATIONAL FLUID EQUATIONS

RETAIN ONLY THE LOW-FREQUENCY SCALES.

LWFA PLASMAS ARE IRROTATIONAL

↓

IRROTATIONAL FLUID EQUATION **for the low-frequency fields**



The EM field side

The EM field eqq. **for the normalized (** $\times e/mc^2$**) potentials** can be written as (see e.g Jackson)

$$\nabla^2\varphi + \frac{1}{c}\partial_t\vec{\nabla}\cdot\vec{a} = k_p^2\left(\frac{n}{n_0} - 1\right)$$

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\vec{a} - \vec{\nabla}\left(\vec{\nabla}\cdot\vec{a} + \frac{1}{c}\partial_t\varphi\right) = k_p^2\frac{n}{n_0}\vec{\beta}$$

n_0 : background density

$k_p = \sqrt{\frac{4\pi n_0 e^2}{mc^2}}$: plasma wavenumber

$\lambda_p = \frac{2\pi}{k_p}$: plasma wavelength

IF WE CHOOSE THE **COULOMB GAUGE** $\vec{\nabla}\cdot\vec{a} = 0$ we get

$$\nabla^2\varphi = k_p^2\left(\frac{n}{n_0} - 1\right)$$

$$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\vec{a} = \frac{1}{c}\partial_t\vec{\nabla}\varphi + k_p^2\frac{n}{n_0}\vec{\beta}$$

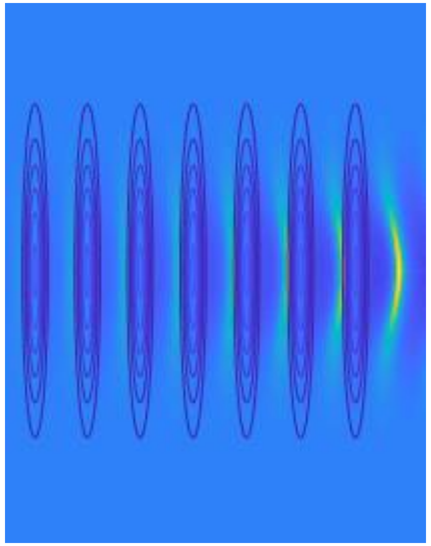
POISSON Eq.
NO time derivative

OUTLOOK

1. Introduction to LWFA
2. Understanding the excitation and the structure of the plasma waves
3. ***The wide spot (1D and QSA) limiting case***
4. Limiting factors to high energy gain accelerators
5. 3D effects on
6. Downramp (or shock) injection
7. Two-Color injection
8. The Resonant Multi-Pulse Ionisation Injection
9. High-Brilliance e-bunches

LWFA in the nonlinear 1D regime

In the 1D limit the radial motion of the plasma electrons can be neglected. ***<u>This happens more frequently than you are expecting now!</u>***
If the radial spot of the laser pulse is «decently» large, the 1D nonlinear regime approximation reveals to be extreme powerful.



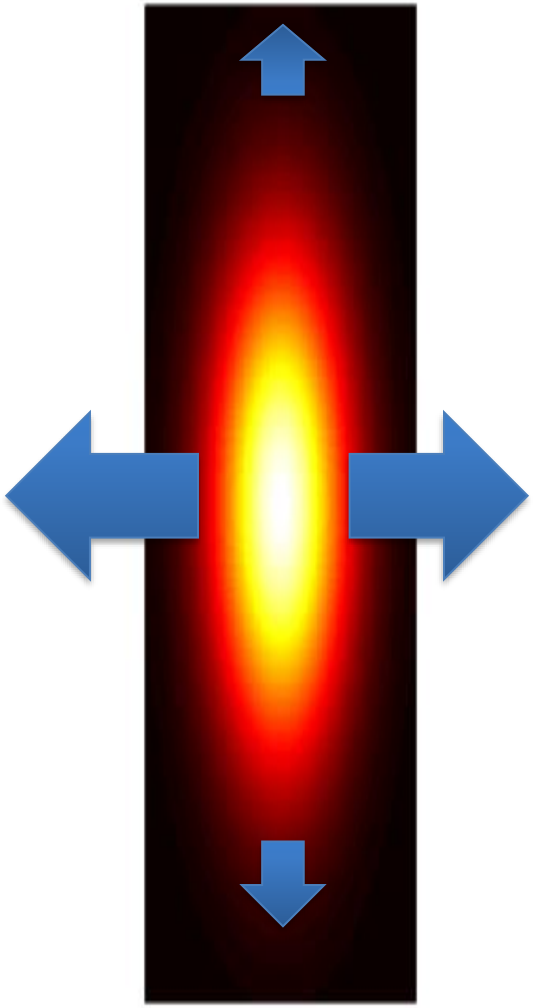
This regime holds when the **radial** ponderomotive force is **negligibile** with respect to the **longitudinal** one, i.e when cT<<w and also when the transverse size of the pulse w is much higher than the plasma wavelength λp .

$$k_p w >> 1 \quad (k_p = 2\pi/\lambda_p)$$

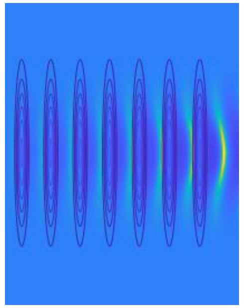
Transverse gradients are negligible

$$\nabla_\perp \ll \nabla_\parallel$$

$$\nabla_\perp \ll \frac{1}{c}\partial_t$$

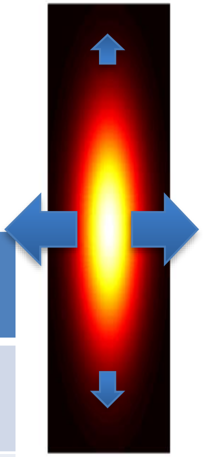


LWFA in the nonlinear 1D regime



$$k_p w >> 1 \;\; (k_p = 2\pi/\lambda_p) \quad \nabla_\perp \ll \nabla_{||} \quad \nabla_\perp \ll \frac{1}{c}\partial_t$$

Transverse gradients are negligible



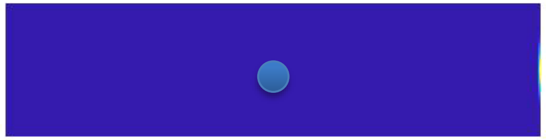
	3D	1D
Continuity equation	$\frac{1}{c}\partial_t + \vec{\nabla}\cdot(n\beta) = 0$	$\frac{1}{c}\partial_t + \partial_z(n\beta_z) = 0$
Fuid eq. for plasma momentum	$\frac{1}{c}\frac{\partial}{\partial t}(\vec{u} - \vec{a}) = \vec{\nabla}(\varphi - \gamma)$	$\frac{1}{c}\frac{\partial}{\partial t}(\vec{u}_\perp - \vec{a}_\perp) = 0$ $\frac{1}{c}\frac{\partial}{\partial t}(u_z - a_z) = \partial_z(\varphi - \gamma)$
Poisson eq. for ϕ	$\nabla^2\varphi = k_p^2\left(\frac{n}{n_0} - 1\right)$	$\partial_z^2\varphi = k_p^2\left(\frac{n}{n_0} - 1\right)$
Wave eq. for a	$\left(\nabla^2 - \frac{1}{c^2}\partial_t^2\right)\vec{a} = \frac{1}{c}\partial_t\vec{\nabla}\varphi + k_p^2\frac{n}{n_0}\vec{\beta}$	$\left(\partial_z^2 - \frac{1}{c^2}\partial_t^2\right)\vec{a}_\perp = k_p^2\frac{n}{n_0}\vec{\beta}_\perp$ $\left(\partial_z^2 - \frac{1}{c^2}\partial_t^2\right)a_z = \frac{1}{c}\partial_t\partial_z\varphi + k_p^2\frac{n}{n_0}\beta_z$
Gauge	$\vec{\nabla}\cdot\vec{a} = 0$	$\partial_z a_z = 0$

Useful descriptions/approximations
The Quasistatic regime

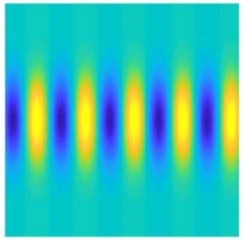
*If you are watching the system by a window that moves vith a certain speed v and you see **almost nothing** happens…well you can safely employ the quasistatic approximation!*

(P. Sprangle et al., 1990)

FOCUS



STATIC WINDOW



MOVING WINDOW

Formal description of QSA:

1) Make the change in variables from the standard (z,ct) to the new (ξ, z_g)

$$\xi = z - z_g(t)$$
$$z_g(t) = -c\int^t d\tau \beta_g(\tau)$$

$$\frac{\partial \xi}{\partial ct} = \beta_g \qquad \frac{\partial \xi}{\partial z} = 1$$
$$\frac{\partial z_g}{\partial ct} = -\beta_g \qquad \frac{\partial z_g}{\partial z} = 0$$

Jacobian

2) Observe that

$$\partial_z f = \partial_\xi f \times \frac{\partial \xi}{\partial z} + \partial_{z_g} f \times \frac{\partial z_g}{\partial z} \longrightarrow \partial_z f = \partial_\xi f$$
$$\frac{1}{c}\partial_t f = \partial_\xi f \times \frac{\partial \xi}{\partial ct} + \partial_{z_g} f \times \frac{\partial z_g}{\partial ct} \longrightarrow \frac{1}{c}\partial_t f = \beta_g\left(\partial_\xi f - \partial_{z_g} f\right)$$

EXACT!

3) Finally, if the case, neglect the zg derivative to employ the QSA

$$\partial_z f = \partial_\xi f \qquad \frac{1}{c}\partial_t f \simeq \beta_g \partial_\xi f$$

EXACT! APPROXIMATE

QSA !

LWFA in the nonlinear 1D+QSA regime

$$\xi = z + \beta_g ct$$

FULL	1D	1D+QSA
$\vec{\nabla}\cdot(n\vec{u}/\gamma) + \frac{1}{c}\partial_t n = 0$	$\partial_z(nu_z/\gamma) + \frac{1}{c}\partial_t n = 0$	$\partial_\xi(n\beta_z + \beta_g n) = 0$
$\frac{\partial}{c\partial t}(\vec{u}-\vec{a}) = \vec{\nabla}(\phi-\gamma)$	$\frac{\partial}{c\partial t}(u_z - a_z) = \partial_z(\phi-\gamma)$	$\partial_\xi(\gamma - \phi + \beta_g u_z) = 0$
$\nabla^2\phi = k_p^2(n/n_0 - 1)$	$\partial_z^2\phi = k_p^2(n/n_0 - 1)$	$\partial_\xi^2\phi = -k_p^2\frac{\beta_z}{\beta_g+\beta_z}$

$$\gamma_\perp^2 = 1 + < a^2 >$$

$$n = n_0\frac{\beta_g}{\beta_g+\beta_z}$$

$$\gamma + \beta_g u_z = 1 + \phi = \Psi$$

$$\Psi \equiv \phi + 1; \quad \partial_\xi^2\Psi = k_p^2\gamma_g^2\left\{\frac{\beta_g}{\sqrt{1-\frac{\gamma_\perp^2}{\gamma_g^2\Psi^2}}} - 1\right\}$$

$$E_0 = mc^2k_p/e \simeq 96\sqrt{n_0(cm^{-3})}\,V/m$$

«nonrelativistic» wave-breaking limit

$$E_z/E_0 = -\frac{1}{k_p}\partial_\xi\Psi$$

In the 1D+QSA limit everything is determined upon the pseudopotential ψ is found by solving the nonlinear ODE

LWFA in the nonlinear 1D regime

BEFORE TO CONTINUE

$$E_0 = mc^2 k_p / e \simeq 96 \sqrt{n_0 (cm^{-3})}\, V/m$$

«nonrelativistic» wave-breaking limit

$$E_z / E_0 = -\frac{1}{k_p} \partial_\xi \Psi$$

Suppose (**as usual**) that we are able to excite waves with $E_z \approx E_0$ in a Plasma with density, say, of $10^{18} cm^{-3}$. In this case the accelerating gradient is:

$$E_z \approx 100 \sqrt{10^{18}} \approx 0.1\, TV/m$$

This means that if we had the possibility to accelerate an electron for a distance of **10m** (and this is at the moment far to be true…) we had build a (many tables-top) **1TeV Accelerator**.

The effort of the scientific community is now on both the **beam-quality side** and the «**long acceleration**» side

LWFA in the nonlinear 1D regime

In a **tenuous plasma** $n_e \ll n_c \rightarrow \beta_{ph} \rightarrow 1$

$$\psi = 1 + \phi$$

$$\partial_\xi^2 \psi = k_p^2 \gamma_p^2 \left\{ \frac{\beta_{ph}}{\sqrt{1 - \frac{\gamma_\perp^2}{\gamma_{ph}^2 \psi^2}}} - 1 \right\}$$

$$E_z / E_0 = -\frac{1}{k_p} \partial_\xi \psi$$

$$\gamma_\perp^2 = 1 + \bar{u_\perp^2} = 1 + \bar{a^2}$$

$$2k_p^{-2} \partial_\xi^2 \psi \simeq \frac{1 + \bar{a^2}}{\psi} - 1$$

$$n/n_0 \simeq \frac{\gamma_\perp^2 + \psi^2}{2\psi^2}$$

$$u_z \simeq \frac{\gamma_\perp^2 - \psi^2}{2\psi}$$

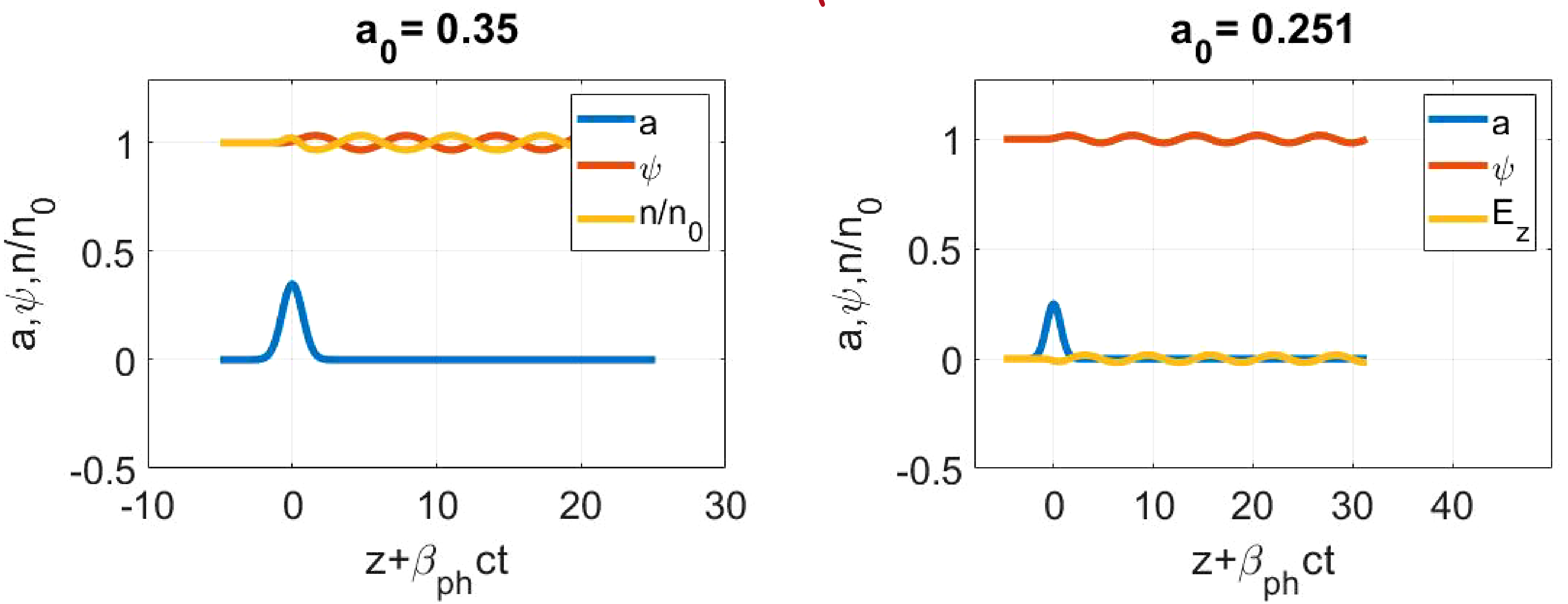
$$\gamma \simeq \frac{\gamma_\perp^2 + \psi^2}{2\psi}$$

PLASMA FIELDS **TEND TO DIVERGE** WHEN $\psi \rightarrow 0$

LWFA in the nonlinear 1D regime

$$E_z/E_0 = -\frac{1}{k_p}\partial_\xi \psi \qquad \partial_\xi^2 \psi = k_p^2 \gamma_p^2 \left\{ \frac{\beta_{ph}}{\sqrt{1-\frac{\gamma_\perp^2}{\gamma_{ph}^2 \psi^2}}} - 1 \right\} \qquad \gamma_\perp^2 = 1 + \bar{u_\perp^2} = 1 + \bar{a^2}$$

What happens if a0 increases



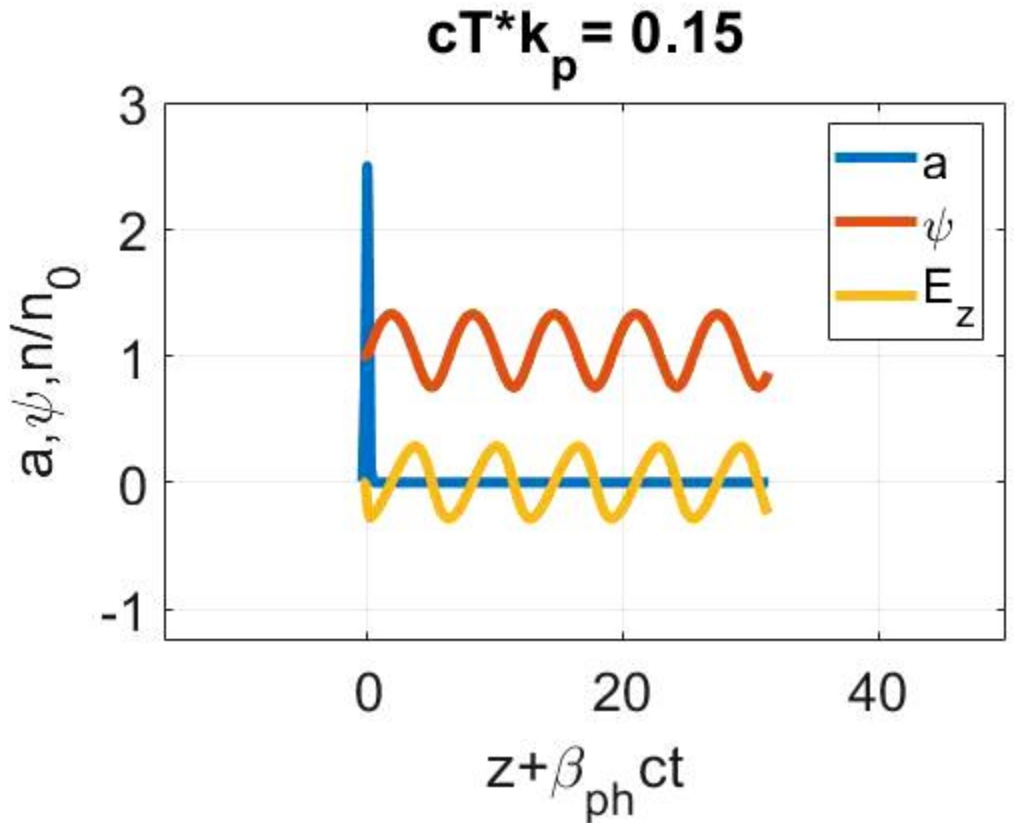
LWFA in the nonlinear 1D regime

$$E_z/E_0 = -\frac{1}{k_p}\partial_\xi \psi \qquad \partial_\xi^2 \psi = -k_p^2 \gamma_p^2 \left\{ \frac{\beta_{ph}}{\sqrt{1-\frac{\gamma_\perp^2}{\gamma_{ph}^2 \psi^2}}} - 1 \right\} \quad \gamma_\perp^2 = 1 + \bar{u}_\perp^2 = 1 + \bar{a}^2$$

What happens if cT varies (a_0=2.5 is constant)

$k_p = \frac{2\pi}{\lambda_p}$

$cT\, k_p = 2\pi \frac{cT}{\lambda_p}$



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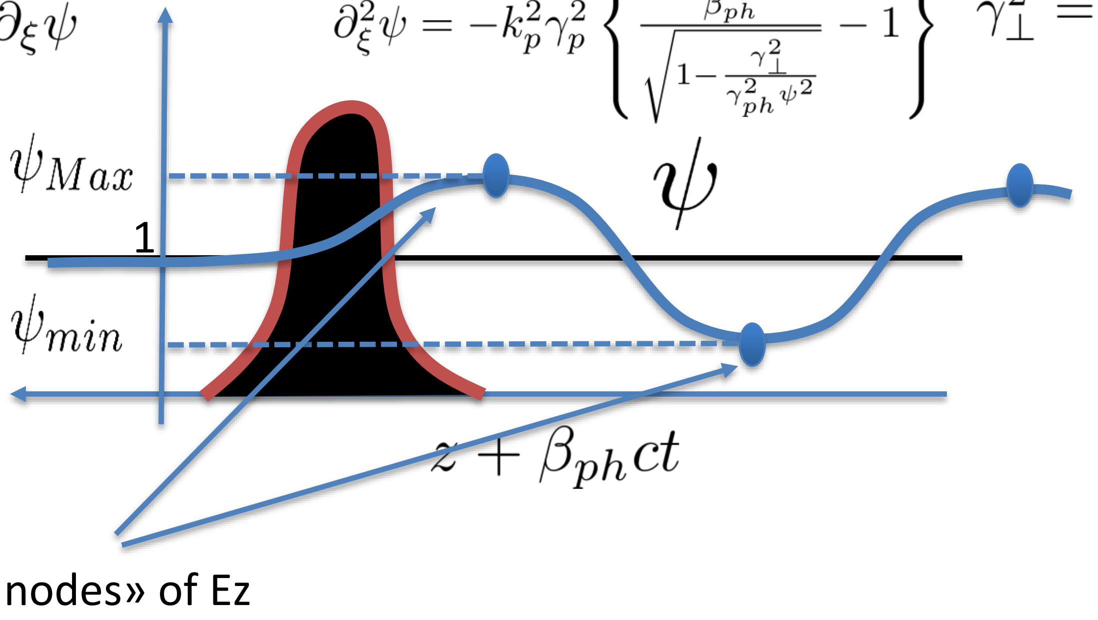
LWFA in the nonlinear 1D regime

$$E_z/E_0 = -\frac{1}{k_p}\partial_\xi \psi$$

$$\partial_\xi^2 \psi = -k_p^2\gamma_p^2 \left\{ \frac{\beta_{ph}}{\sqrt{1-\frac{\gamma_\perp^2}{\gamma_{ph}^2\psi^2}}} - 1 \right\} \qquad \gamma_\perp^2 = 1 + \bar{u_\perp^2} = 1 + \bar{a^2}$$



After straightforward algebraic manipulation we can find the Max and min values of the potential once the maximum value of the normalized electric field is known:

$$\psi_{Max,min} = \mathcal{F} \pm \beta_{ph}\sqrt{(1+\mathcal{F})^2 - 1}$$

$$\mathcal{F} \equiv \frac{1}{2}(E_z/E_0)|_{max}^2$$

This relation is useful to set the **wave-breaking** limit and to deal with particles **trapping** in the plasma wave.

Wave breaking in the nonlinear 1D regime

Particles with $v > v_{ph}$ leave the plasma fluid WAVEBREAKING

A. I. Akhiezer and R. V. Polovin, Zh. Eksp. Teor. Fiz. 30, 915 (1956) [Sov. Phys. JETP 3, 696 (1956)];

$$E_{WB} = E_0 \sqrt{2(\gamma_{ph} - 1)}$$

$$E_0 = mc^2 k_p / e \simeq 96 \sqrt{n_0 (cm^{-3})}\, V/m$$

$n_e = 10^{17} cm^{-3}$ $\gamma_g = \omega_0/\omega_p = \sqrt{n_c/n_e} \simeq 100$ $E_{WB} \approx 15 E_0$

UNDERSTANDING
Nonlinear Relativistic Wave breaking Threshold

A. I. Akhiezer and R. V. Polovin, Zh. Eksp. Teor. Fiz. 30, 915 (1956) [Sov. Phys. JETP 3, 696 (1956)];

$$v > v_{ph}$$

@WB the term $1-\gamma_\perp^2/(\Psi\gamma_g^2)$ is null !

EXACT solution of the fluid motion in 1D+QSA

$$\begin{cases} \gamma = \gamma_g^2 \left\{ \psi - \beta_g \sqrt{\psi^2 - \frac{\gamma_\perp^2}{\gamma_g^2}} \right\} \\ u_z = \gamma_g^2 \left\{ -\beta_g \psi + \sqrt{\psi^2 - \gamma_\perp^2/\gamma_g^2} \right\} \\ n/n_0 - 1 = -\gamma_g^2 \left(1 - \beta_g \frac{1}{\sqrt{1 - \frac{\gamma_\perp^2}{\gamma_g^2 \psi^2}}} \right) \end{cases}$$

$$\beta_g \text{ (laser)} = \beta_\phi \text{ (wave)}$$

REMEMBER that $\gamma + \beta_g u_z - \Psi = 0$, therefore the (fluid) speed is_

$$\beta_z = u_z/\gamma = \frac{1}{\beta_g}(\Psi/\gamma - 1)$$

THEREFORE the wavebreaking is reached when ($-\beta_z \geq \beta_g$, laser moves to z-)

$$-\beta_z = -\frac{1}{\beta_g}(\Psi/\gamma - 1) \geq \beta_g$$

$$\beta_z(\psi = \psi_{WB}) = -\beta_g$$

(after some manipulation) $\Psi \leq \gamma_\perp/\gamma_g \rightarrow \Psi_{WB} = \gamma_\perp/\gamma_g$ **WB potential**

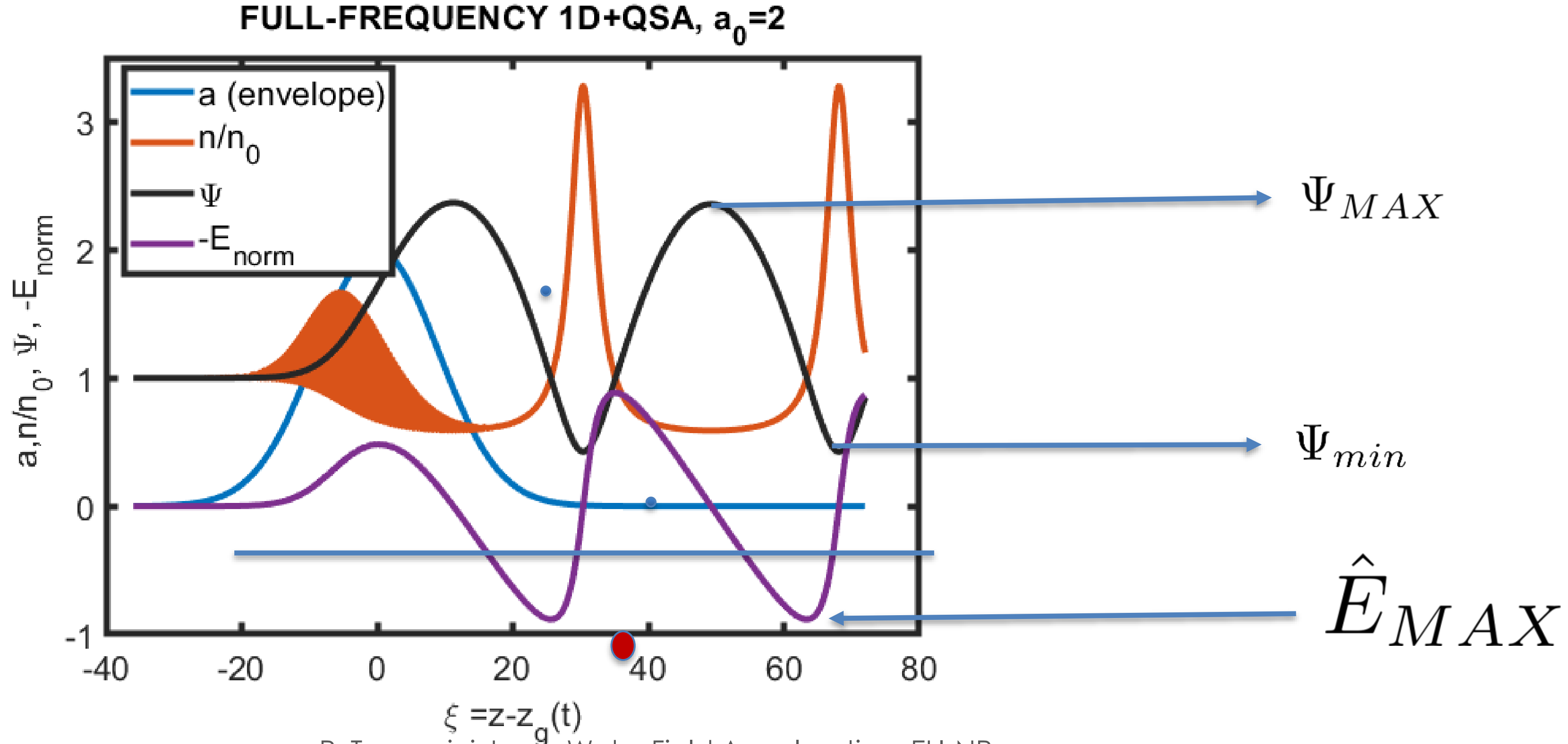
Nonlinear Relativistic Wave breaking Threshold

A. I. Akhiezer and R. V. Polovin, Zh. Eksp. Teor. Fiz. 30, 915 (1956) [Sov. Phys. JETP 3, 696 (1956)];

This can be used to express the **maximum an minimum** values of Ψ as a function of **the MAXIMUM acceleratring gradient**

$$\Psi_{MAX,min} = (\hat{E}^2_{MAX}/2 + \gamma_\perp) \pm \beta_g \sqrt{(\hat{E}^2_{MAX}/2 + \gamma_\perp)^2 - \gamma_\perp^2}$$

$\hat{E} = \frac{E_z}{E_0}$



Nonlinear Relativistic Wave breaking Threshold

A. I. Akhiezer and R. V. Polovin, Zh. Eksp. Teor. Fiz. 30, 915 (1956)

We found earlier that the fluid velocity reaches the plasma phase speed (WAVE BREAKING) when

$$\Psi_{WB} = \gamma_\perp / \gamma_g$$

This means that the MINIMUM potential satiesfies (at WB)

$$\Psi_{min} = \gamma_\perp / \gamma_g$$

NOTE

$$\Psi_{min} \to 0$$

as $n_0 \ll n_c$

$\gamma_g \to \infty$

Therefore:

$$\Psi_{min} = (\hat{E}^2_{MAX}/2 + \gamma_\perp) - \beta_g \sqrt{(\hat{E}^2_{MAX}/2 + \gamma_\perp)^2 - \gamma_\perp^2} = \gamma_\perp/\gamma_g$$

After some manipulation (and supposing $\gamma_\perp = 1$) we can find

$$\hat{E}_{MAX} = E_{WB}/E_0 = \sqrt{2(\gamma_g - 1)}$$

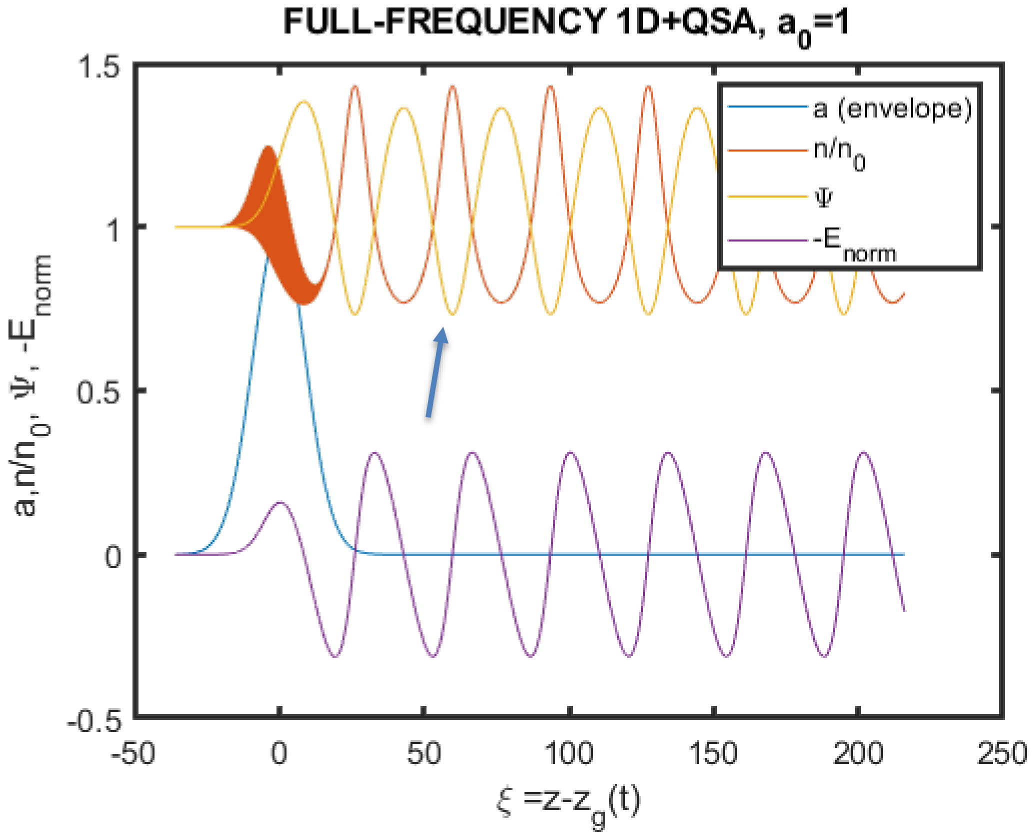
← Maximum E that can be sustained by the wake

$$E_0 = mc^2 k_p / e \simeq 96\sqrt{n_0(cm^{-3})}\, V/m$$

$n_e = 10^{17} cm^{-3}$ $\quad \gamma_g = \omega_0/\omega_p = \sqrt{n_c/n_e} \simeq 100$ $\quad E_{WB} \approx 15 E_0$

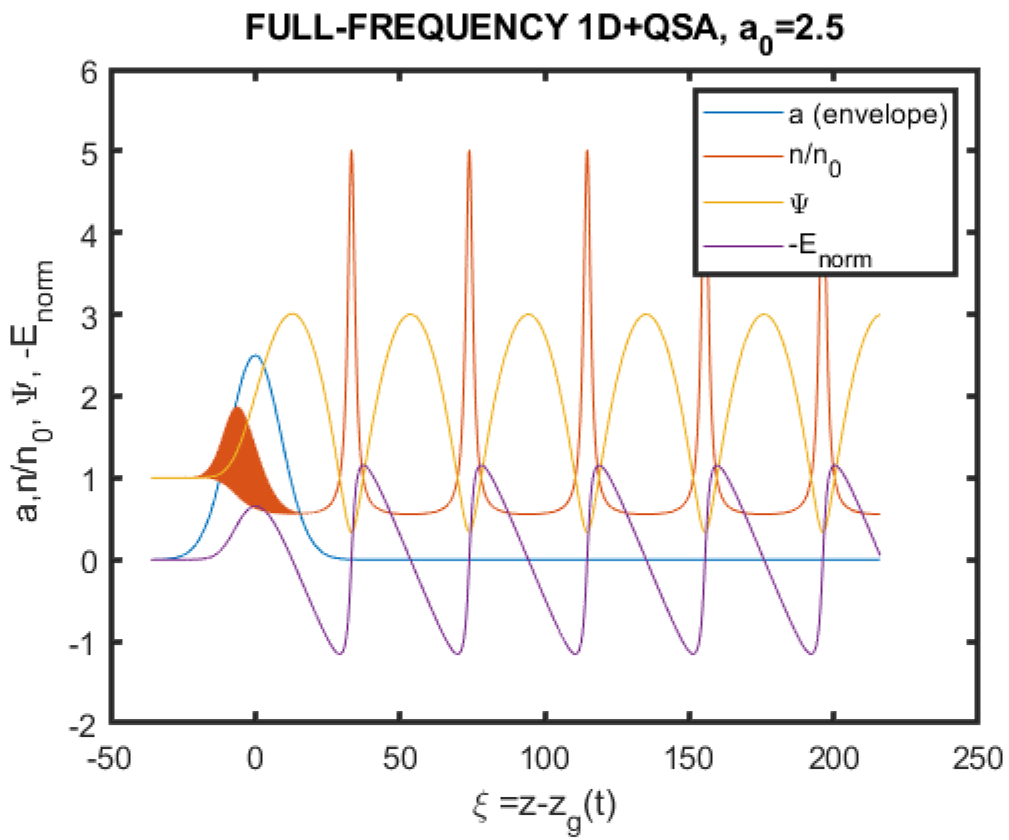
APPROACHING
Nonlinear Relativistic Wave breaking Threshold

As a_0 increases…



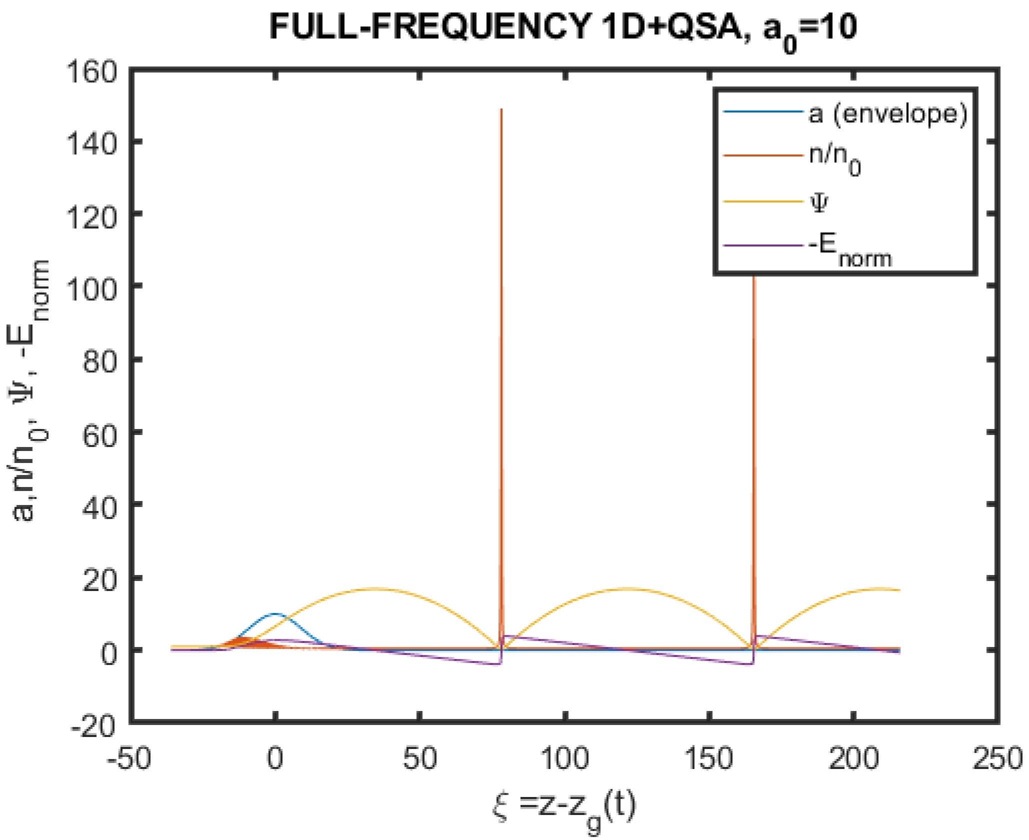
APPROACHING
Nonlinear Relativistic Wave breaking Threshold

As a_0 increases…



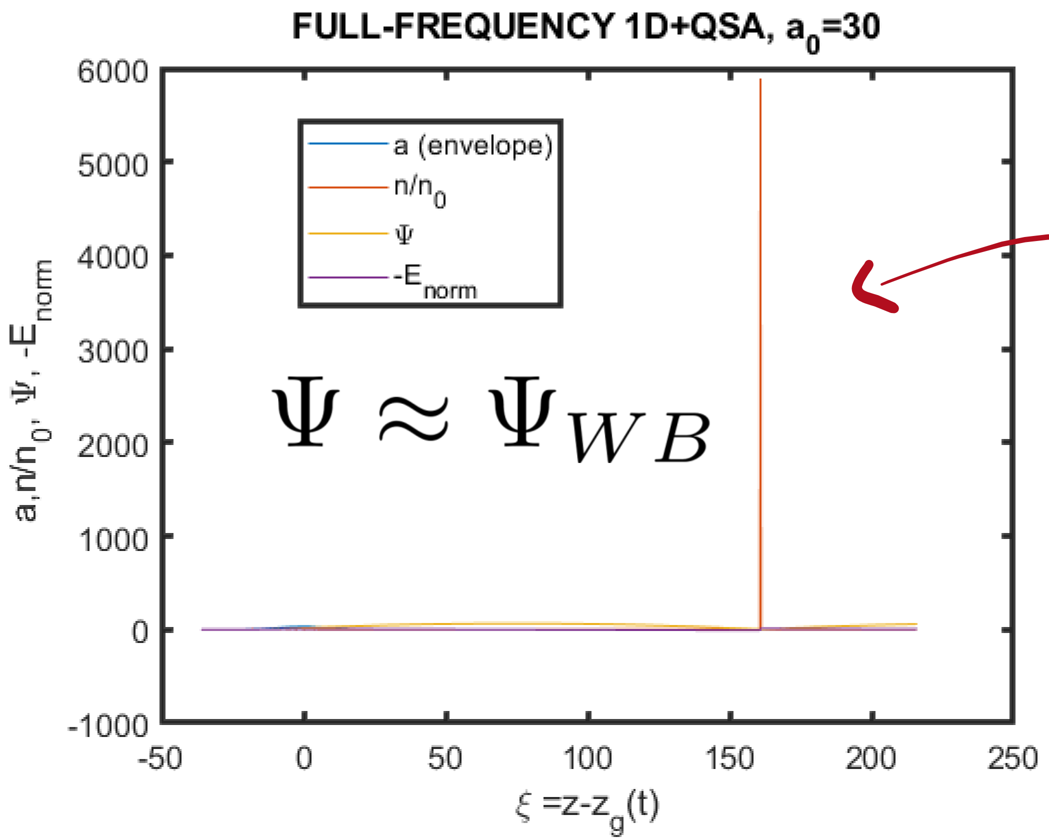
APPROACHING
Nonlinear Relativistic Wave breaking Threshold

As a_0 increases…



APPROACHING
Nonlinear Relativistic Wave breaking Threshold

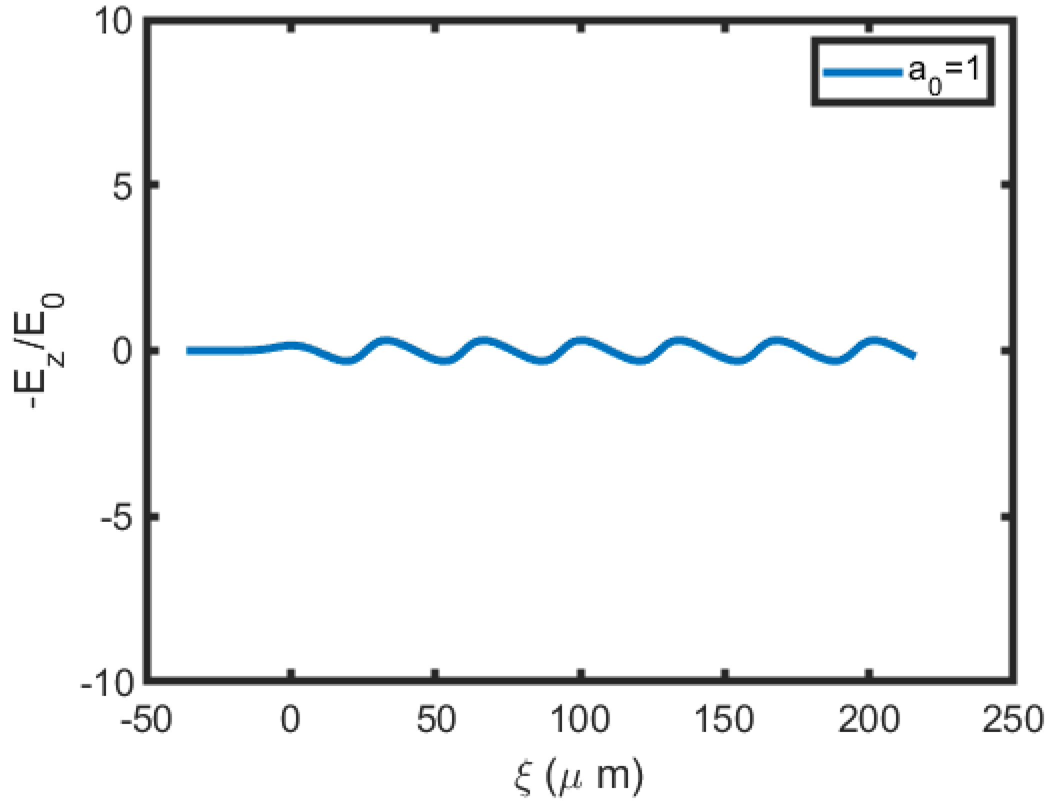
As a_0 increases…



n tends to diverge @ wavebreaking

APPROACHING
Nonlinear Relativistic Wave breaking Threshold

What happens to the accelerating gradient
as the wave approaches its nonlinear breaking?



APPROACHING
Nonlinear Relativistic Wave breaking Threshold

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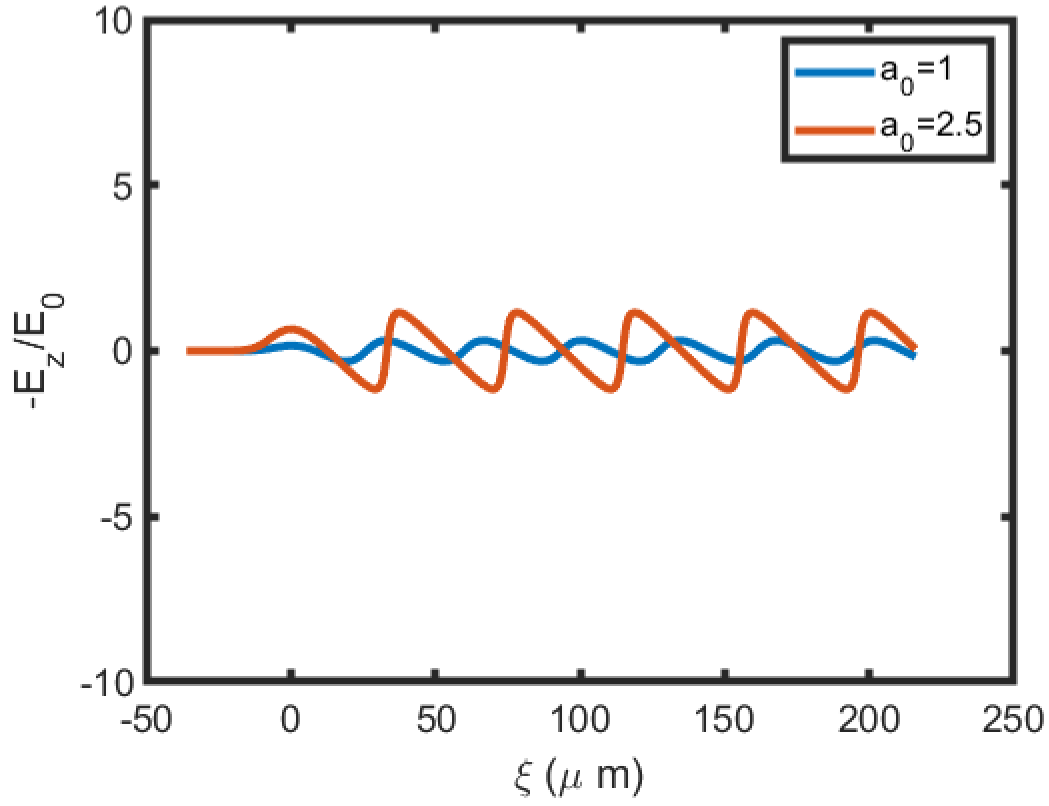

APPROACHING
Nonlinear Relativistic Wave breaking Threshold

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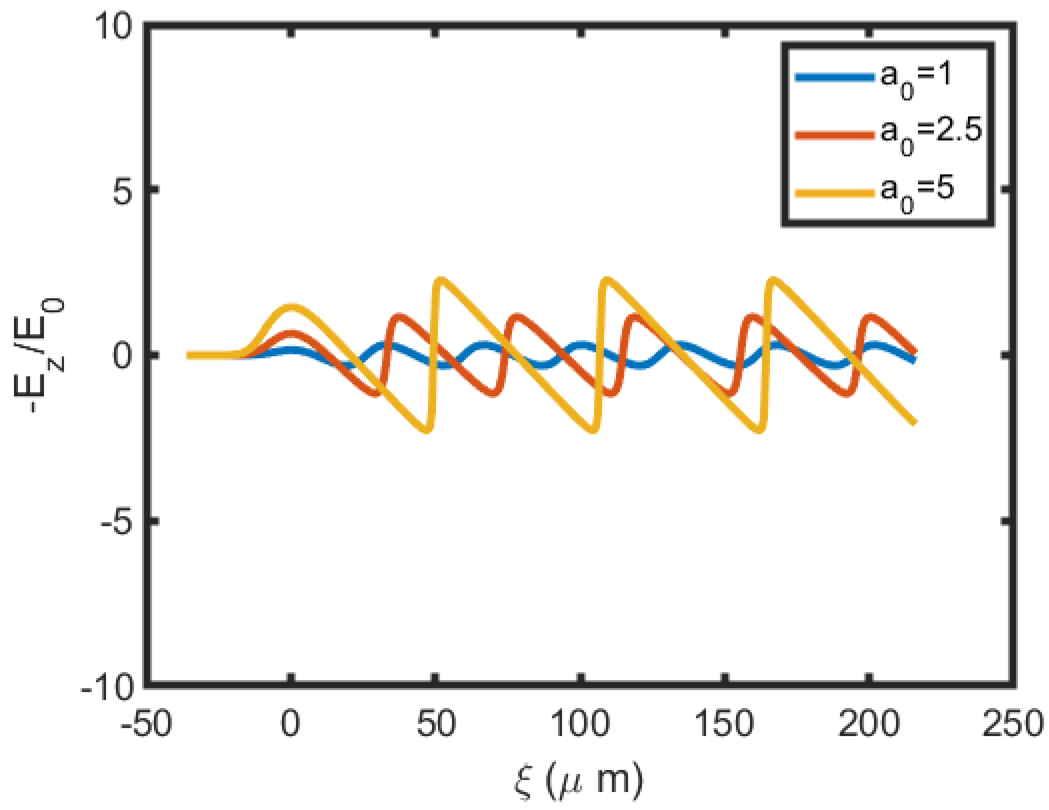

APPROACHING
Nonlinear Relativistic Wave breaking Threshold

What happens to the accelerating gradient
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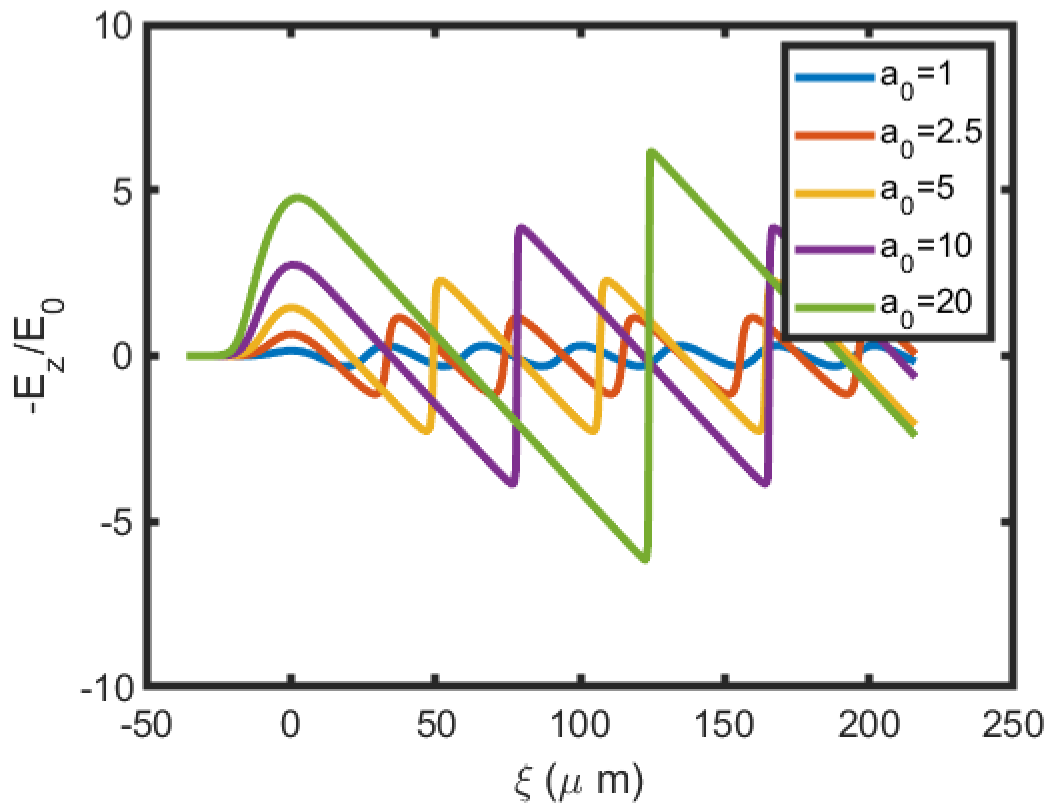
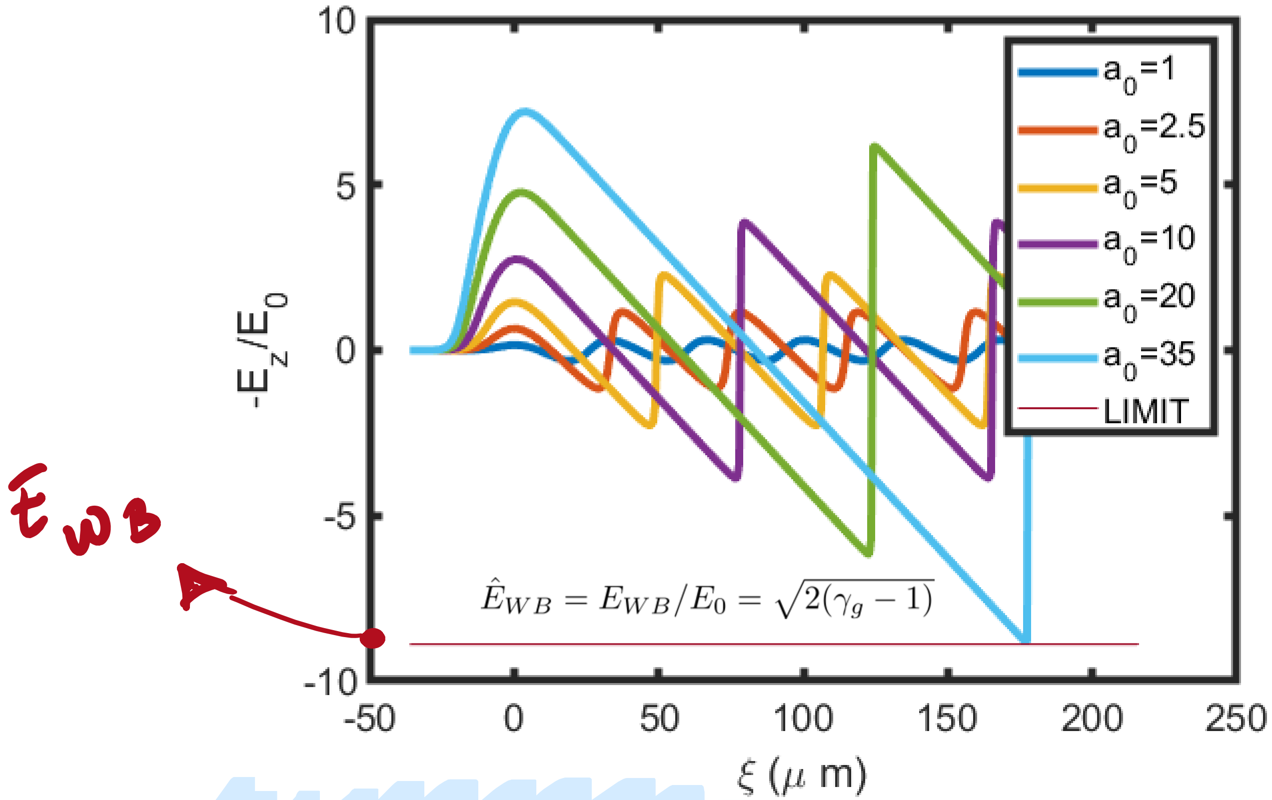


AT Relativistic Wave breaking Threshold

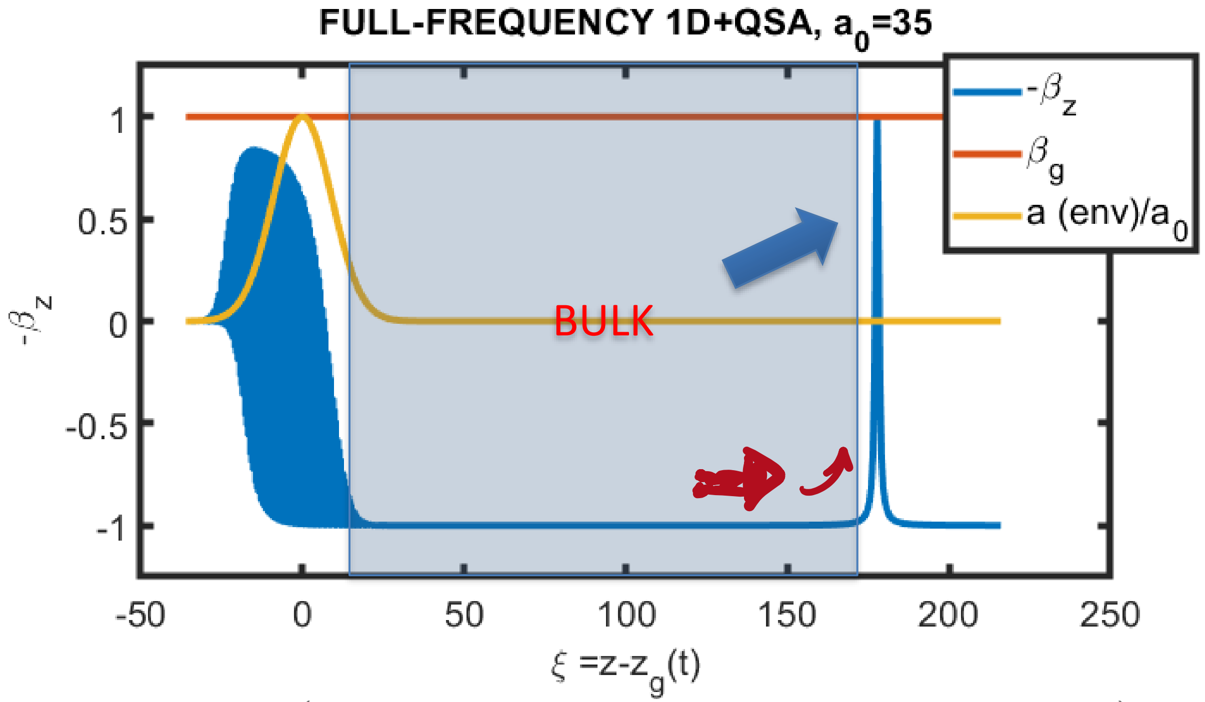
What happens to the accelerating gradient
as the wave approaches its nonlinear breaking?



WE CANNOT GO FURTHER, from now on the wave is able
to catch electrons from the plasma (on the top of the plasma spike)

Nonlinear Relativistic Wave breaking Threshold

What happens to fluid **speed** as the wave approaches its nonlinear breaking?

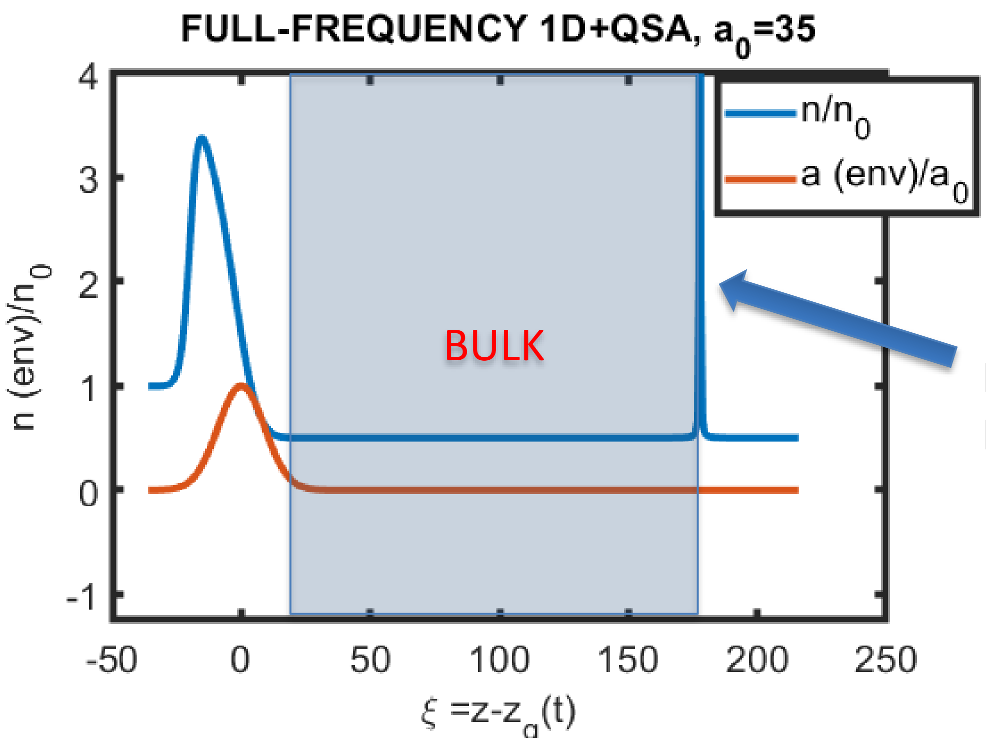


NOTE: in the bulk (up to just before the velocity peak) $\beta_z \simeq 1$ *i.e.* the plasma particles move at (about) the speed of light ON THE LASER PULSE OPPOSITE direction !!!

This means that IN THE BULK $(\beta_g + \beta_z) \simeq 2$

Nonlinear Relativistic Wave breaking Threshold

What happens to the **density** as the wave approaches its nonlinear breaking?



Max(n/n_0) is about 40000
In the simulation

We need 3D effects

...but if IN THE BULK $(\beta_g + \beta_z) \simeq 2$, this implies that (there) the electron plasma density

$$n/n_0 = \beta_g/(\beta_z + \beta_g) \simeq 1/2$$

THEREFORE the electron plasma density never drops below ½ of the background Value.

In 1D we have no chance to create a fully depleted zone

Trapping (and acceleration) theory in STABLE 1D+QSA wakefields

Approx: STABLE wakefield in 1D+QSA

LOOKING AT PARTICLE ACCELERATION, not fluid plasma evolution

Effective Hamiltonian for «**passive**» particle dynamics (**no plasma reaction**)

In a moving window $\xi = z + \beta_{ph} t$ NO BEAM-LOADING

$$\vec{\pi} \equiv \frac{\partial \mathcal{L}}{\partial \vec{\beta}} = \vec{u} - \vec{a}$$

$$a_x \equiv a(\xi); \quad a_y = 0; \quad a_z = 0$$

$$\mathcal{H} = \sqrt{1 + (\pi + a)^2} + \beta_{ph}\pi_z - \phi$$

$$\begin{cases} \frac{1}{c}\partial_t \xi = \frac{\partial \mathcal{H}}{\partial \pi_z} \rightarrow \frac{1}{c}\partial_t \xi = \beta_z + \beta_{ph} \\ \frac{1}{c}\partial_t \vec{\pi} = -\frac{\partial \mathcal{H}}{\partial \vec{x}} \rightarrow u_x = a; \ \frac{1}{c}\partial_t u_z = -\partial_\xi(\gamma - \phi) \end{cases}$$

← Position

← Momentum

GIVES «**space-time**» **in the** $(\xi(ct), u_z(ct))$ **phase space plane**

Also, the potential Ψ satisfies the Poisson equation

$$\partial_\xi^2 \Psi = -k_p^2 \gamma_p^2 \left\{ \frac{\beta_{ph}}{\sqrt{1 - \frac{\gamma_\perp^2}{\gamma_{ph}^2 \Psi^2}}} - 1 \right\}$$

To be solved once

And must be solved numerically (but it's trivial to do it !!!)

Trapping (and acceleration) theory in STABLE 1D+QSA wakefields

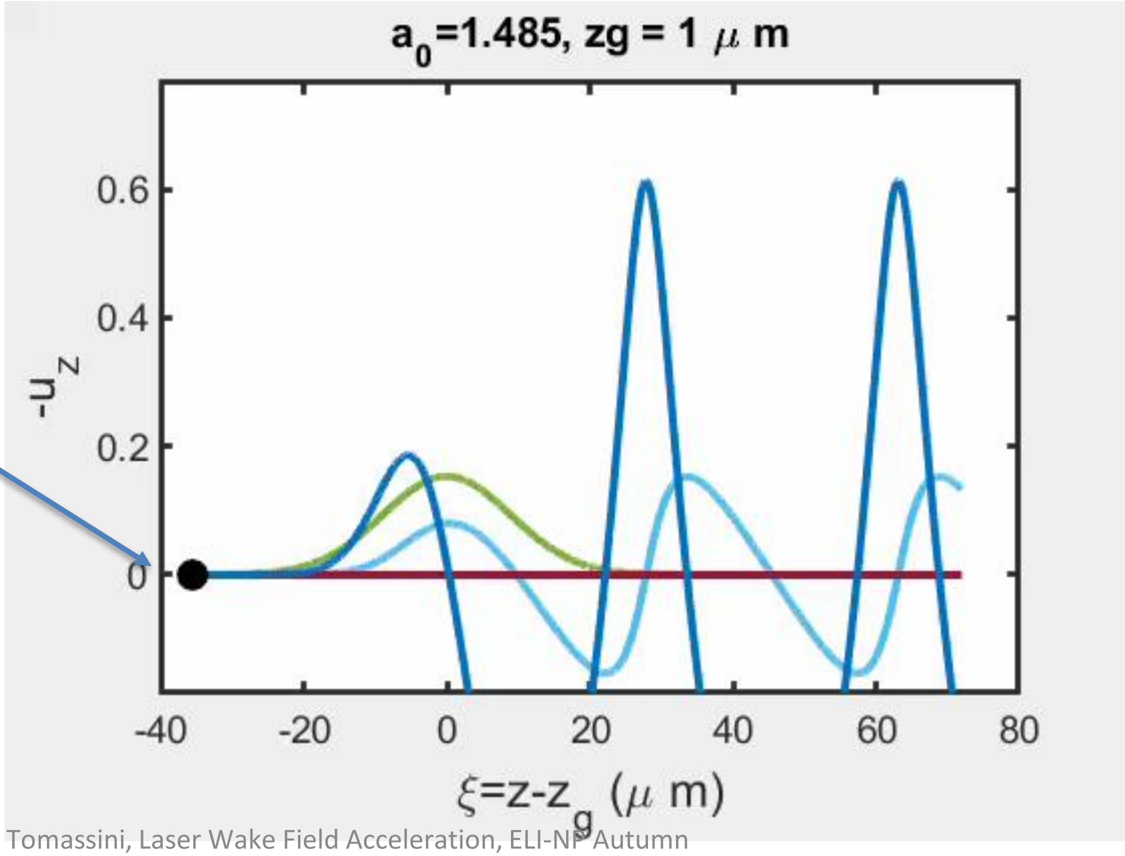
Approx: STABLE wakefield in 1D+QSA

LOOKING AT PARTICLE ACCELERATION, not fluid plasma evolution

Effective Hamiltonian for «**passive**» particle dynamics (no plasma reaction)

In a moving window $\xi = z + \beta_{ph} t$

Particle as a generic plasma electron



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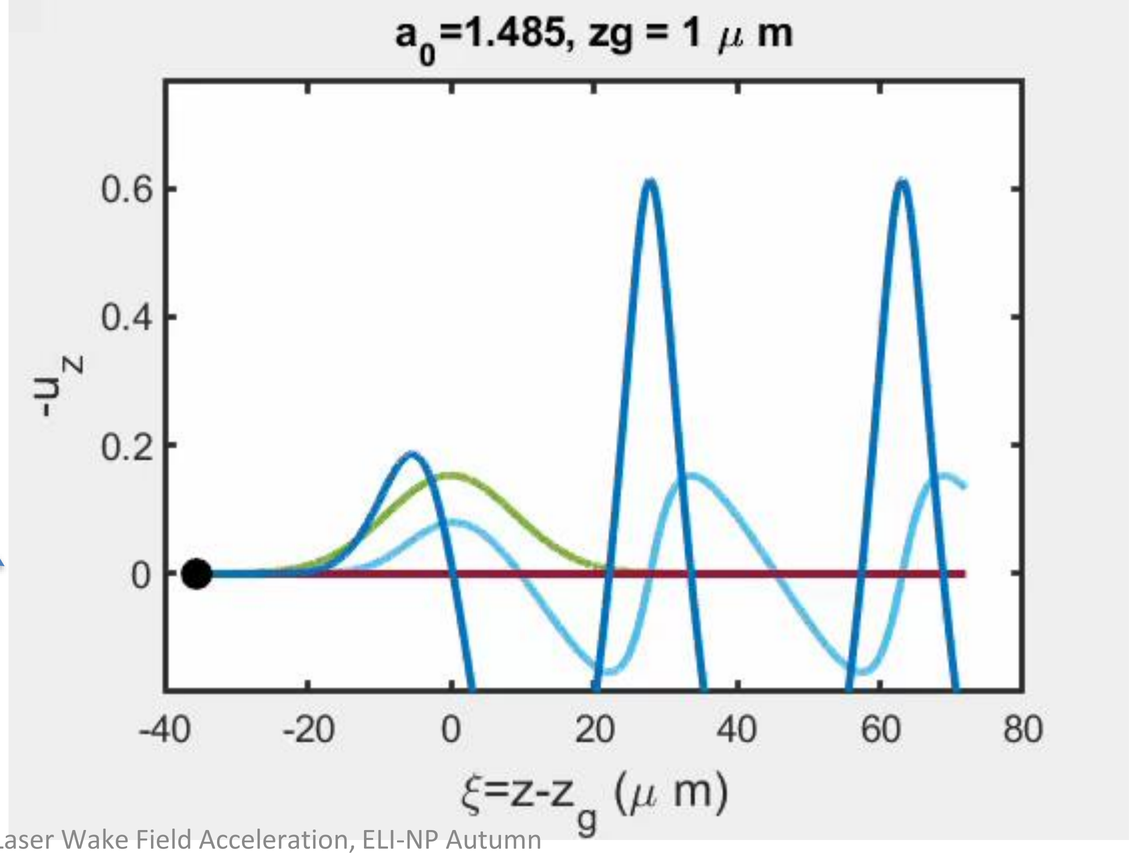


Trapping (and acceleration) theory in STABLE 1D+QSA wakefields

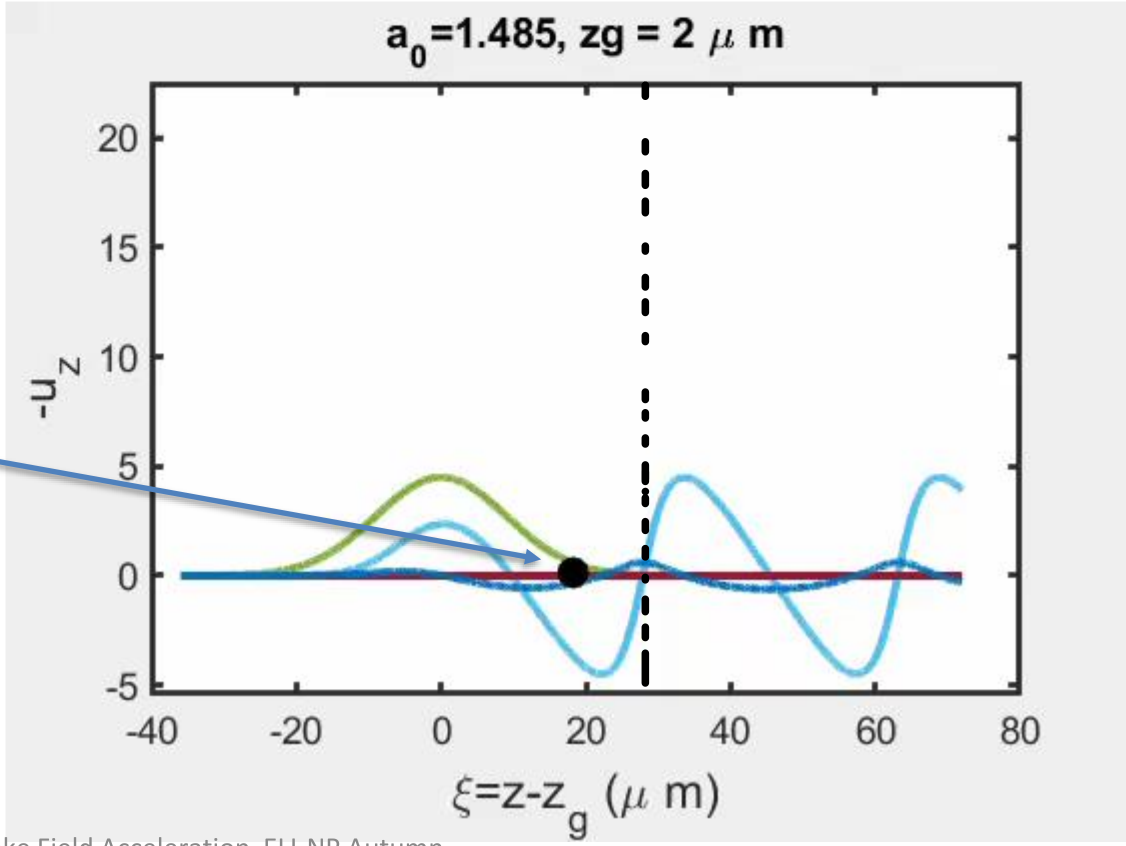
Approx: STABLE wakefield in 1D+QSA

LOOKING AT PARTICLE ACCELERATION, not fluid plasma evolution

Effective Hamiltonian for «**passive**» particle dynamics (no plasma reaction)

In a moving window $\xi = z + \beta_{ph} t$

Particle «BORN» inside the wave (e.g. by ionization)



Trapping (and acceleration) theory in STABLE 1D+QSA wakefields

Approx: STABLE wakefield in 1D+QSA

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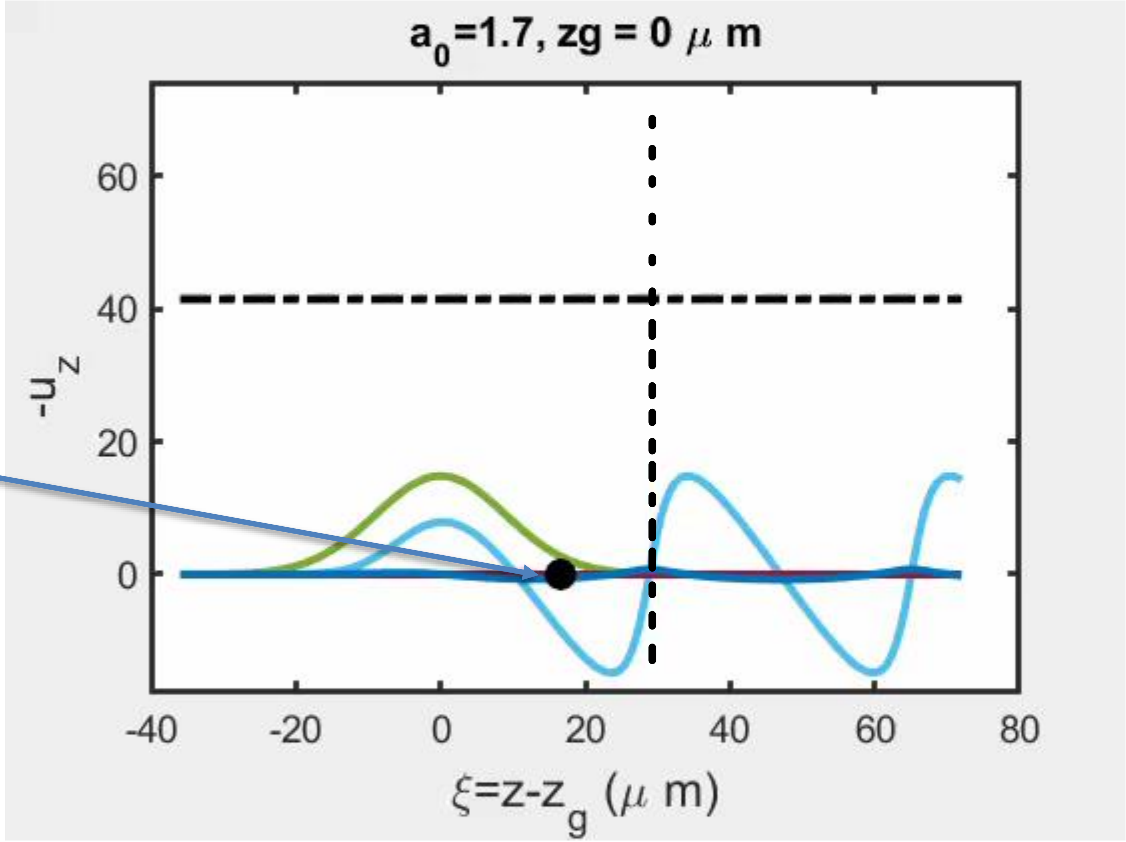

Trapping (and acceleration) theory in STABLE 1D+QSA wakefields

Approx: STABLE wakefield in 1D+QSA

LOOKING AT PARTICLE ACCELERATION, not fluid plasma evolution

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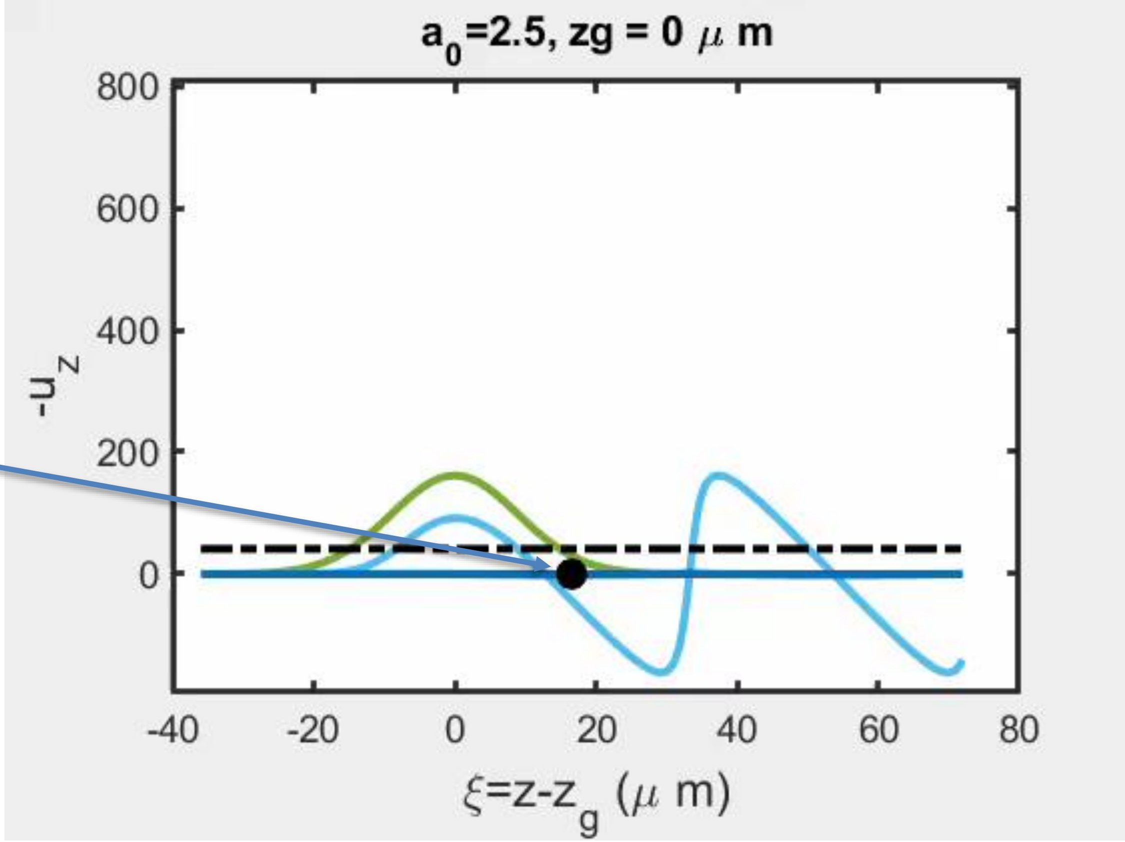


Trapping (and acceleration) theory in STABLE 1D wakefields

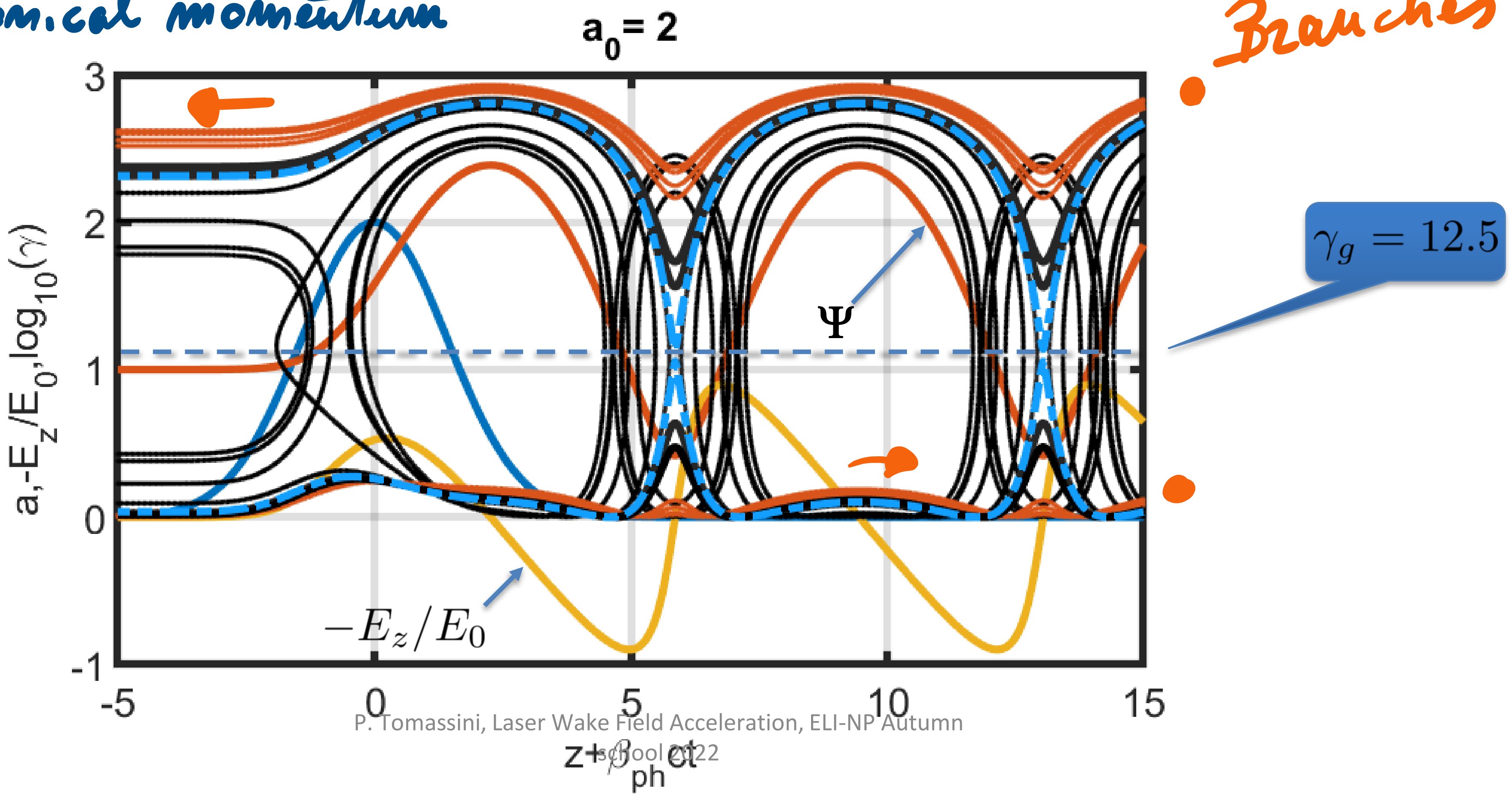
PHASE-SPACE TRAJECTORIES

$\vec{\pi} = \vec{u} - \vec{a}$ → Canonical momentum

$$\mathcal{H} = \sqrt{1 + (\pi + a)^2} + \beta_{ph}\pi_z - \phi$$

$$\mathcal{E}(ct) = \sqrt{1 + a^2(ct) + u_z^2(ct)} + \beta_{ph}u_z(ct) - \phi(ct) = \gamma(ct) + \beta_{ph}u_z(ct) - \phi(ct)$$

-



Trapping (and acceleration) theory in STABLE 1D wakefields

PHASE-SPACE TRAJECTORIES: Once the Ψ map is known, the (infinite) set of trajectories can be analytically found, **one trajectory for each value of the (constant) energy**

$H = h_0$

Despite the nonlinear behaviour of the plasma wave and of the relativistic effects on the particles, **conditions for trapping a surprisingly simple.**

$$d_t H = \partial_t H = 0 \quad \Rightarrow \quad \gamma + \beta_{ph} u_z - \phi = (\gamma + \beta_{ph} u_z - \phi)_0 \equiv h_0$$

$$\gamma_\perp^2 = 1 + a^2/2; \quad \gamma^2 = \gamma_\perp^2 + u_z^2;$$

TRAJECTORIES in the (ϕ, u_z) plane (*found after some manipulation*)

$$u_z(\phi) = \gamma_{ph}^2 (h_0 + \phi) \left\{ -\beta_{ph} \pm \sqrt{1 - \frac{\gamma_\perp^2}{\gamma_{ph}^2 (h_0+\phi)^2}} \right\}$$

And with the aid of the $\phi(\xi) = \Psi(\xi) - 1$ map, **trajectories can be finally remapped in the** (ξ, u_z) **phase-space plane**

$$u_z(\xi) = \gamma_{ph}^2 (h_0 + \phi) \left\{ -\beta_{ph} \pm \sqrt{1 - \frac{\gamma_\perp^2(\xi)}{\gamma_{ph}^2 (h_0+\phi(\xi))^2}} \right\}$$

TWO branches!

!!

SOUNDS FAMILIAR?

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Trapping (and acceleration) theory in STABLE 1D wakefields

PHASE-SPACE TRAJECTORIES: Once the Ψ map is known, the (infinite) set of trajectories can be analytically found, **one trajectory for each value of the (constant) energy**

In the case of a **generic plasma particle** (at rest before the arrival of the pulse)

…for a plasma particle trajectory (**it is just ONE of the possible trajectories**)

$$h_0 = (\gamma + \beta_{ph} u_z - \phi)_0 = 1$$

initial value

SOUNDS FAMILIAR?

$$u_z(\xi) = \gamma_{ph}^2 (h_0 + \phi) \left\{ -\beta_{ph} \pm \sqrt{1 - \frac{\gamma_\perp^2(\xi)}{\gamma_{ph}^2 (h_0 + \phi(\xi))^2}} \right\}$$

$$u_z(\xi) = \gamma_{ph}^2 \Psi \left\{ -\beta_{ph} + \sqrt{1 - \frac{\gamma_\perp^2(\xi)}{\gamma_{ph}^2 \Psi^2}} \right\}$$

Describes the fluid plasma momentum

Plasma particle $h_0 = 1$

We saw this for the fluid plasma momentum

Trapping (and acceleration) theory in STABLE 1D wakefields

TRAPPING THEORY (1D+QSA, of course)

- We saw that trapping occurs when the particles reaches the wave's speed (**in a still accerating region**)

$$\beta_z = -\beta_{ph}$$

- The **separatrix** between the closed (periodic) orbit and the open ones is found at the extremal point for the accelerating region, i.e when

$$u_{z,tr} = -\beta_{ph}\gamma_{tr} \rightarrow \gamma_{tr} = \sqrt{1+\beta_{ph}^2\gamma_{tr}^2} \rightarrow \gamma_{tr} = \gamma_{ph}\gamma_{\perp,tr}$$

- Therefore, **the particle gets trapped when** $\gamma_{tr} = \gamma_{ph}\gamma_{\perp}$

i.e. when

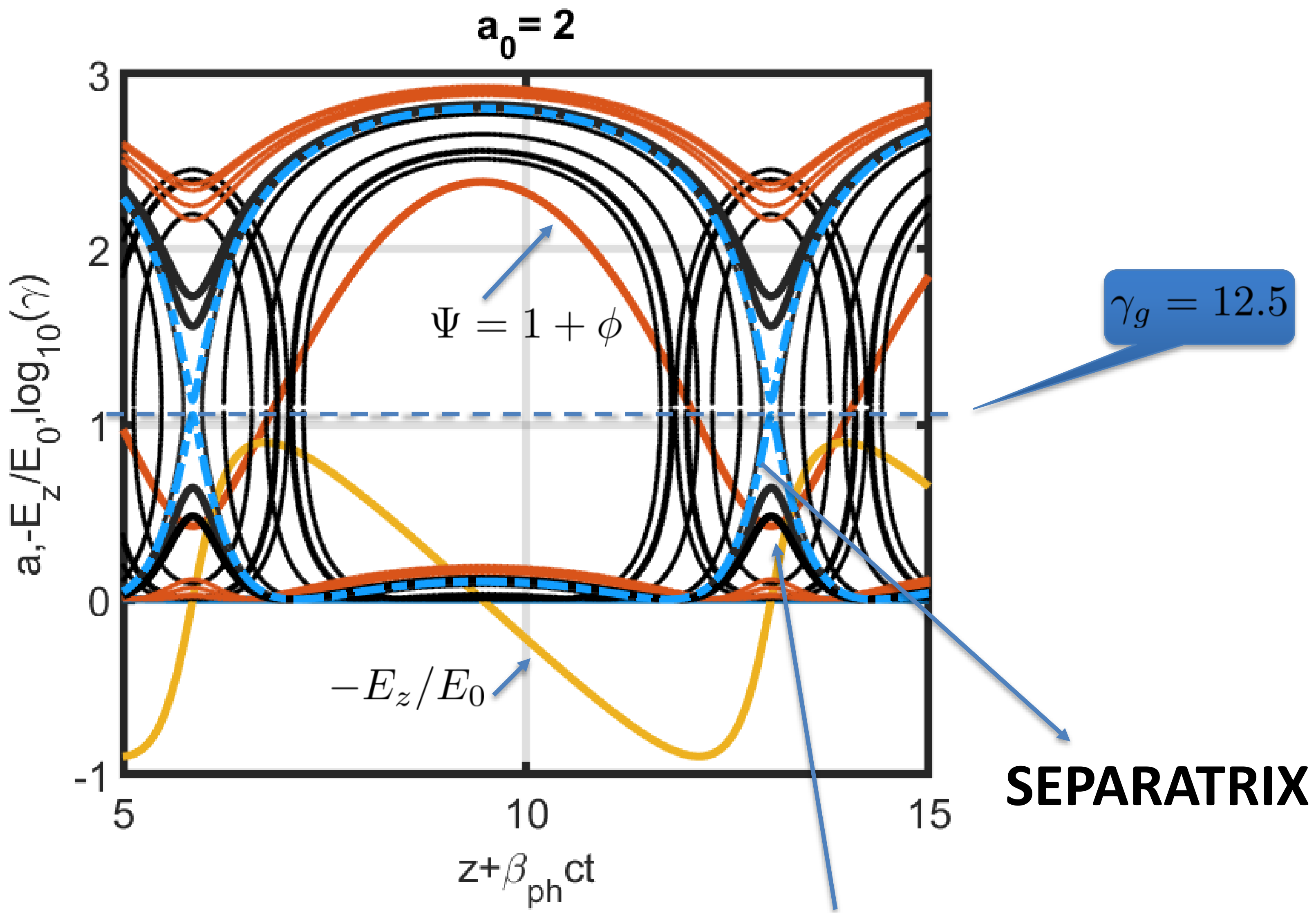
$$h_{0,separ} \equiv \gamma_{tr} + \beta_{ph}u_{z,tr} - \phi_{tr}$$
$$h_{0,separ} = \gamma_{\perp,tr}/\gamma_{ph} - \phi_{tr}$$

SEPARATRIX VALUE

AT THE END, particles initially having $a_0 = a(t=0), \phi_0, u_{z,0}$ are trapped if

$$h_0 \leq h_{0,separ} \rightarrow \gamma_0 + \beta_{ph}u_{z,0} - \phi_0 \leq \gamma_{\perp}/\gamma_{ph} - \phi_{tr}$$

Trapping (and acceleration) theory in STABLE 1D wakefields



$$h_{0,trapping} = \gamma_{\perp,tr}/\gamma_{ph} - \phi_{min}$$

Trapping (and acceleration) theory in STABLE 1D wakefields

Standard trapping «weak trapping» vs «strong trapping»

$$(1+\phi_{Max,min}) = \mathcal{F} \pm \beta_{ph}\sqrt{(1+\mathcal{F})^2 - 1}$$

$$\mathcal{F} \equiv \frac{1}{2}(E_z/E_0)|^2_{max}$$

- Standard «**weak trapping**» [**E. Esarey et al.; Phys. Plasmas 2 (1995)**]: the particles are trapped at the minimum of the potential (their trajectories lie in the **separatix**), where the **electic field is null**.

$$\gamma_0 - \phi_0 \leq \gamma_\perp/\gamma_{ph} - \phi_{min}$$

$$2|\beta_{ph}|\sqrt{(1+\mathcal{F})^2 - 1} \geq 1 - 1/\gamma_{ph}$$

Potential @trapping position

- «**Strong trapping**»: **P.Tomassini et al.; Phys. Plasmas 24 (2017)**]: the particles are trapped where the potential is null (and the **accelerating field is maximum**).

$$\gamma_0 - \phi_0 \leq \gamma_\perp/\gamma_{ph} - 0$$

$$\mathcal{F} + |\beta_{ph}|\sqrt{(1+\mathcal{F})^2 - 1} \geq 1 - 1/\gamma_{ph}$$

Trapping (and acceleration) theory in STABLE 1D wakefields

Conserved Hamiltonian in 1D+QSA

$$\mathcal{H}(\psi,\gamma) = \gamma(1-\beta\beta_{ph}) - \phi(\psi)$$

$$\psi \equiv k_p(z-\beta_\phi ct)$$

[E.Esarey & M. Pillov, PoP 2 1432 (1995)]

$$E_{norm} = E_z/E_0;\ \ E_0 = mc\omega_p/e$$

$$\mathcal{F} \equiv \frac{1}{2} E_{norm}|^2_{max}$$

$$(1+\phi_{Max,min}) = \mathcal{F} \pm \beta_{ph}\sqrt{(1+\mathcal{F})^2-1}$$

$$2|\beta_{ph}|\sqrt{(1+\mathcal{F})^2-1} \geq 1 - 1/\gamma_{ph}$$

Weak (standard) trapping threshold

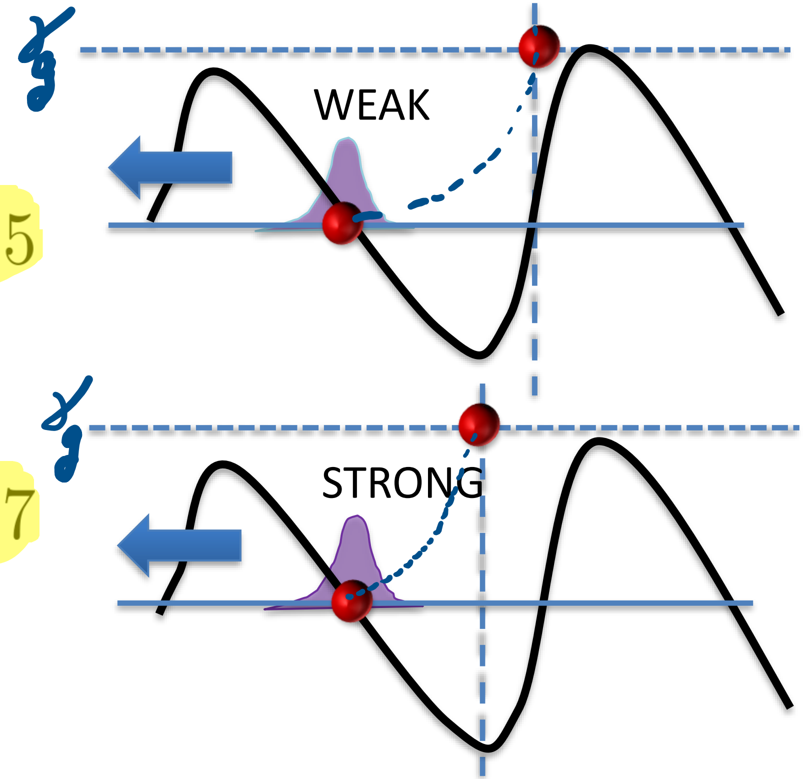
$$E_{norm} \simeq 0.5$$

Strong trapping threshold

$$\mathcal{F} + |\beta_{ph}|\sqrt{(1+\mathcal{F})^2-1} \geq 1 - 1/\gamma_{ph}$$

$$E_{norm} \simeq 0.7$$

[Tomassini et al.; Phys. Plasmas 24 (2017)]

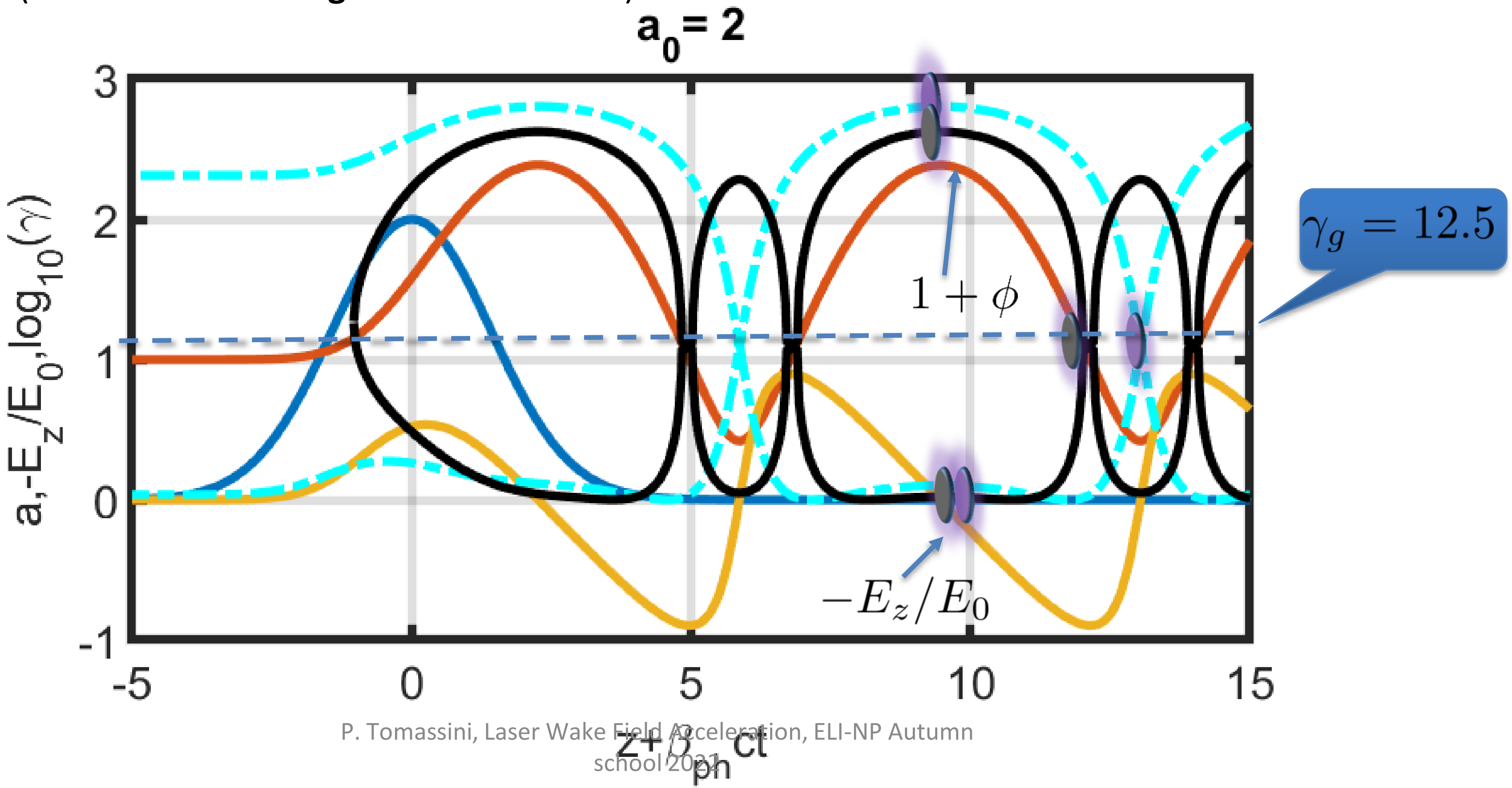


Strong trapping condition is more demanding than the standard «weaker» one

Trapping (and acceleration) theory in STABLE 1D wakefields

Standard trapping «weak trapping» vs «strong trapping»

Standard «weak trapping» [**E. Esarey et al.; Phys. Plasmas 2 (1997)**]: the particles are trapped at the minimum of the potential (their trajectories lie in the separatix), where the **electic field is null**.
«Strong trapping»: **P.Tomassini et al.; Phys. Plasmas 24 (2017)**]: the particles are trapped where the potential is null (and the **accelerating field is maximum**).



Trapping (and acceleration) theory in STABLE 1D wakefields

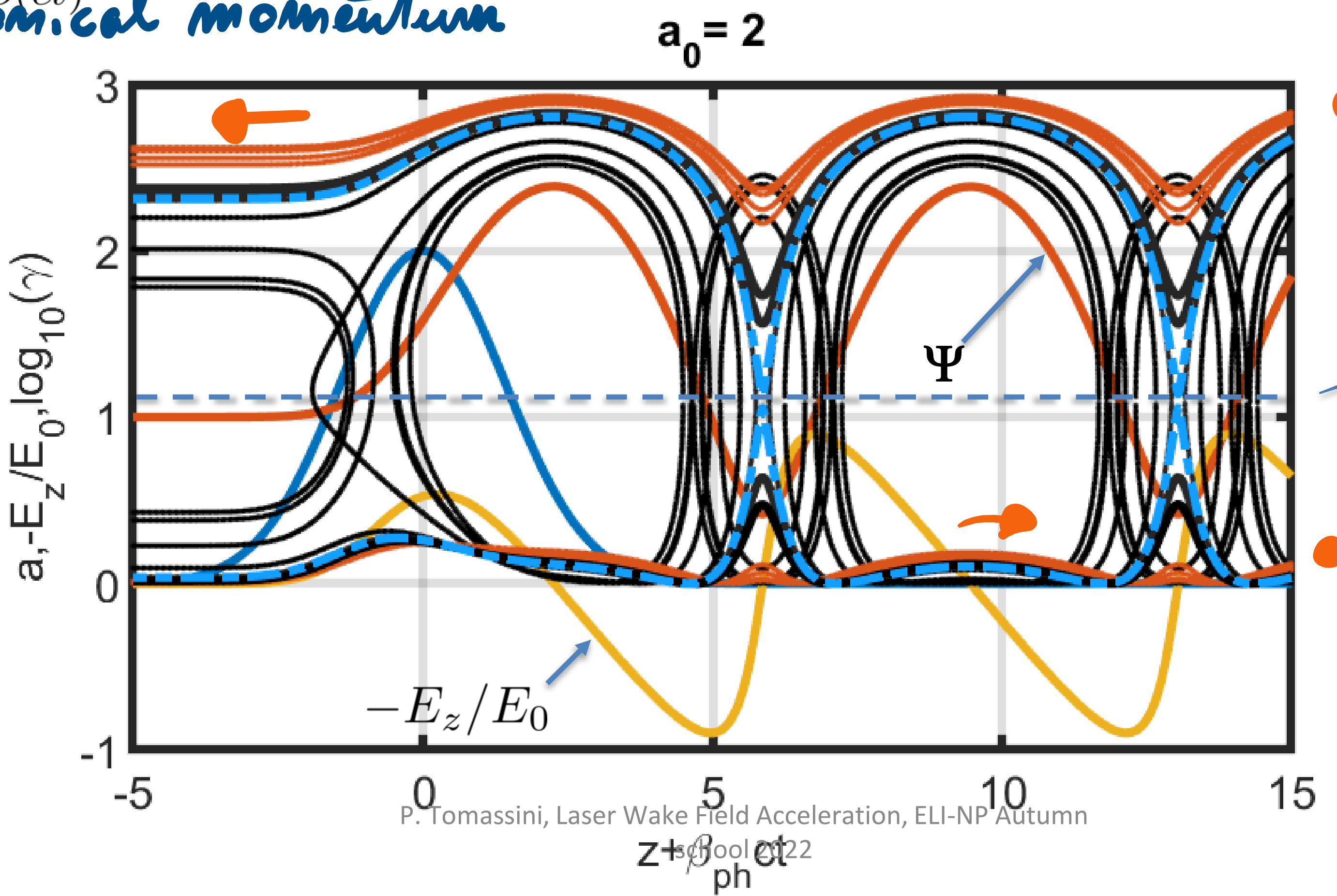
PHASE-SPACE TRAJECTORIES

$\vec{p} = \vec{u} - \vec{a}$ → Canonical momentum

$$\mathcal{H} = \sqrt{1 + (P + a)^2} + \beta_{ph} P_z - \phi$$

$$h_0 = \mathcal{H}(ct) = \sqrt{1 + a^2(ct) + u_z^2(ct)} + \beta_{ph} u_z(ct) - \phi(ct) = \gamma(ct) + \beta_{ph} u_z(ct) - \phi(ct)$$

Branches



$\gamma_g = 12.5$

Trapping (and acceleration) theory in STABLE 1D wakefields

$$u_z(\xi) = \gamma_{ph}^2 (h_0 + \phi) \left\{ -\beta_{ph} \pm \sqrt{1 - \frac{\gamma_\perp^2(\xi)}{\gamma_{ph}^2 (h_0+\phi(\xi))^2}} \right\}$$

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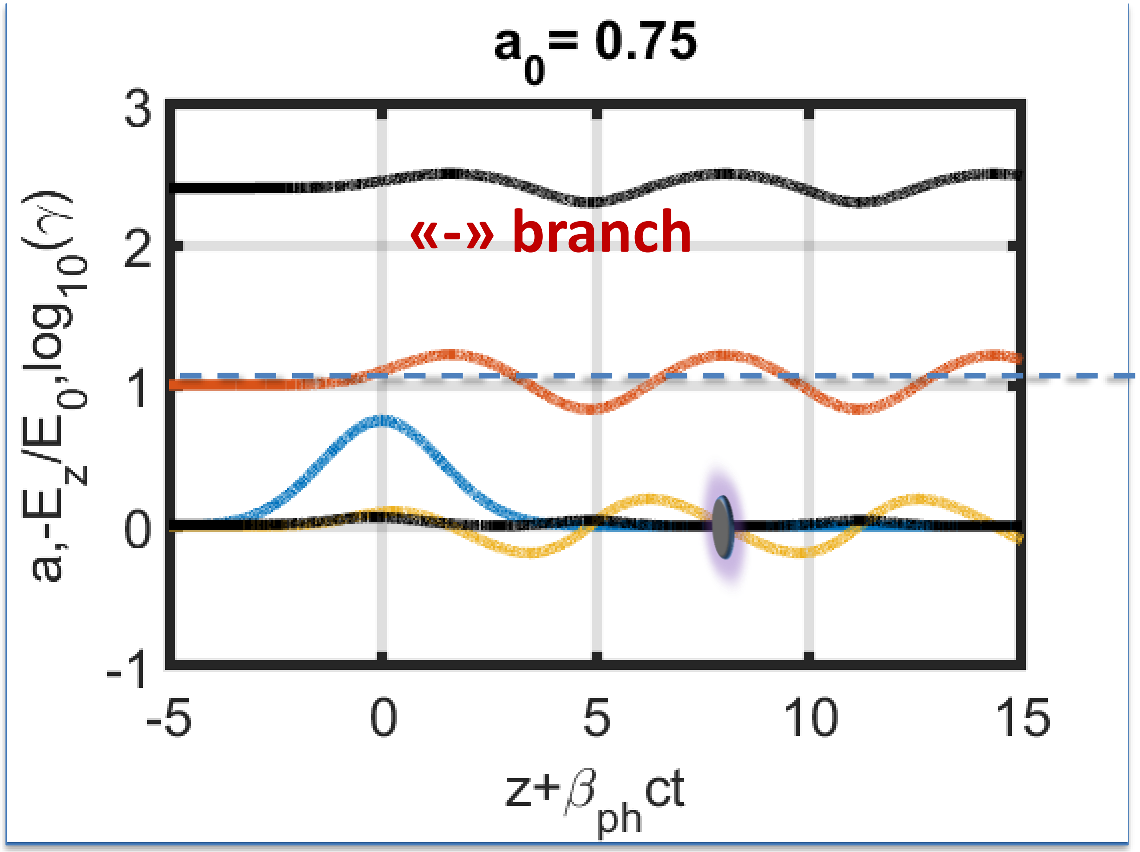

$\gamma_{ph} = 12.5$

Trapping (and acceleration) theory in STABLE 1D wakefields

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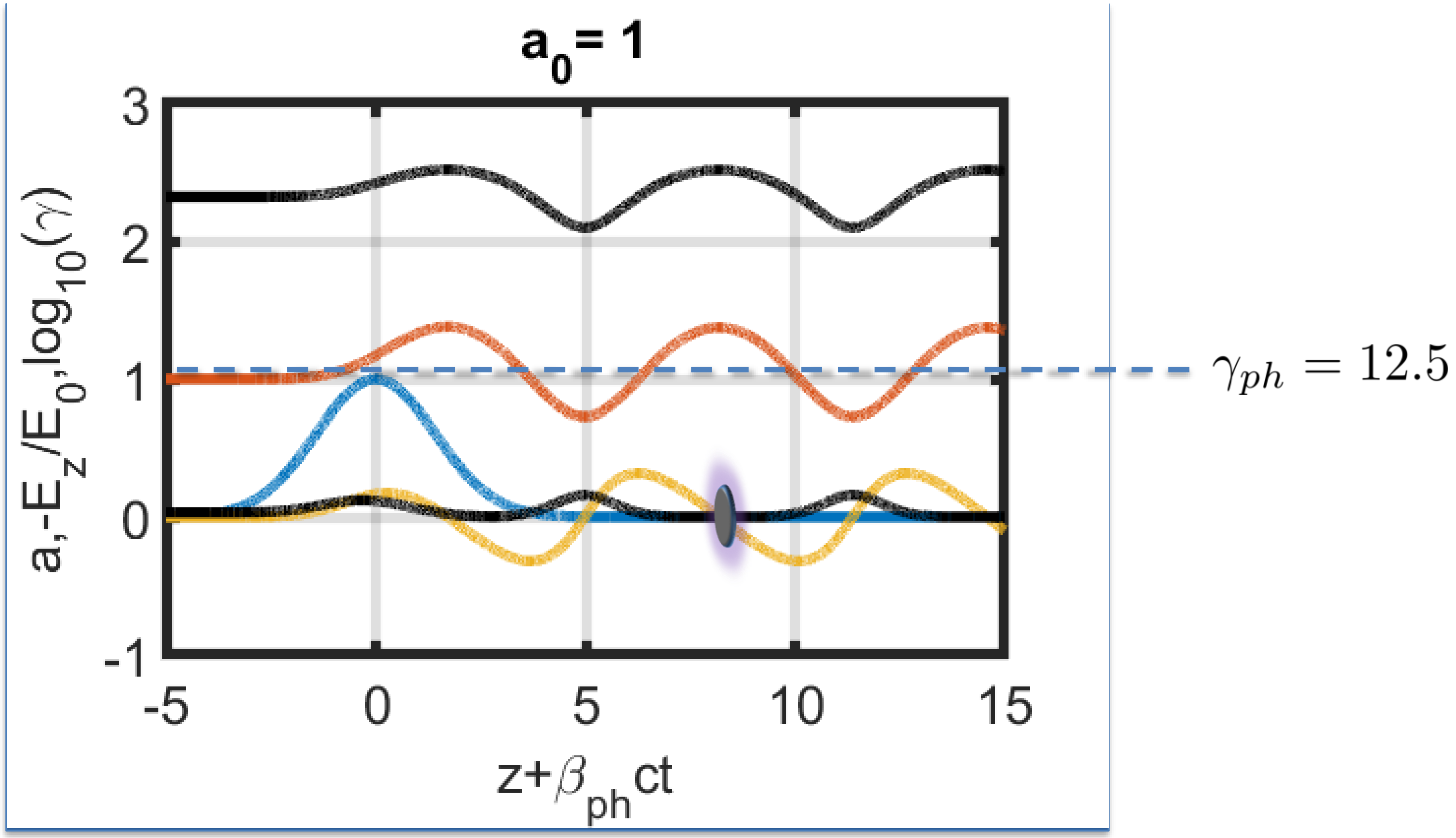
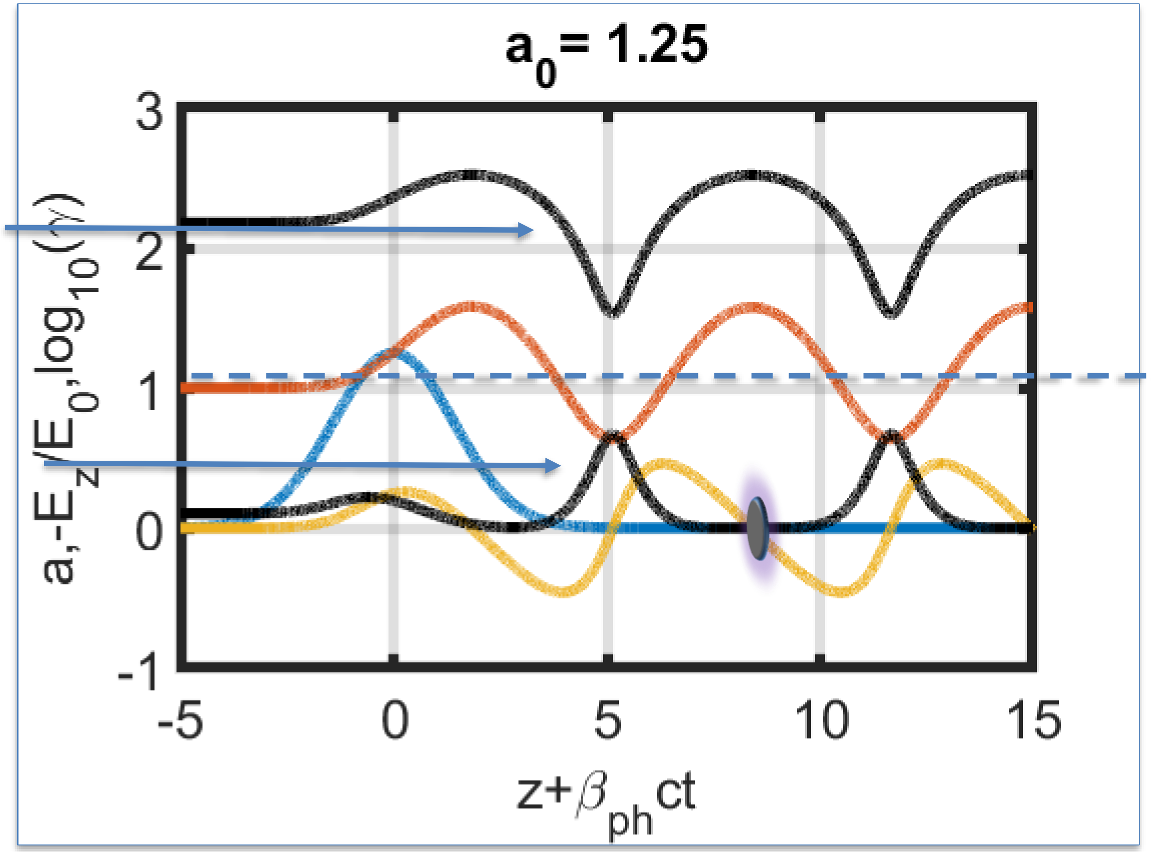


Trapping (and acceleration) theory in STABLE 1D wakefields

«-» branch

•

«+» branch



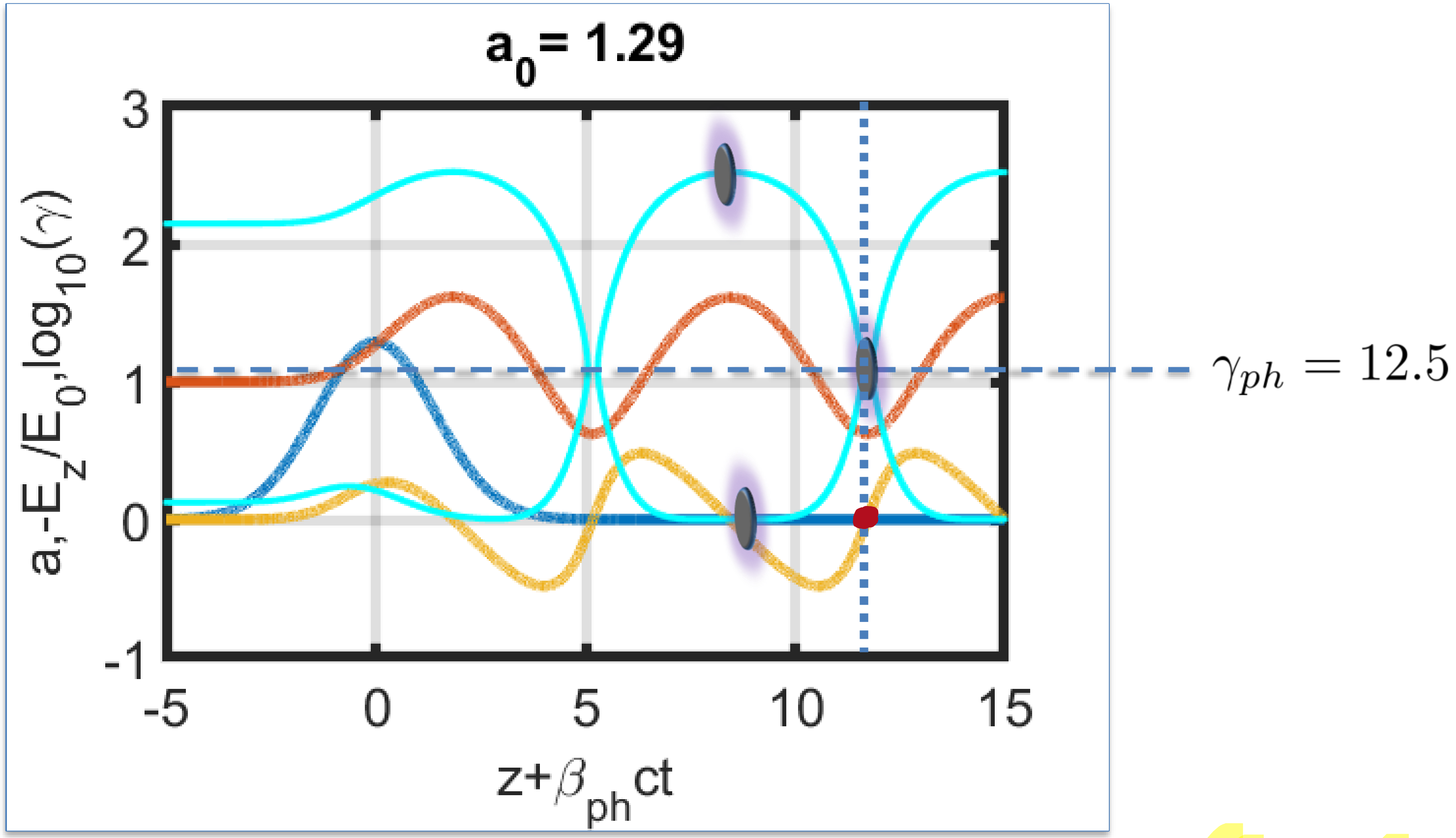
$\gamma_{ph} = 12.5$

Trapping (and acceleration) theory in STABLE 1D wakefields

!

Weak Trapping

Branches merging



$$u_z(\xi) = \gamma_{ph}^2 (h_0 + \phi) \left\{ -\beta_{ph} \pm \sqrt{1 - \frac{\gamma_\perp^2(\xi)}{\gamma_{ph}^2 (h_0 + \phi(\xi))^2}} \right\}$$

The radical is zero!

Trapping (and acceleration) theory in STABLE 1D wakefields

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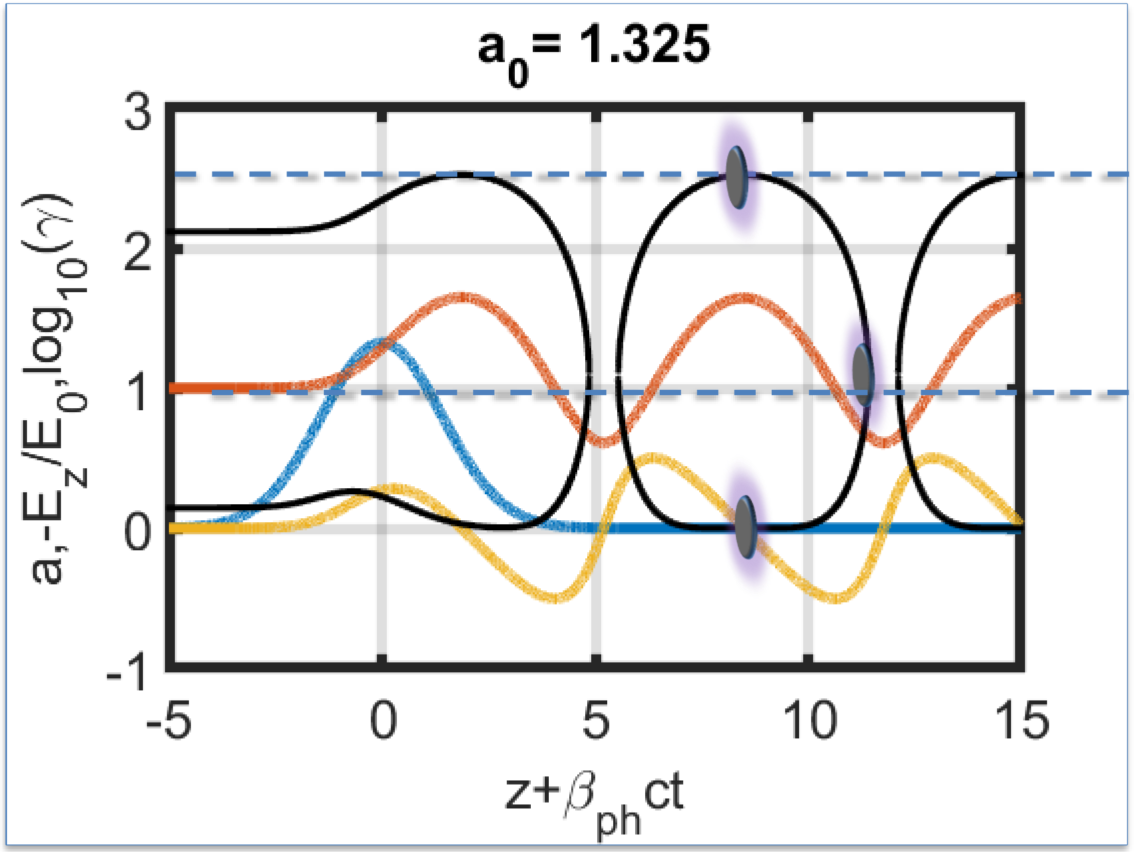
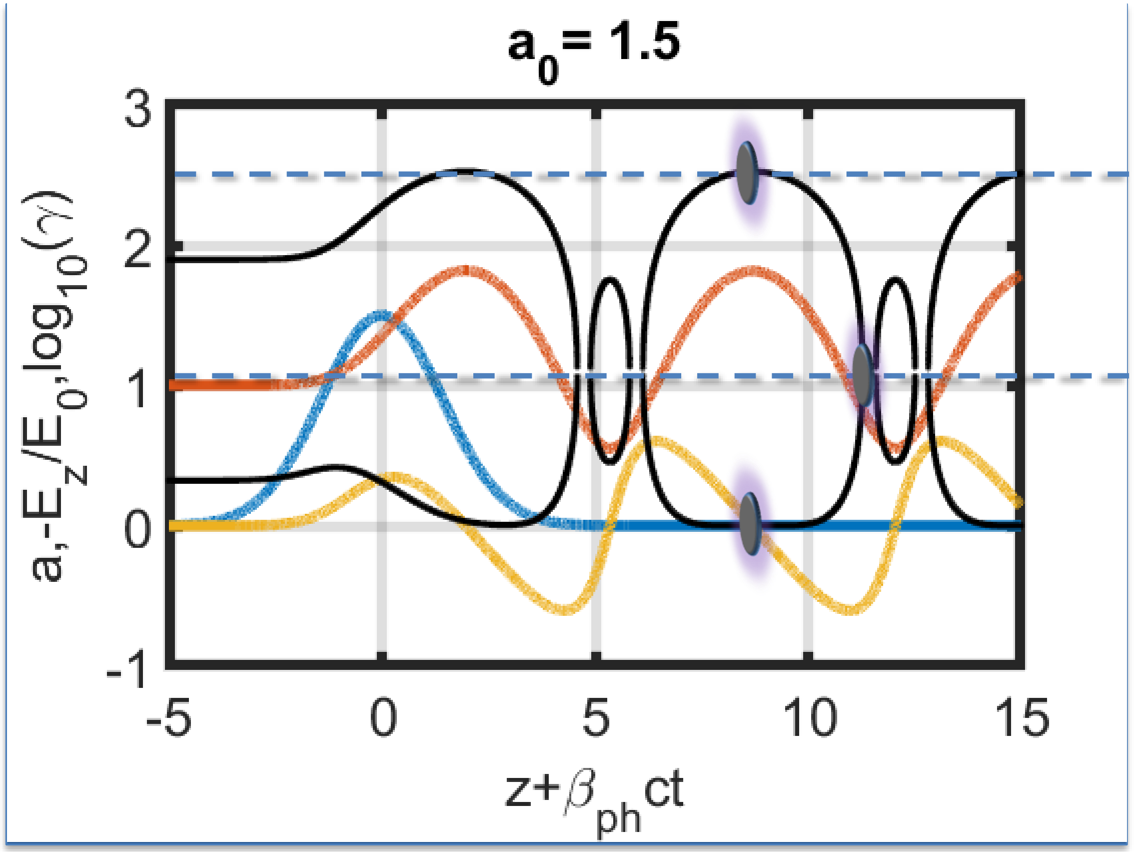


$\gamma_{max} \simeq 300$

$\gamma_{ph} = 12.5$

Trapping (and acceleration) theory in STABLE 1D wakefields

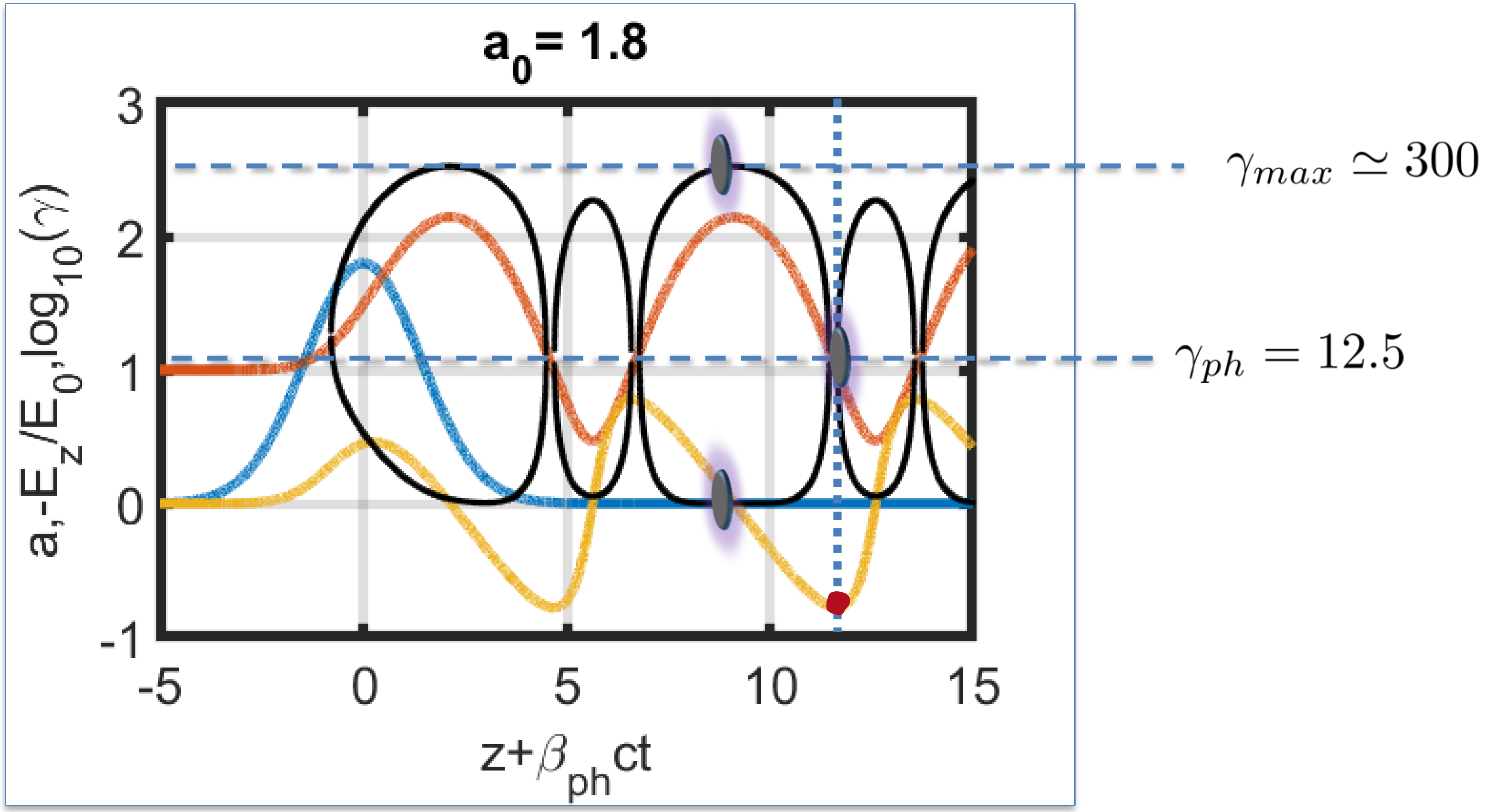


$\gamma_{max} \simeq 300$

$\gamma_{ph} = 12.5$

Trapping (and acceleration) theory in STABLE 1D wakefields

Strong Trapping



Trapping (and acceleration) theory in STABLE 1D wakefields

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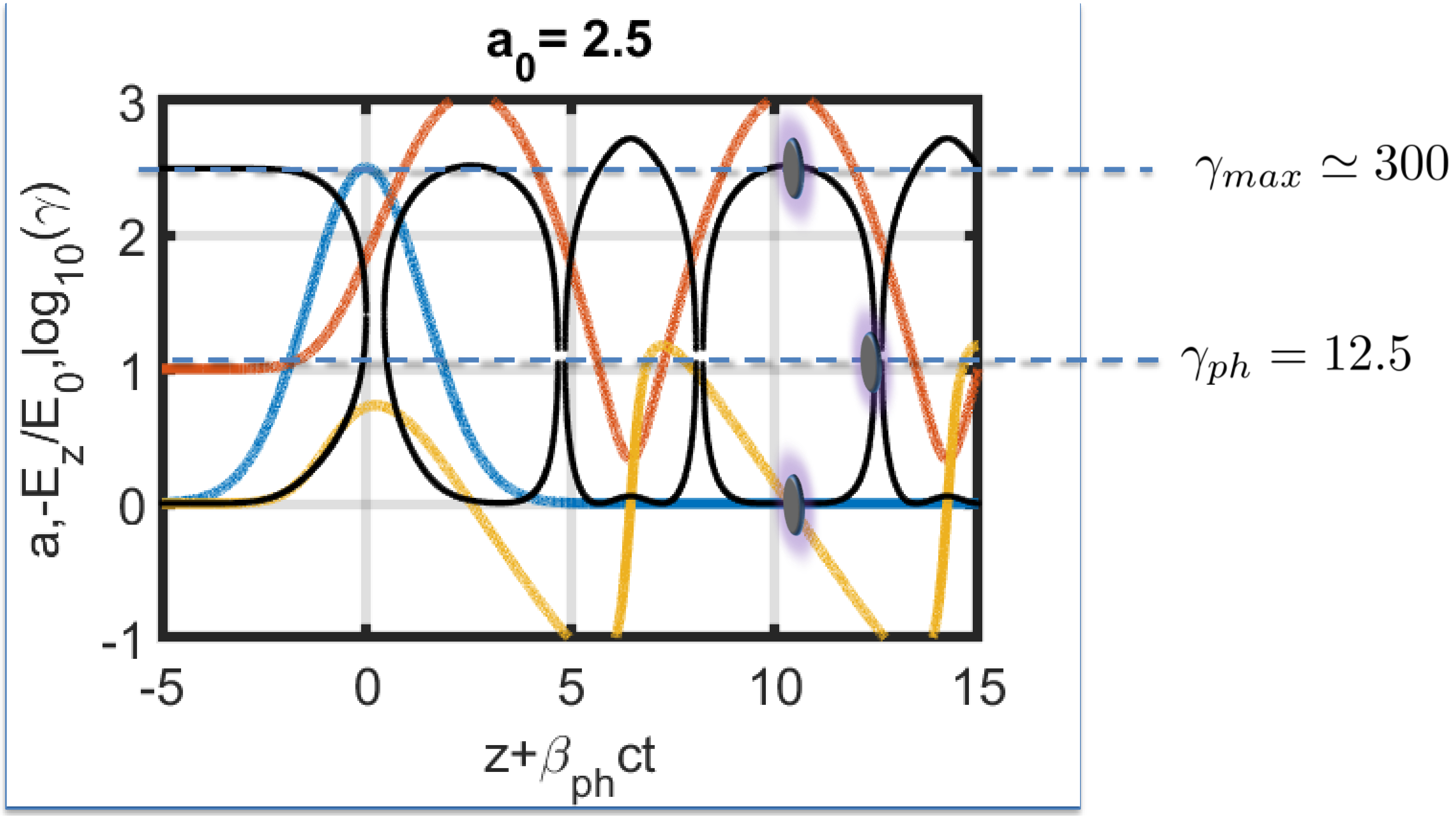

Trapping (and acceleration) theory in STABLE 1D wakefields

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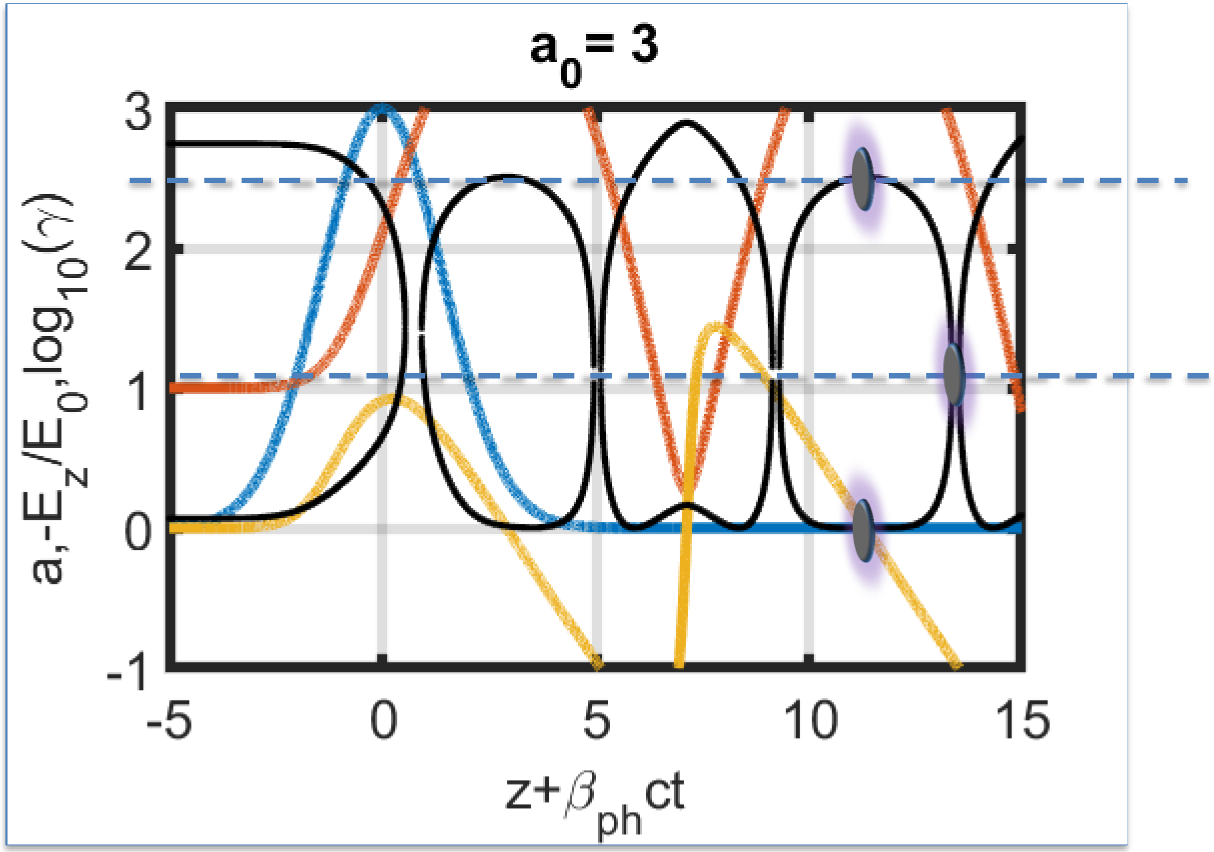


$\gamma_{max} \simeq 300$

$\gamma_{ph} = 12.5$

Trapping (and acceleration) theory in STABLE 1D wakefields

MAXIMUM ENERGY GAIN

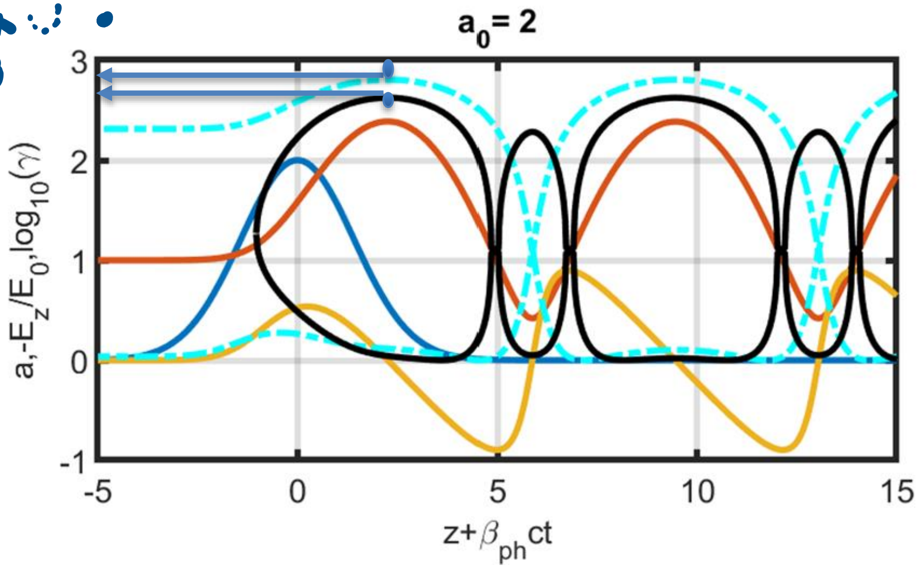
Starting again with the **energy trajectory**:

$$\gamma(\xi) = \gamma_{ph}^2(h_0 + \phi)\left\{1 \pm \beta_{ph}\sqrt{1 - \frac{\gamma_\perp^2(\xi)}{\gamma_{ph}^2(h_0+\phi(\xi))^2}}\right\}$$

We got the maximum value (supposing $\gamma_{ph} \gg 1$

$$\gamma_{MAX} = max\left(\gamma_{ph}^2(h_0 + \phi)\left\{1 + \beta_{ph}\sqrt{1 - \frac{\gamma_\perp^2(\xi)}{\gamma_{ph}^2(h_0+\phi(\xi))^2}}\right\}\right)$$

10^10 !



It depends on the initial conditions, of course

$$\gamma_{MAX} \simeq 2\gamma_{ph}^2 max(h_0 + \phi)$$

$$\gamma + \beta_{ph}u_z - \phi = (\gamma + \beta_{ph}u_z - \phi)_0 \equiv h_0$$

Trapping (and acceleration) theory in STABLE 1D wakefields

MAXIMUM ENERGY GAIN

$$\gamma + \beta_{ph} u_z - \phi = (\gamma + \beta_{ph} u_z - \phi)_0 \equiv h_0$$

$$\gamma_{MAX} \simeq 2\gamma_{ph}^2 max(h_0 + \phi)$$

ϕ_{MAX}

$$h_0 \leq h_{0,tr} = \gamma_\perp/\gamma_g - \phi_{min}$$

The maximum possible value, compatible with trapping

Therefore, the best trajectory is the separatrix

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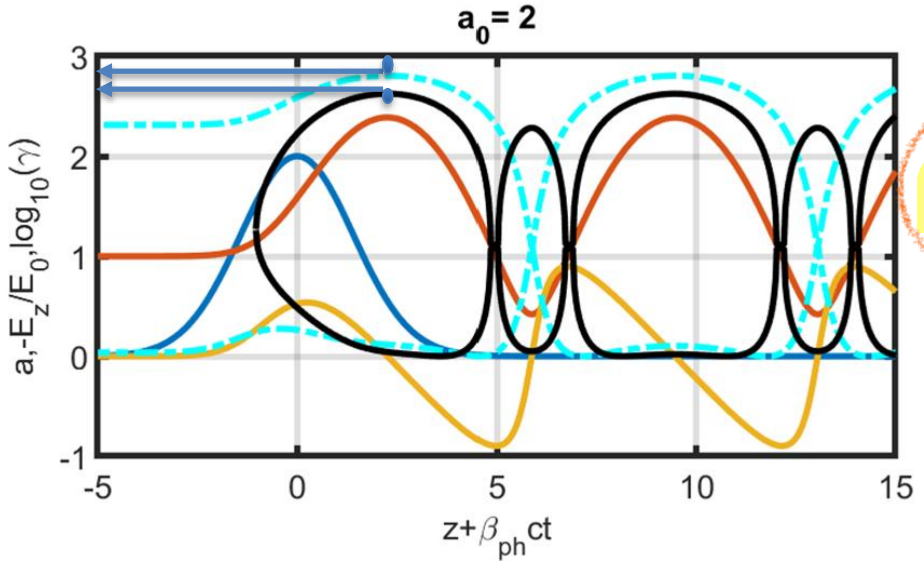


$$\gamma_{MAX} \simeq 2\gamma_{ph}^2 (\phi_{MAX} - \phi_{min})$$

Of course!
It takes advance of all the allowable potential difference

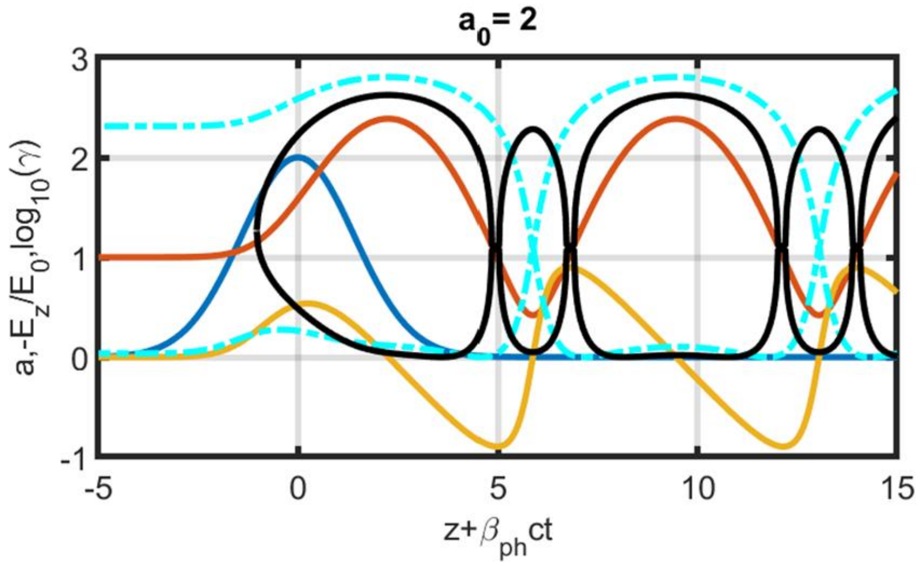
Trapping (and acceleration) theory in STABLE 1D wakefields

MAXIMUM ENERGY GAIN

$$\Psi_{MAX} - \Psi_{min} = \phi_{MAX} - \phi_{min} = 2\beta_g \sqrt{(\hat{E}^2_{MAX}/2 + \gamma_\perp)^2 - \gamma_\perp^2}$$

Therefore the maximum energy gain is

$$\gamma_{MAX} \simeq 4\gamma_g^2 \sqrt{(\hat{E}^2_{MAX}/2 + \gamma_\perp)^2 - \gamma_\perp^2}$$

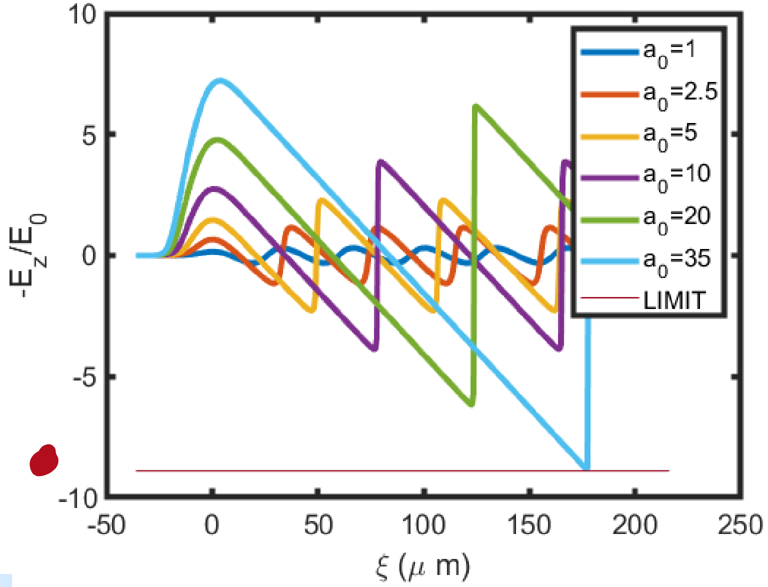


![ELI-NP Autumn School logo]

Trapping (and acceleration) theory in STABLE 1D wakefields

THE HIGHEST POSSIBLE ENERGY GAIN

$$\hat{E}_{MAX} \gg 1 \qquad \gamma_{MAX} \simeq 2\gamma_g^2 \hat{E}_{MAX}^2$$



@ wave breaking we have the highest E_z

$$\hat{E}_{MAX}^2 = 2(\gamma_g - 1)$$

1D Wavebreaking limit

$$n_e = 10^{17} cm^{-3}$$

$$E_{WB} \approx 15 E_0$$

$$\gamma_g = \omega_0/\omega_p = \sqrt{n_c/n_e} \simeq 100$$

$$\gamma_{MAX} \simeq 4\gamma_g^3 \approx 4 \cdot 10^6$$

$$\mathcal{E} \simeq 2TeV$$!

Unrealistic!

OUTLOOK

1. Introduction to LWFA
2. Understanding the excitation and the structure of the plasma waves
3. *The wide spot (1D and QSA) limiting case*
4. **<u>Limiting factors to high energy gain accelerators</u>**
5. 3D effects on
6. Downramp (or shock) injection
7. Two-Color injection
8. The Resonant Multi-Pulse Ionisation Injection
9. High-Brilliance e-bunches

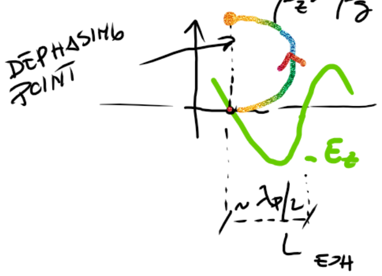
MAIN LIMING FACTORS FOR HIGH ENERGY GAIN

- The electric field cannot exceed the highest possible value sustainable by the wave without partial or complete wave breaking

$$E_{MAX} = E_0 \cdot \sqrt{2(\gamma_g - 1)}$$

- During the acceleration particles move advancing their position in the wake, after some while experiencing a decelerating field.

  The effective acceleration length is named DEPHASING LENGTH

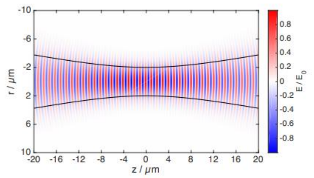


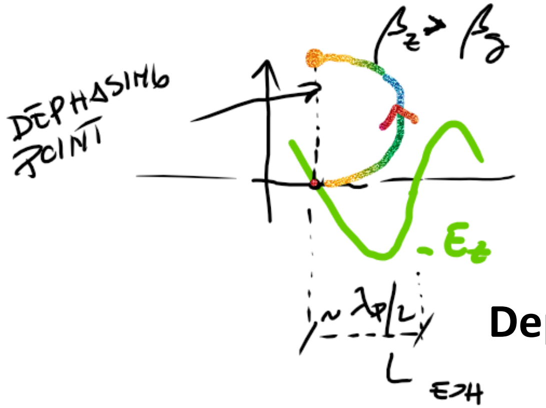
This is fully included in the 1D model

- The laser pulse propagates into the plasma and continuously delivers part of its energy on it after exciting the plasma wave. The energy decay length is named PUMP DEPLETION LENGTH

This is fully included in the 1D model that faces with the 1D pulse evolution [not shown in this lecture]

- The pulse diffracts and/or experience a series of linear and nonlinear 3D effects [more on next slides]





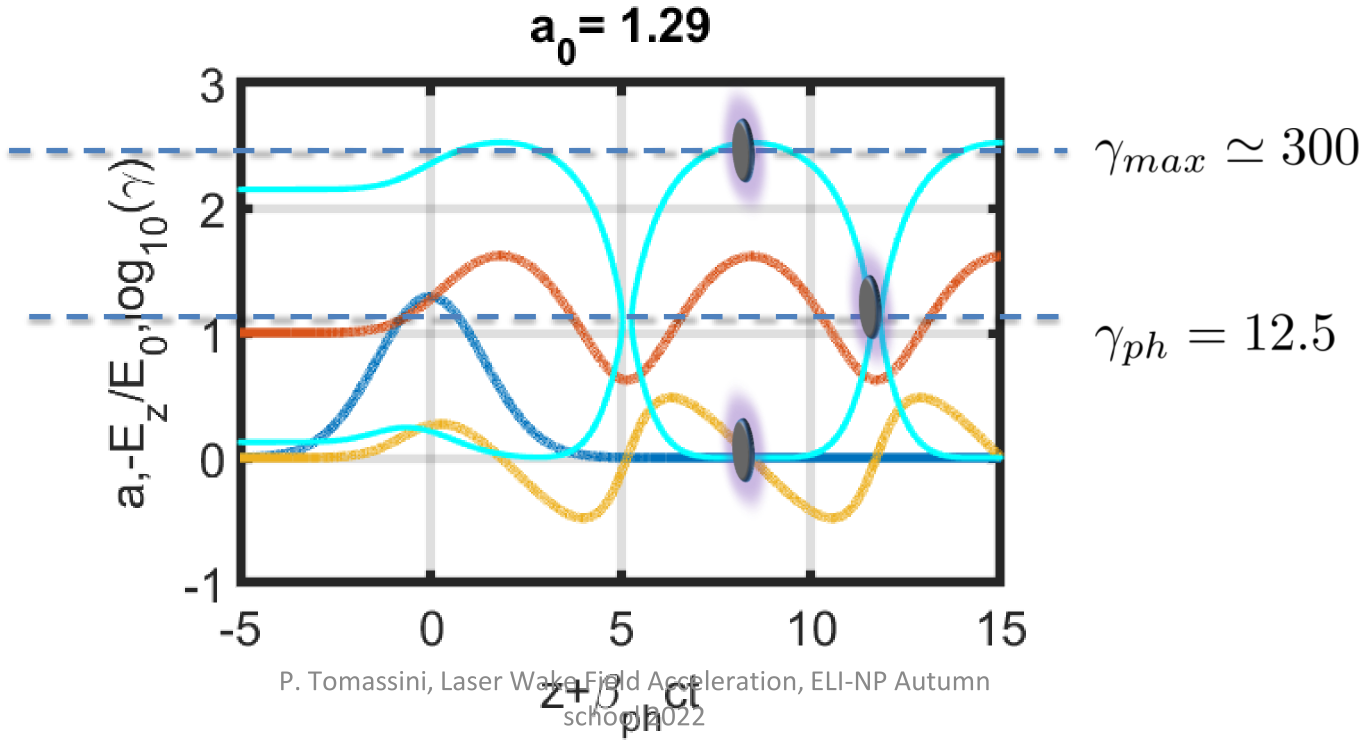
$L_{DEPH} = c\, t_{DEPH}$; $\delta\xi \simeq -\frac{\lambda_p}{2} = (\beta_g + \beta_z)\cdot c\, t_{DEPH}$

$\rightarrow$ $L_{DEPH} \simeq \frac{\lambda_p}{2(-\beta_z - \beta_g)}$

Dephasing length $L_D \approx \frac{\omega^2}{\omega_p^2}\lambda_p \simeq \gamma_{ph}^2 \lambda_p \propto n_0^{-3/2}$

In our example $n_e \simeq 1.7 \cdot 10^{19} cm^{-3}$ (quite a large density) $\lambda_p = 10\mu m; \rightarrow L_D \approx 2mm$

-

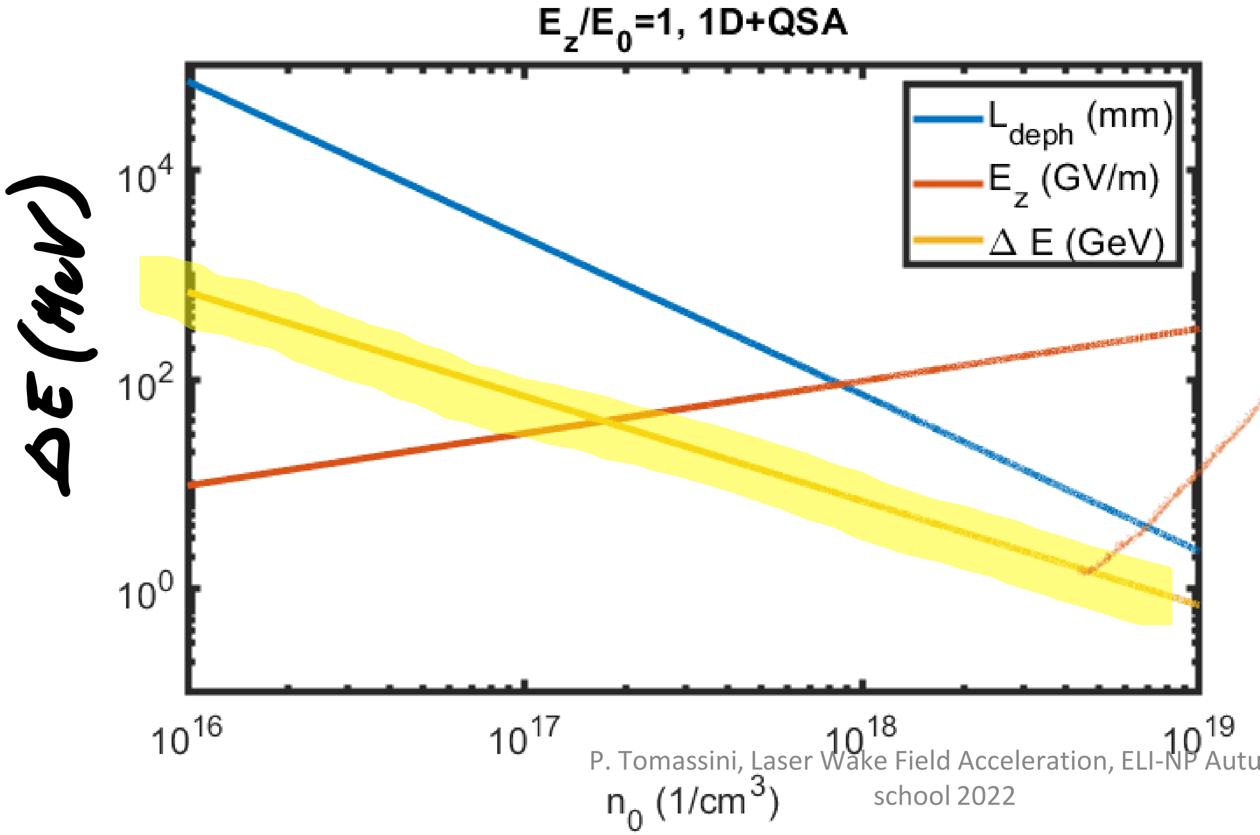


DEPHASING
as a limiting factor to high energy gain

Dephasing length $L_D \approx \frac{\omega^2}{\omega_p^2}\lambda_p \simeq \gamma_{ph}^2\lambda_p \propto n_0^{-3/2}$

Dawson field E0: $E_0[GeV] \simeq 96\sqrt{n_0[10^{18}cm^{-3}]}$

Energy gain (supposing a **very realistic mean field** $E_z \approx E_0$)



So, at the very end

$$\Delta E \propto n_0^{-1}$$

The final energy gain is just prop. to n_0^{-1} if the pulse propagated in a stable way!

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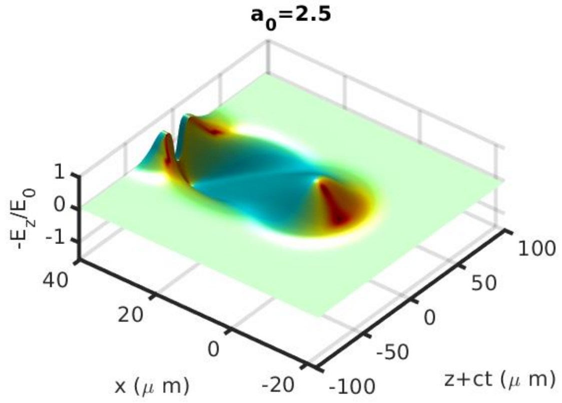
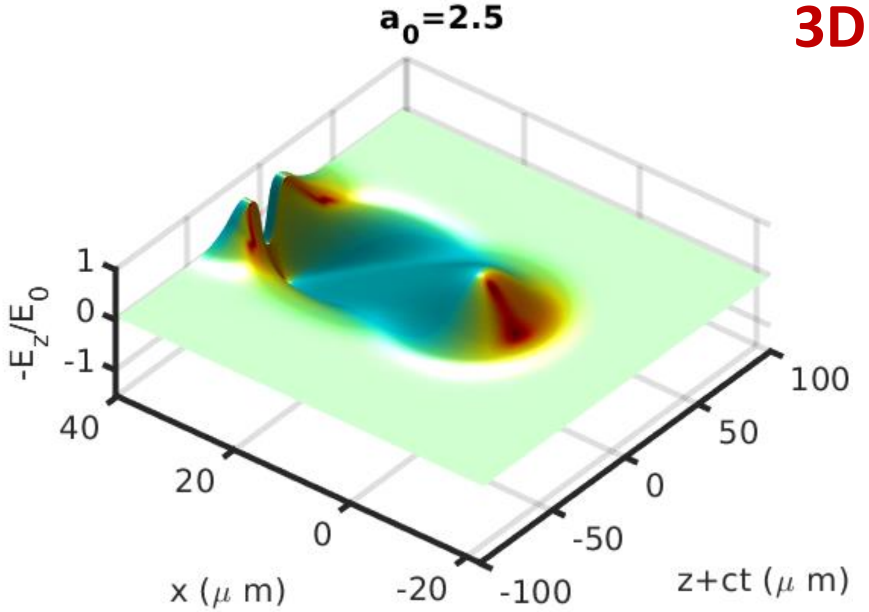
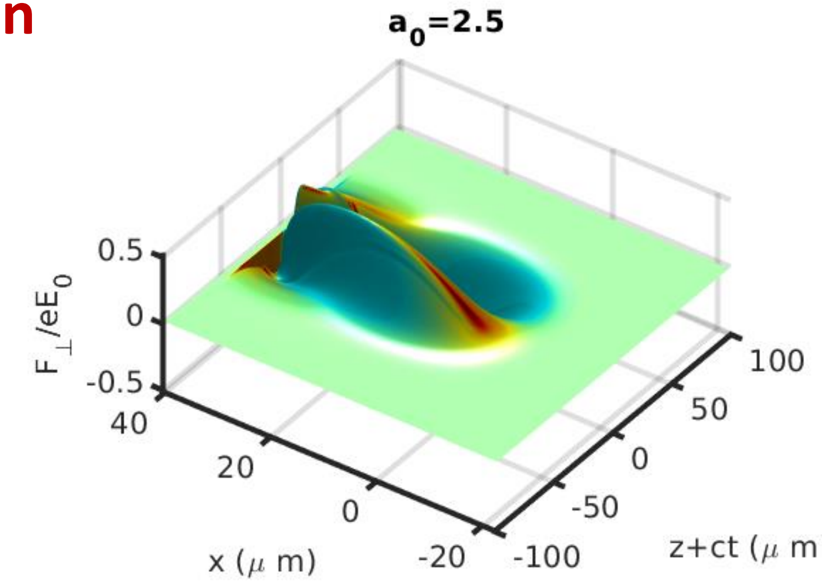


3D effects on

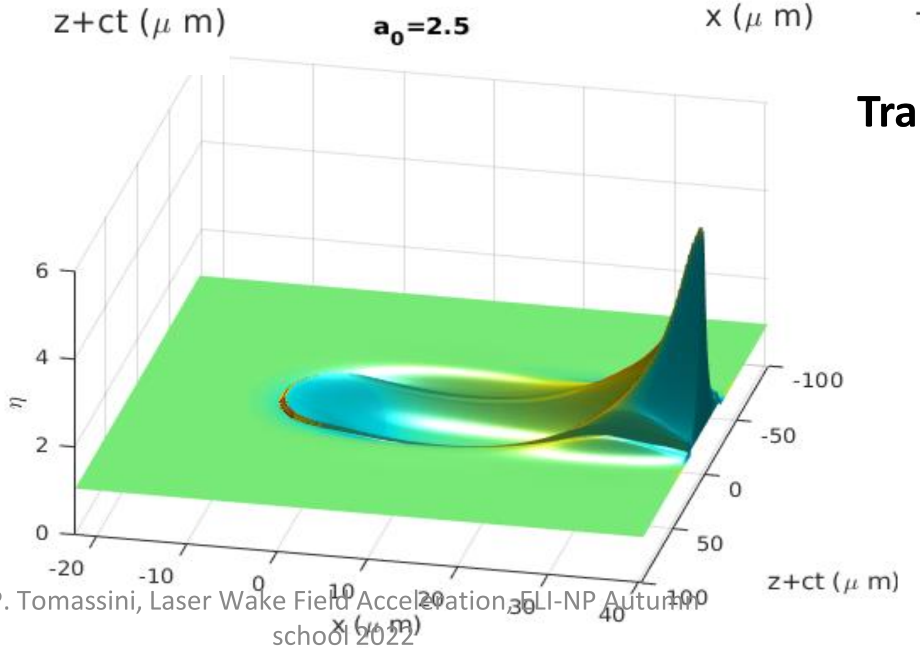
3D effects on



Longitudinal E



Transverse force



Plasma response

3D effects on

3D Effects on laser pulse propagation

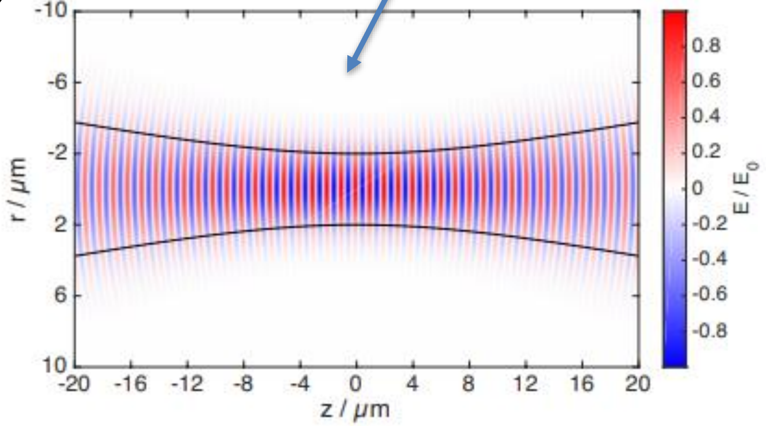
Basically **3D**

$$\left\{\nabla_\perp^2 + \frac{1}{\gamma_g^2}\partial_\xi^2 + 2\beta_g^2\partial_\xi\partial_{z_g} - \beta_g^2\partial_{z_g}\right\} a = k_p^2 \eta a$$

Plasma response [Current]

$\xi = k_0(z - z_g)$

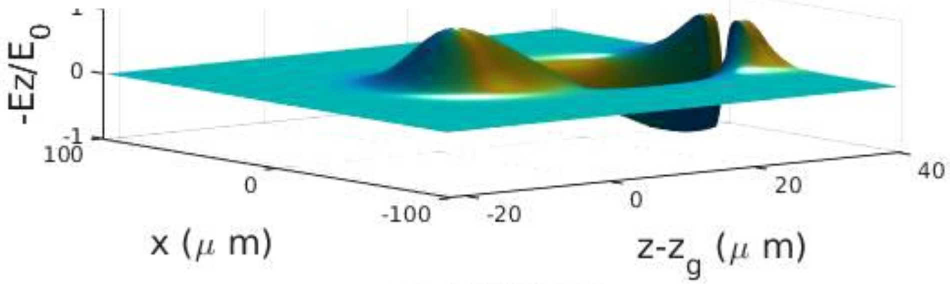
$\eta = \frac{n}{n_0 \gamma}$

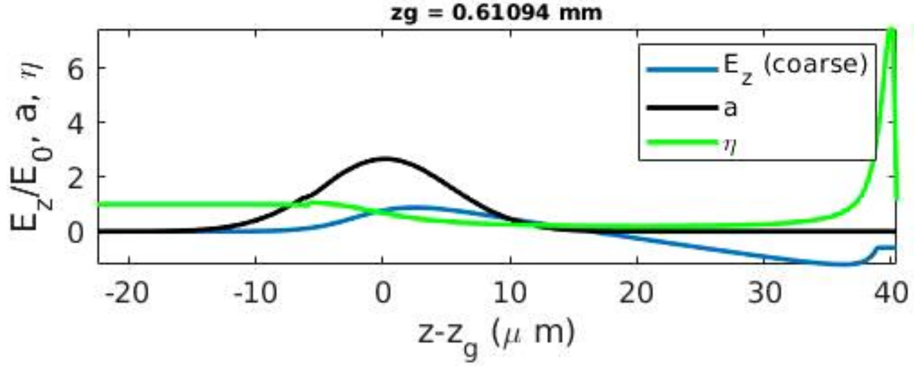


$$Z_R = \pi w_0^2/\lambda$$

$$w(z) = w_0\sqrt{1 + (z/Z_R)^2}$$

$$I(z) = I_0 \frac{1}{1+(z/Z_R)^2}$$





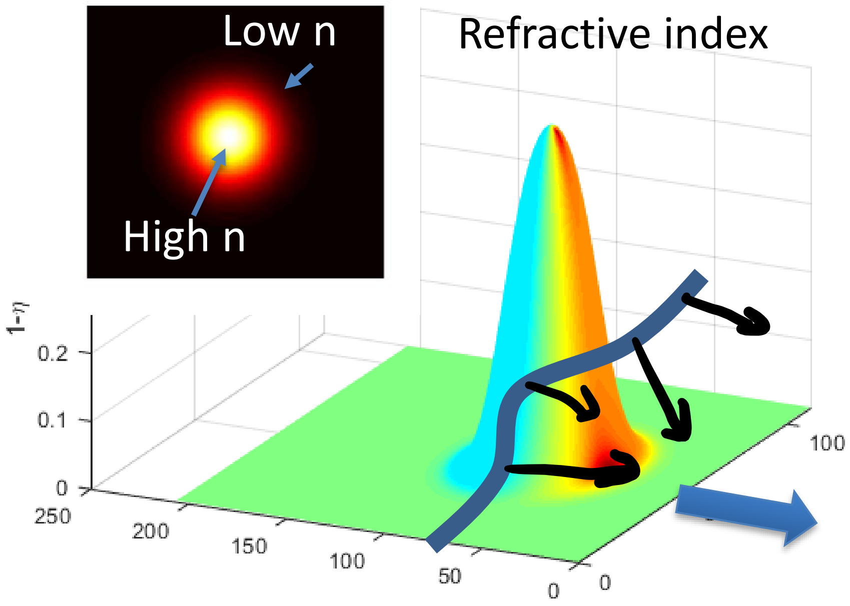
NOTE:
From now on simulations are performed in a moving window with v=-c

3D effects on

RELATIVISTIC SELF-FOCUSING

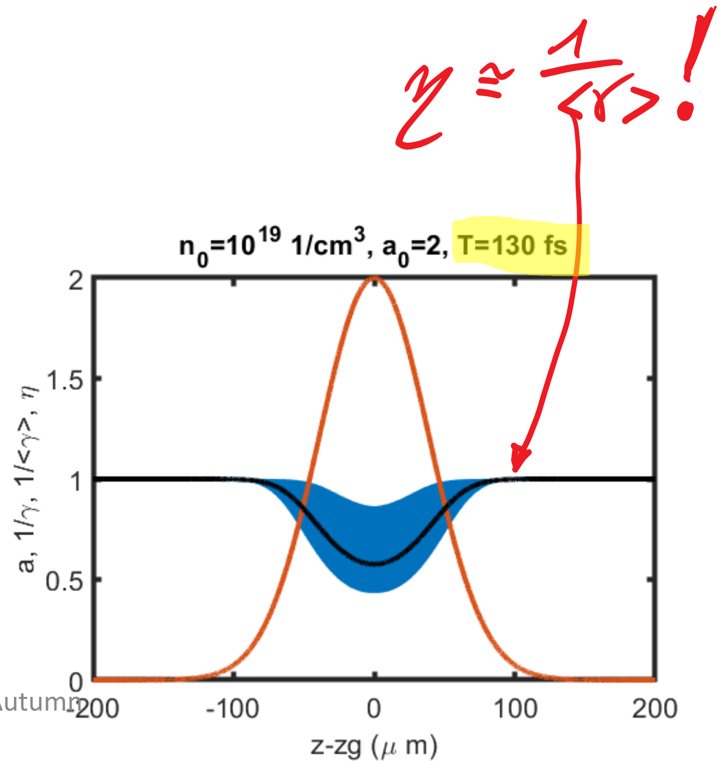
A nonlinear plasma response can arise from the «**relativistic mass increase**» caused by quivering. This, in turn, generates a **POSITIVE** lens effect…

$$\eta = n/(n_0\gamma) \simeq 1/\gamma = 1/\sqrt{1+a^2/2}$$



$$n = c/v_\phi = \sqrt{1 - \frac{n_e}{\gamma n_c}}$$

$\approx$

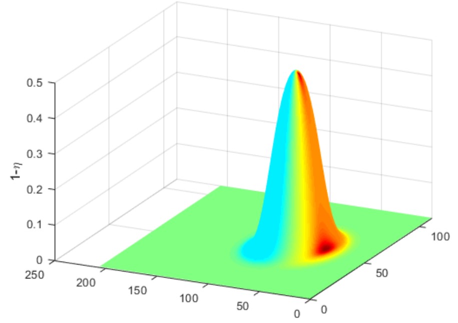


3D effects on

RELATIVISTIC SELF-FOCUSING

$$\eta = n/(n_0\gamma) \simeq 1/\gamma = 1/\sqrt{1+a^2/2}$$

$$\left(\nabla_\perp^2 - k_p^2 \delta\eta \right) a \cong 0$$



Relativistic Power Threshold

Since $k_p = k_0/\gamma_g$ and $P[GW] = 0{,}545 (a w k_0)^2$ we get :

$$P_{CRIT}[GW] \cong 17 \cdot \gamma_g^2 = 17 \left(\frac{\mu_c}{\mu_0} \right)$$

I'ts a matter of **POWER**, not INTENSITY!

STABLE 3D PROPAGATION for matched relativistic self focusing

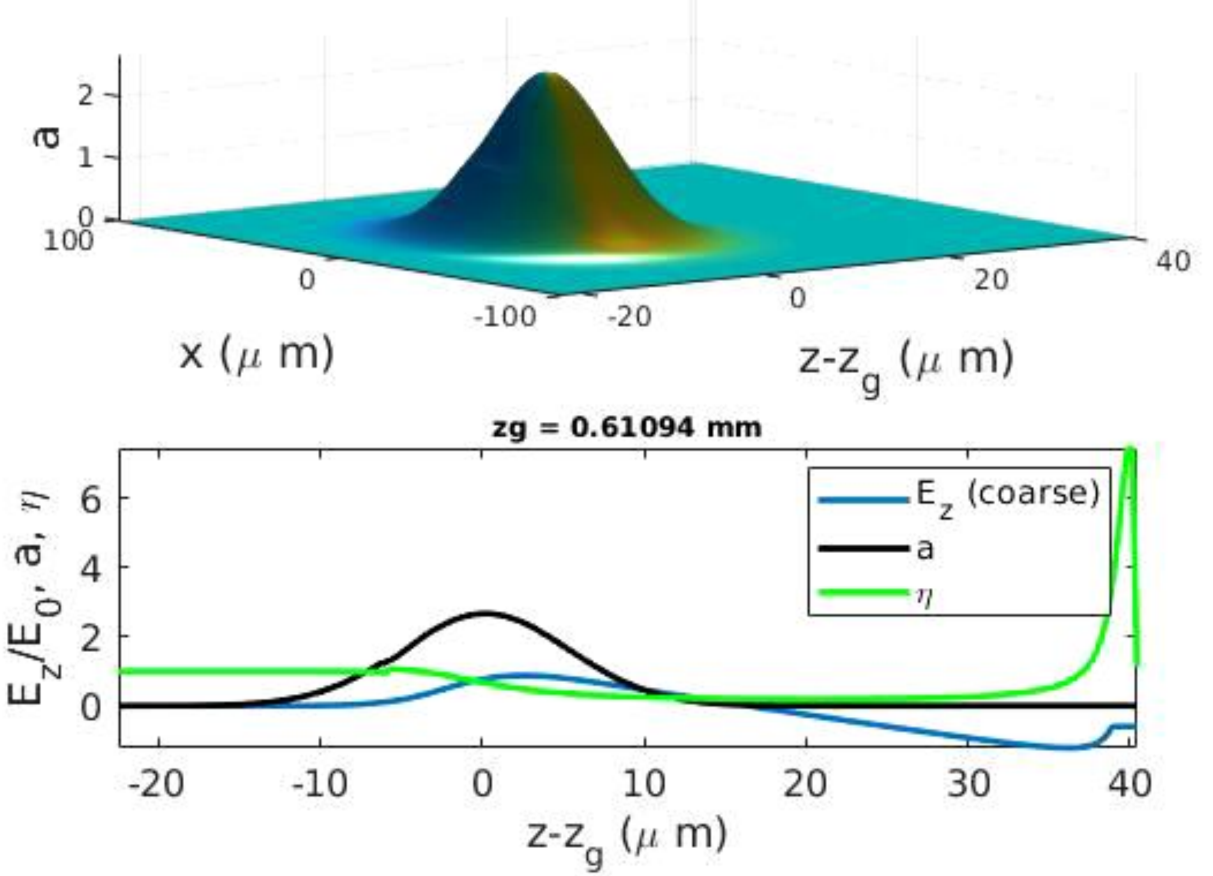
$$P_C(TW) = 1.7 \cdot 10^{-2} \gamma_{ph}^2$$

Example

$$n_e \simeq 1.7 \cdot 10^{19} cm^{-3} \quad \gamma_{ph} = 12.5$$

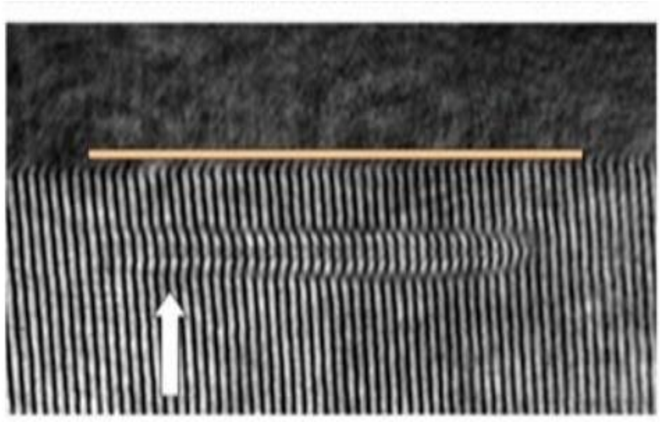
$$P_C(TW) \simeq 2.5 TW$$

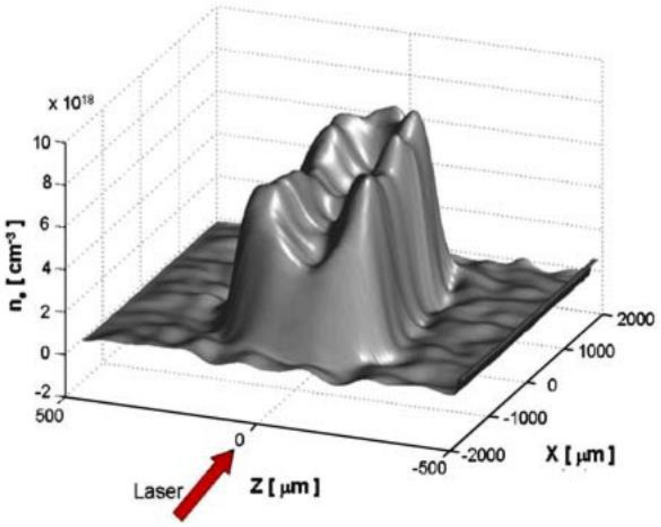
RELATIVISTIC SELF-FOCUSING+
PONDEROMOTIVE CHANNEL FOCUSING

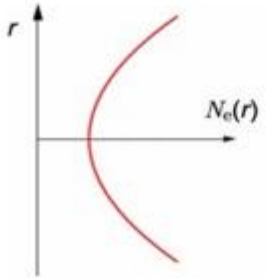


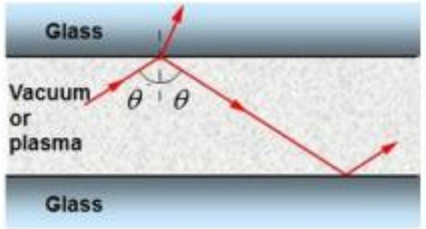
a0=2.4; n0=1e18 1/cm^3

PREFORMED CHANNEL FOCUSING









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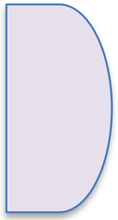


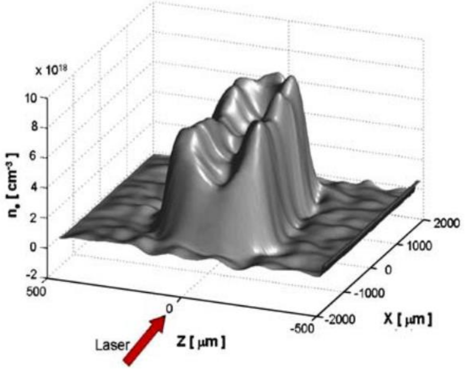
Preformed channels or capillaries act as a positive lens with **parabolic decrease** of the refraction index leaving the axis

3D effects on

PREFORMED CHANNEL FOCUSING

We have seen that the diffractive term acts as a **PARABOLIC** function on a Gaussian pulse,

$$\nabla_\perp^2 a = -\frac{4}{w^2}\left(1 - \frac{r^2}{w^2}\right) a$$



therefore, to compensate it we need a parabolic profile of the refractive index, too, which results in a radial density profile as

$$n(r) = n_0 \left(1 + \Delta \cdot r^2/w^2\right)$$

↳ DEPTH TO BE MATCHED!

$$\eta = \frac{n}{n_0 \gamma} \simeq 1 + \Delta\, r^2/w^2 \Rightarrow \delta\eta = \Delta\, r^2/w^2$$

($\gamma \sim 1$)

Matching occurs when

$$\frac{4}{w^2} \cdot \frac{r^2}{w^2} = k_p^2 \Delta\, r^2/w^2 \Rightarrow \Delta = \frac{4}{k_p^2 w^2}$$

$$\left(\nabla_\perp^2 - k_p^2 \delta\eta\right) a = 0$$

MATCHED CHANNEL

State-of-the-art for High-energy acceleration

A.j. Gonsalves et al.,

PHYSICAL REVIEW LETTERS **122**, 084801 (2019)

Laser power
P=800TW
(0.8PW)

8GeV in 20cm,
(40 GeV/m)

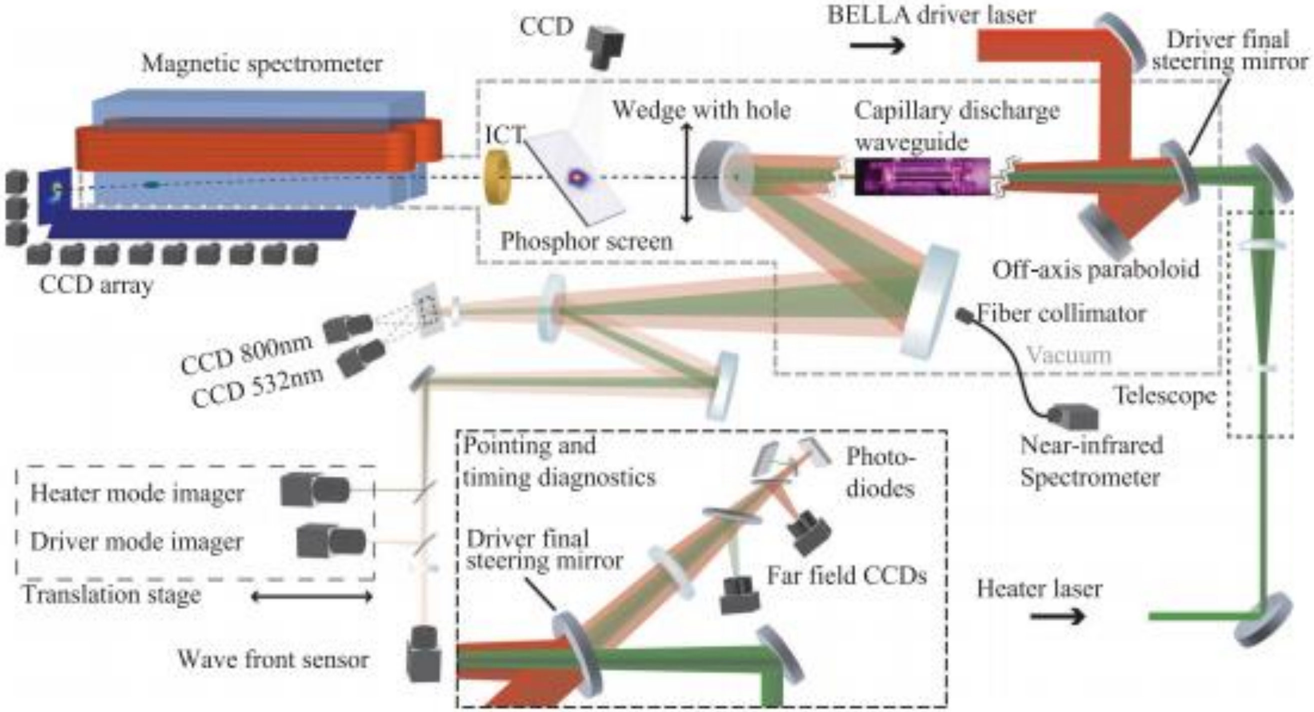
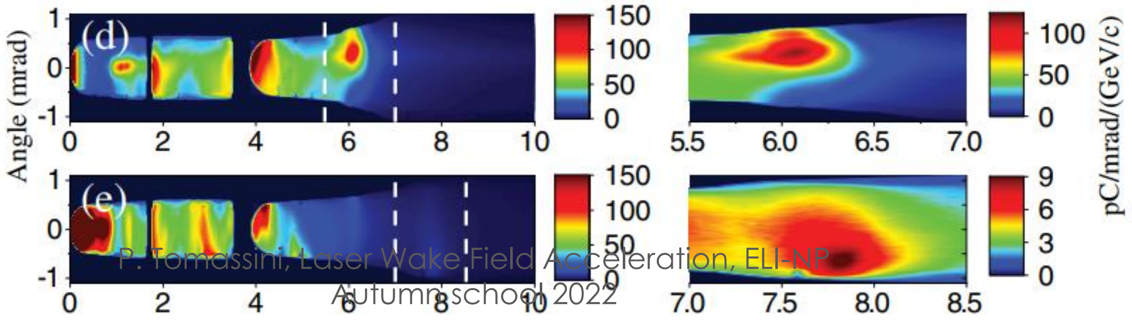


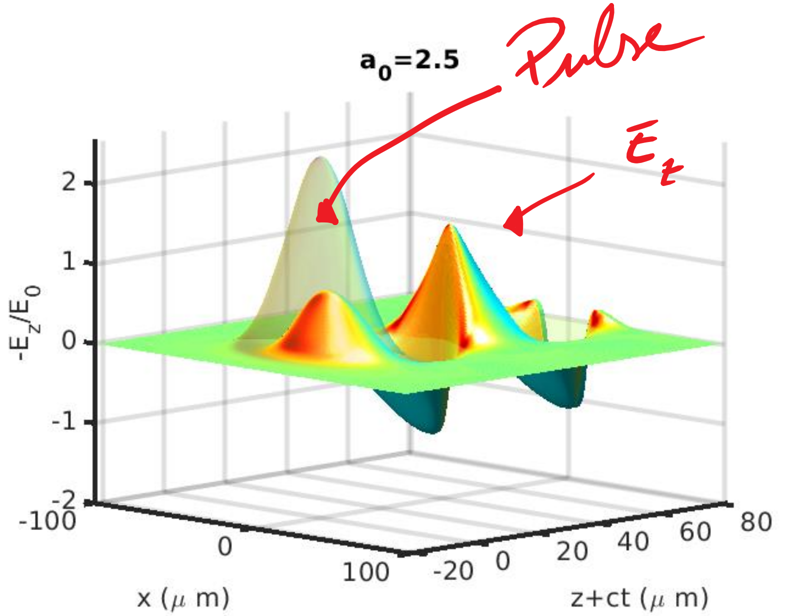
FIG. 2. Schematic layout of the BELLA LPA, including the heater laser system for enhancing the capillary discharge waveguide.



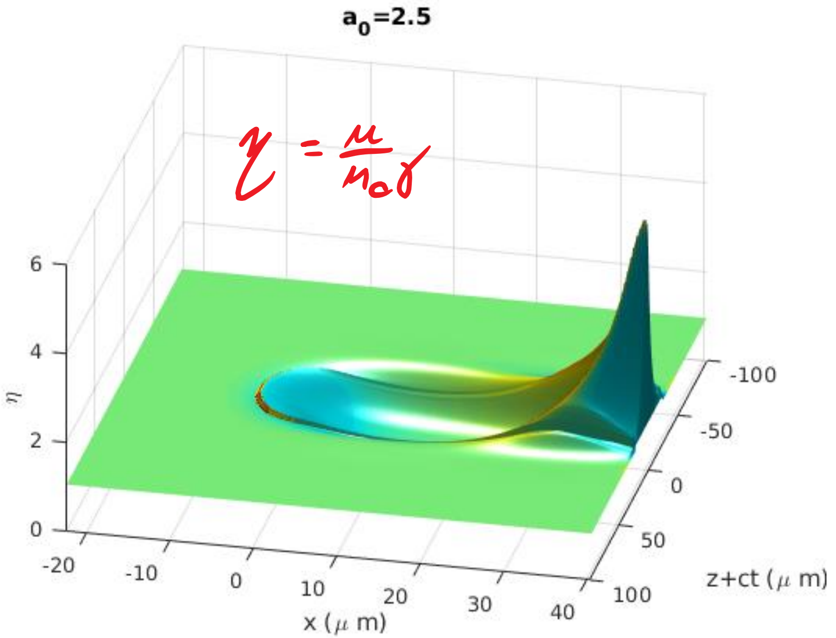
3D effects on

The equations of motion for the plasma are generally solved by means of simulations (there's an analytical solution, but the linear case solely).

This is because longitudinal and transverse motions do mix together **nonlinearly.**



What accelerated electrons see



What laser pulses see

3D effects ON

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~~$\nabla_\perp \ll \nabla_\parallel$~~

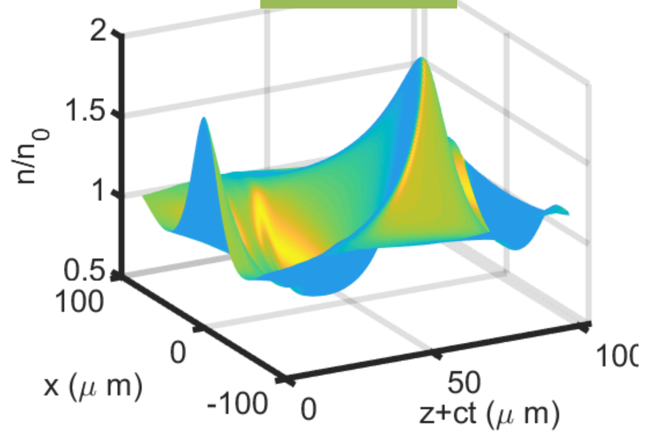
$$\frac{\partial}{c\partial t}(\vec{u}-\vec{a}) = \vec{\nabla}(\phi-\gamma) \qquad \vec{\nabla}\cdot(n\vec{u}/\gamma) + \frac{1}{c}\partial_t n = 0$$

$$\frac{\partial}{c\partial t}(\vec{u}_\perp-\vec{a}_\perp) = \vec{\nabla}_\perp(\phi-\gamma) \qquad \vec{\nabla}_\perp\cdot(n\vec{u}_\perp/\gamma) + \partial_z(nu_z/\gamma) + \frac{1}{c}\partial_t n = 0$$

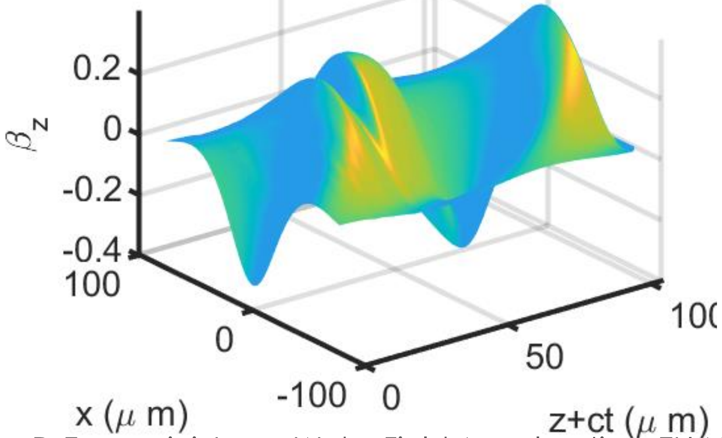
! ~~$(\vec{u}_\perp-\vec{a}_\perp) \simeq (\vec{u}_\perp-\vec{a}_\perp)_0$~~ No more conserved!

WHEN $k_p w_0 < 1$ A NUMERCAL APPROACH IS NECESSARY

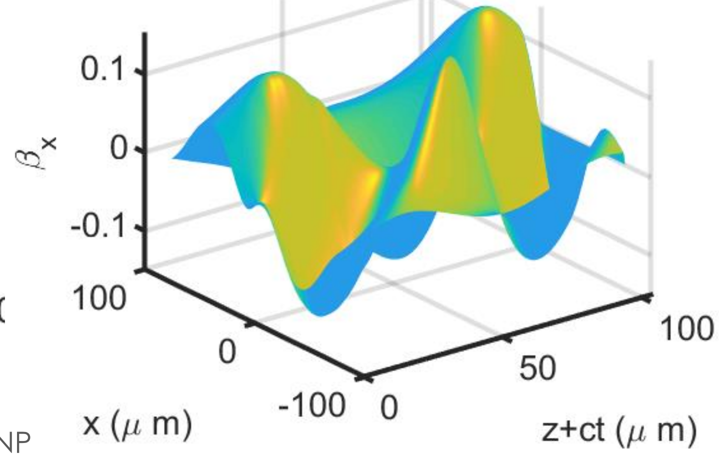
n/n_0



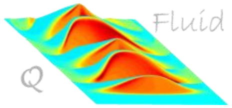
β_z



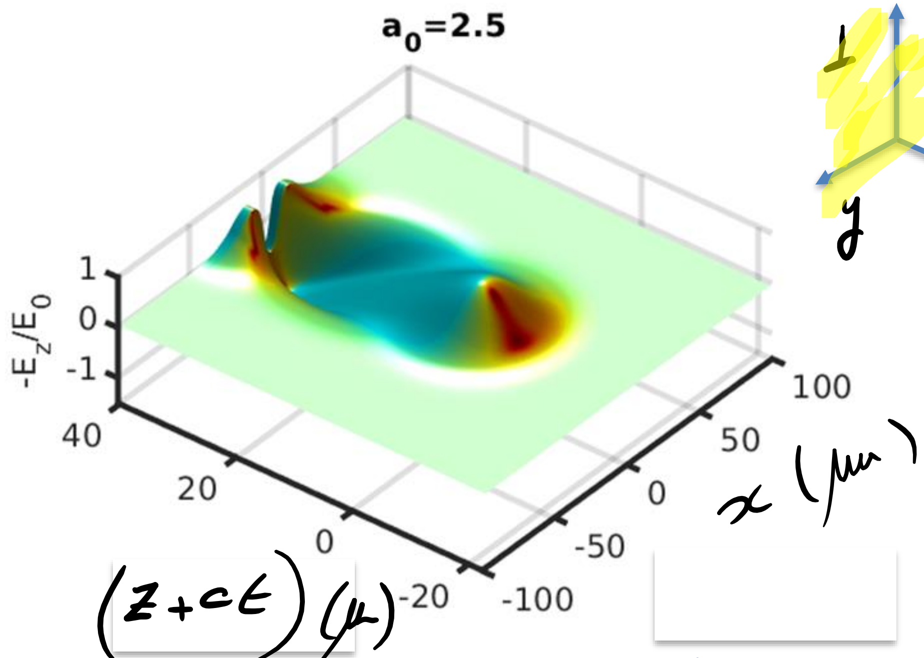
β_x



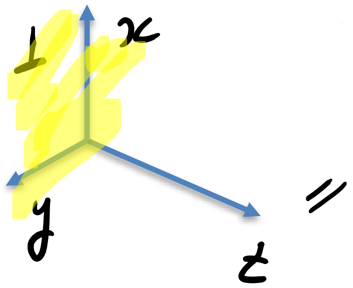
3D effects ON

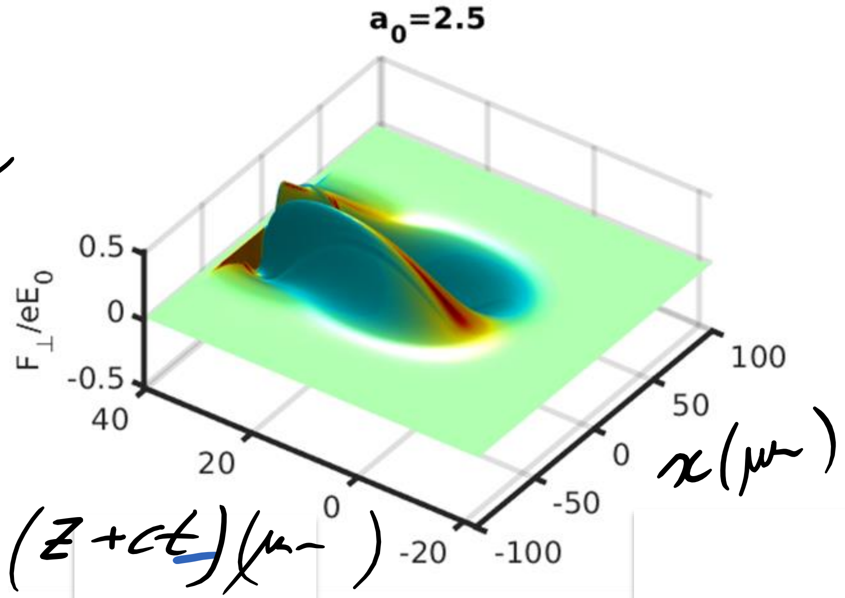


The (basically) 3D effect we never faced with is the **TRANSVERSE FORCE** (focusing/defocusing)

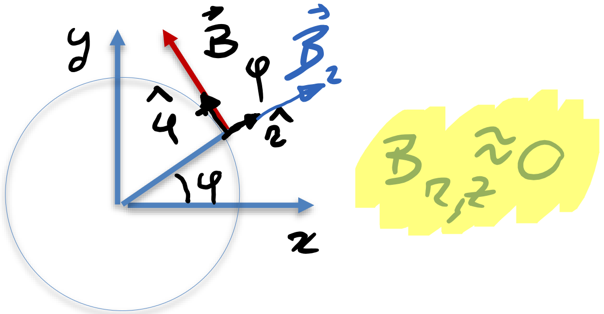


Longitudinal E





Transverse force



$$\vec F_\perp = -e\left(\vec E + \vec\beta \times \vec B\right)_\perp$$

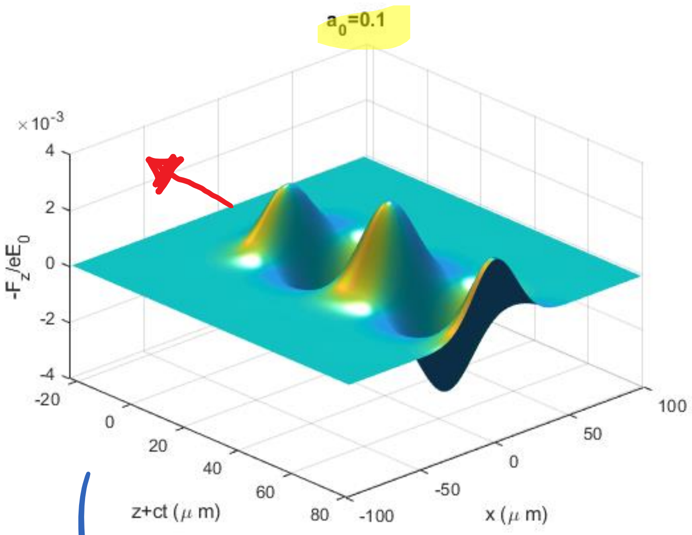
$$(\hat r, \hat\varphi, \hat z)$$

$$\perp \begin{cases} F_r = -e\left(E_r + \beta_\phi B_z - \beta_z B_\phi\right) \simeq -e\left(E_r - \beta_z B_\phi\right) \\ F_\varphi = -e\left(E_\varphi + \beta_z B_r - \beta_r B_z\right) = 0 \end{cases}$$

3D effects on

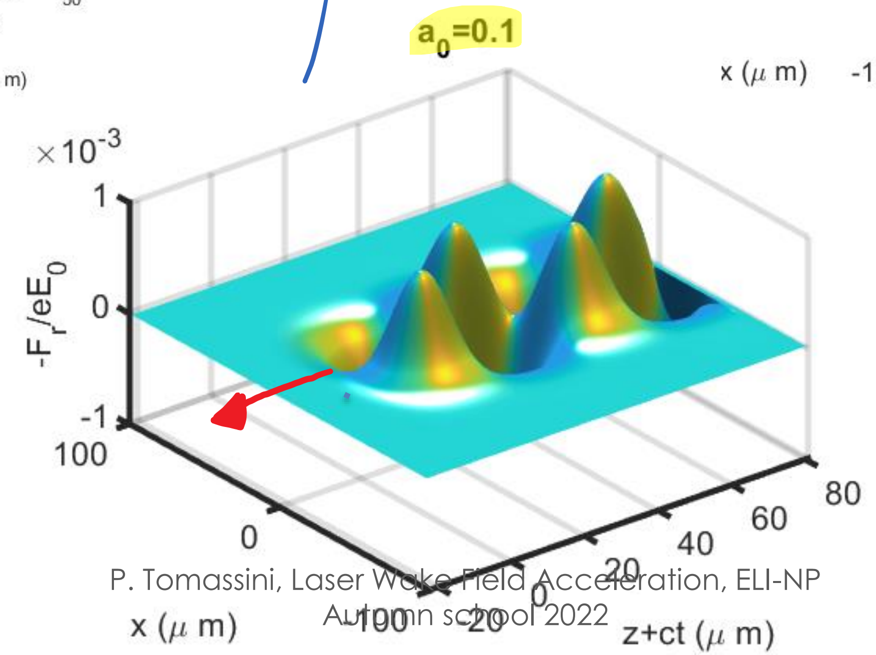
In a LINEAR REGIME the longitudinal profile is sinusoidal and plasma velocities are nonrelativistic

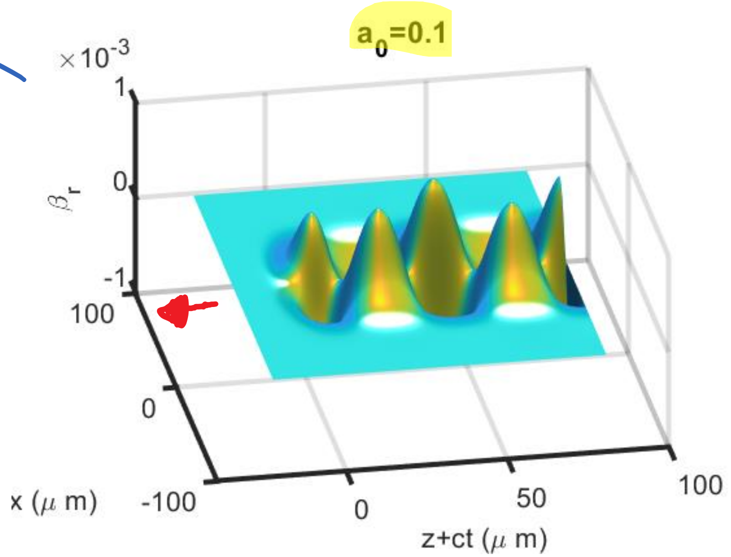
$\beta_z, \beta_r \ll 1$

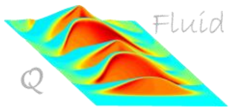


ACCELERATING FORCE

TRANSVERSE FORCE



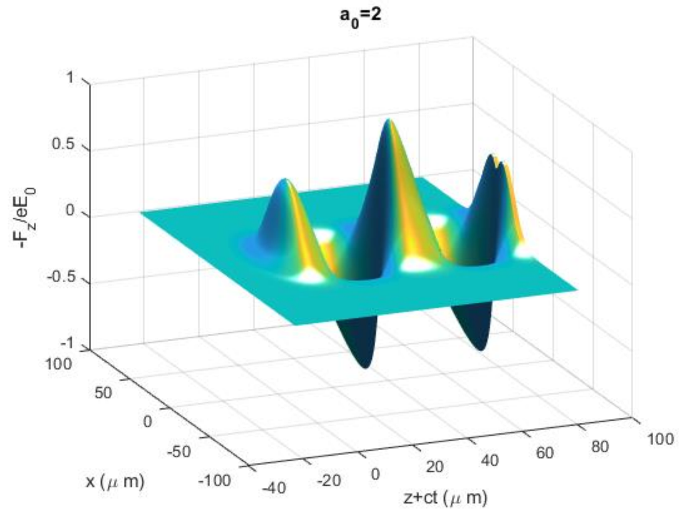


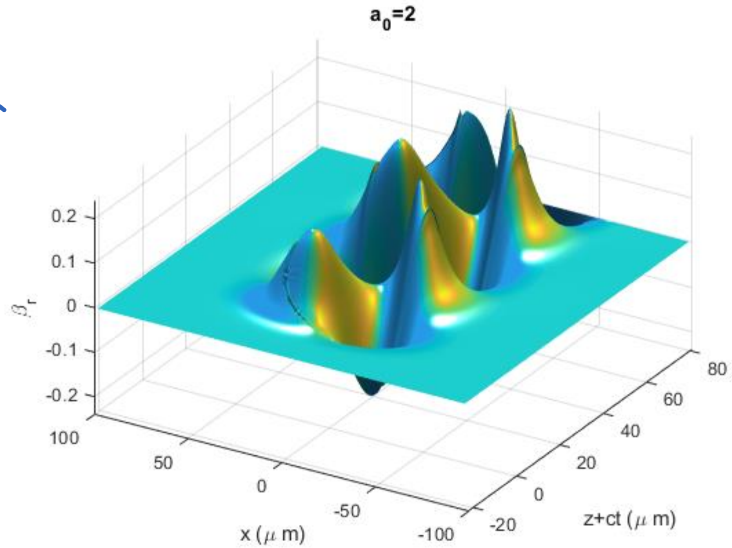


3D effects on

In a LINEAR REGIME the longitudinal profile is sinusoidal and plasma velocities are nonrelativistic

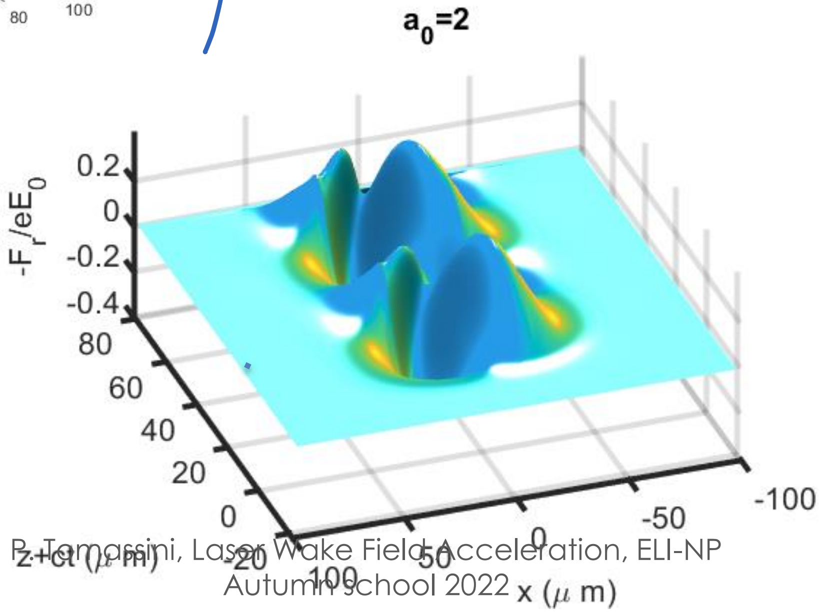
$\beta_z \leq 1$; $\beta_r << 1$

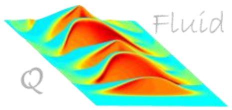




TRANSVERSE FORCE

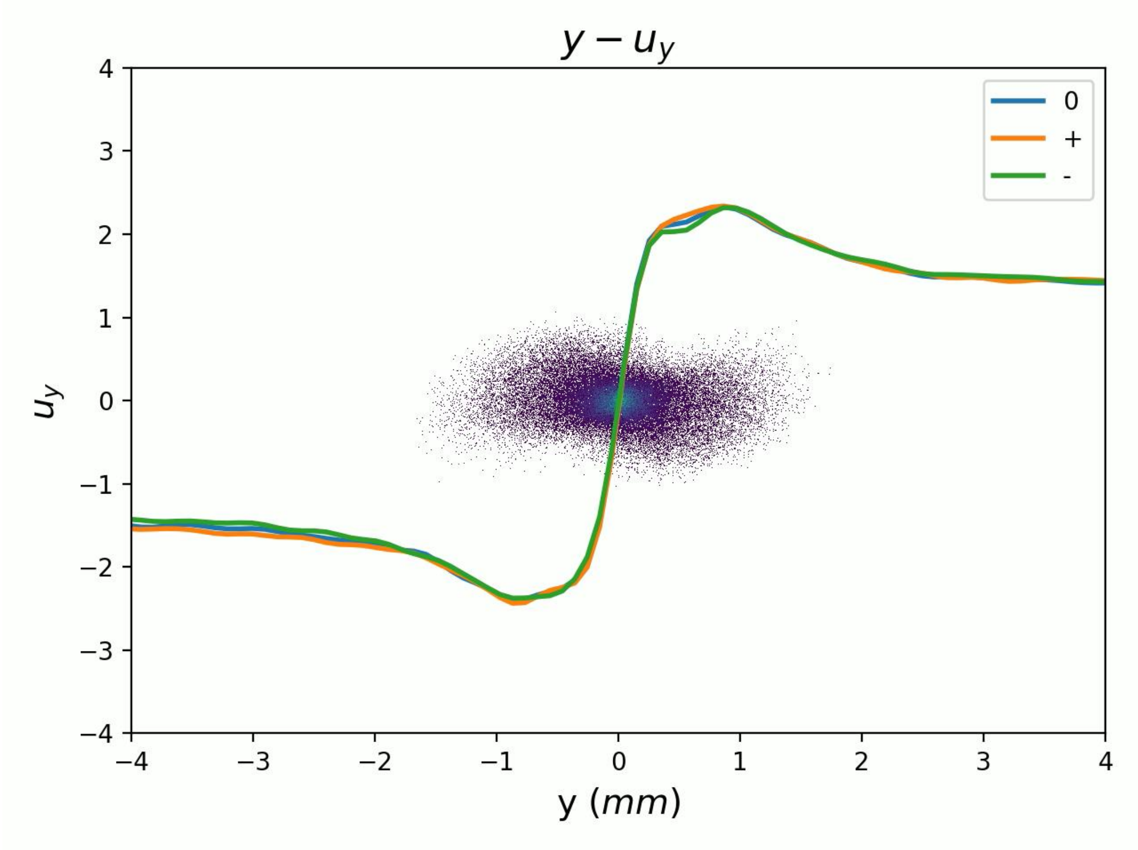
ACCELERATING FORCE



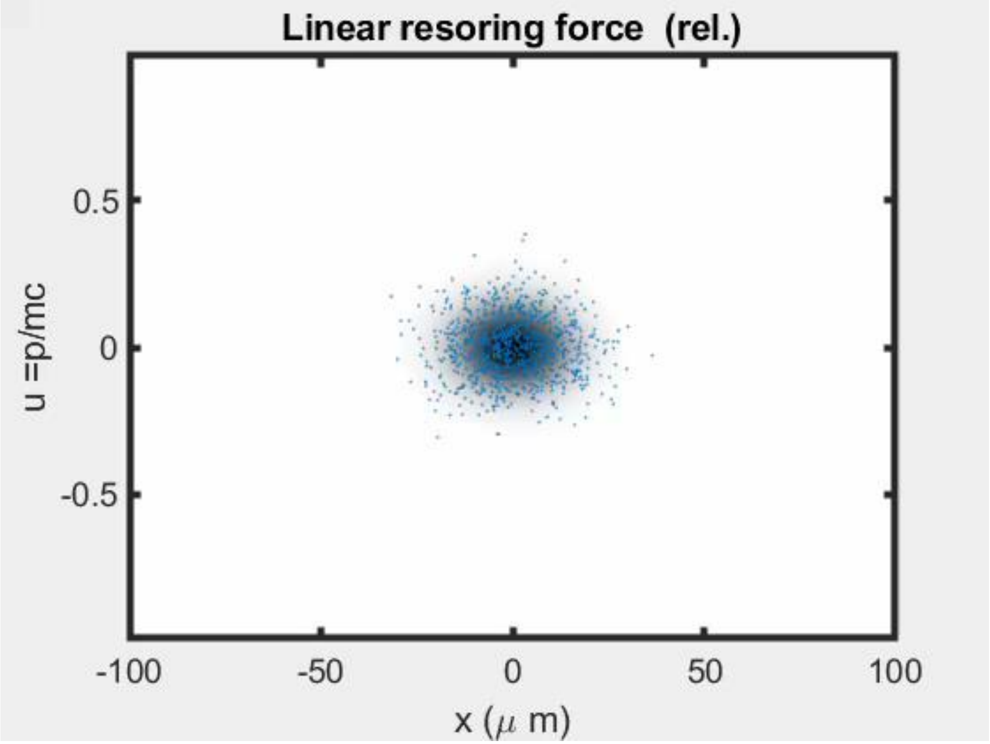


3D effects on

Transverse dynamics (PIC simulation)



EXAMPLE Linear restoring force **(F=-kx)**



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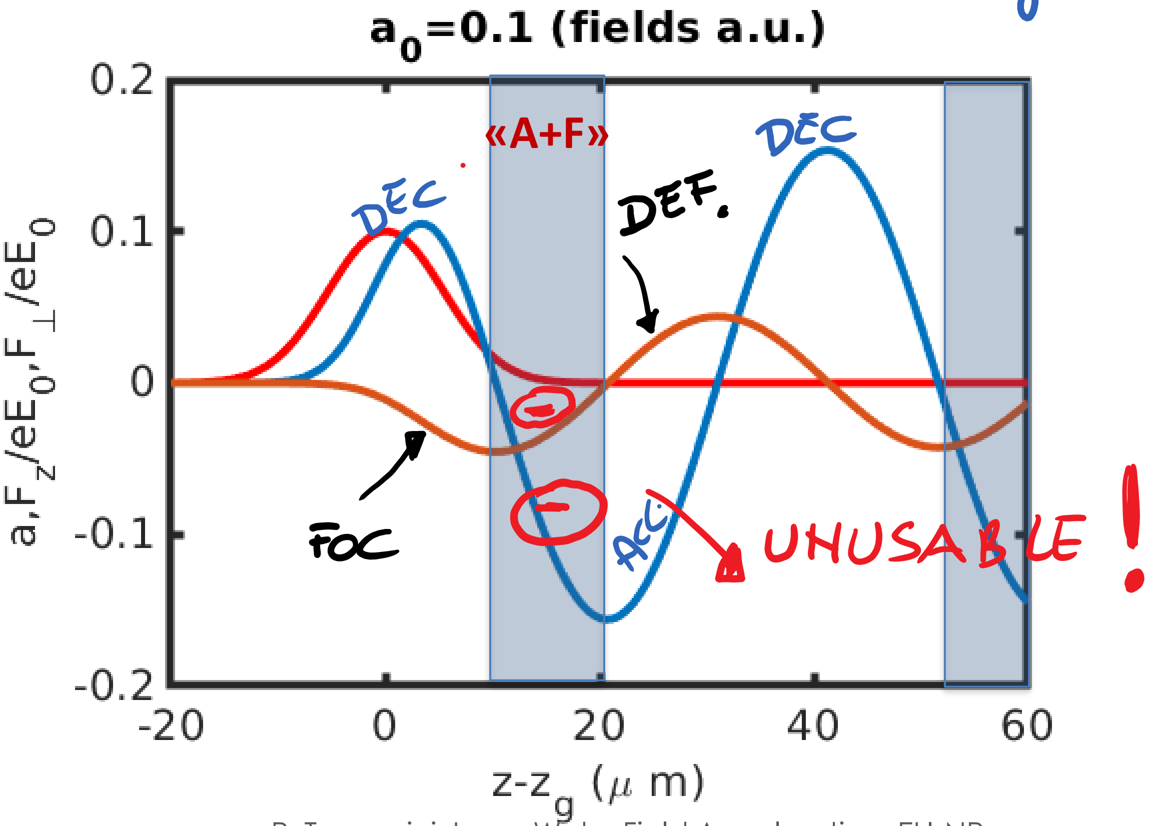
3D effects on

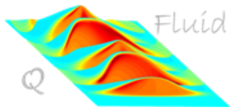
The «A+F» portion of the wave strongly depends on the regime!

ACCELERATING + FOCUSING

⊖ FOCUSING ACCELERATING

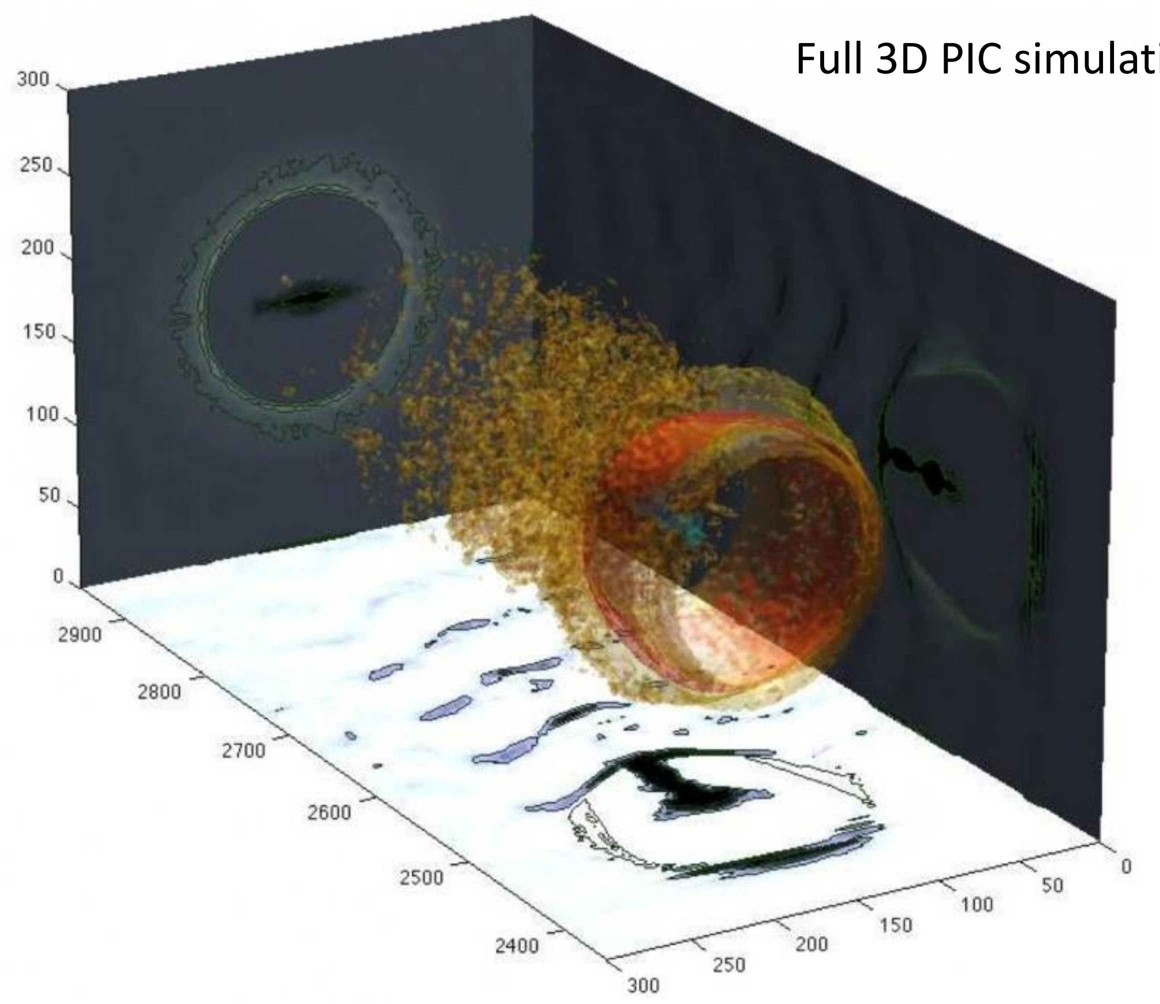
Linear regime





3D effects on

A typical scenario:



https://cuos.engin.umich.edu/researchgroups/hfs/research/theory-and-computation/

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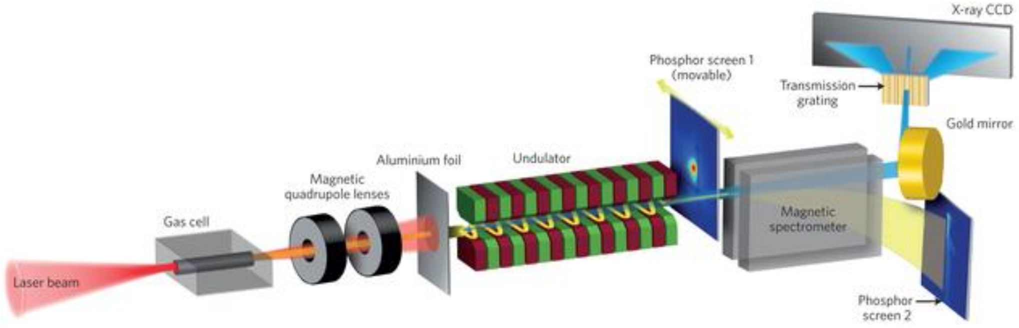
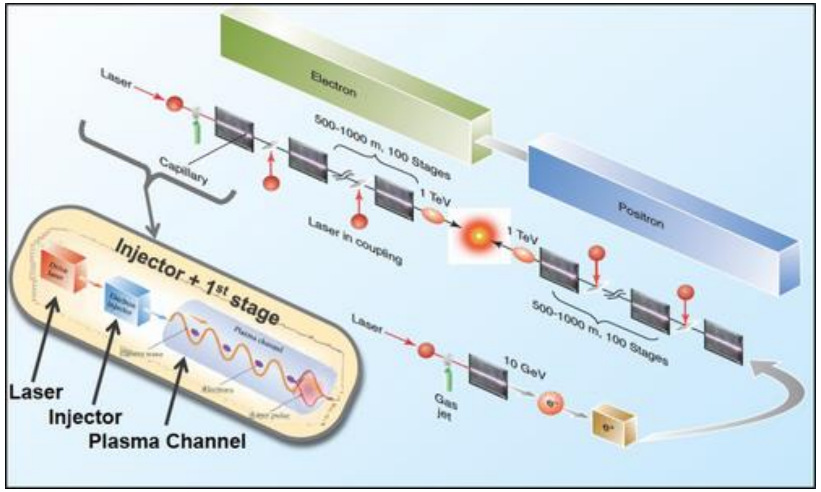
When is beam quality crucial?

1. DRIVER FOR FREE ELECTRON LASER
(lasing requires **very demanding** quality)



2. STAGING, i.e. MULTIPLE PLASMA ACCELERATION MODULES TO OBTAIN VERY LARGE FINAL ENERGIES (TeV?)



3. STABLE or **Monochromatic Compton/Thomson Scattering sources** or direct use of beams for medical purposes…

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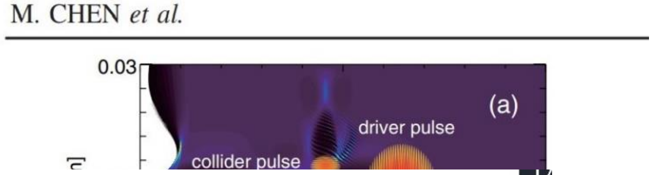
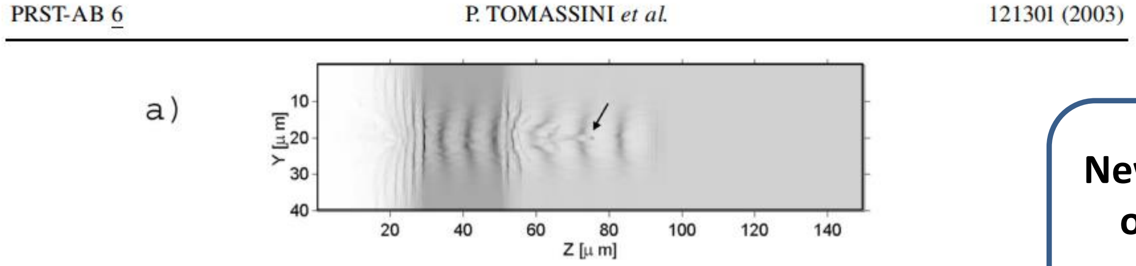
Injection schemes (some of)

- **Colliding pulses injection (E. Esarey et al., 1997; M.Chen et al, 2014) by ponderomotive assisted trapping**

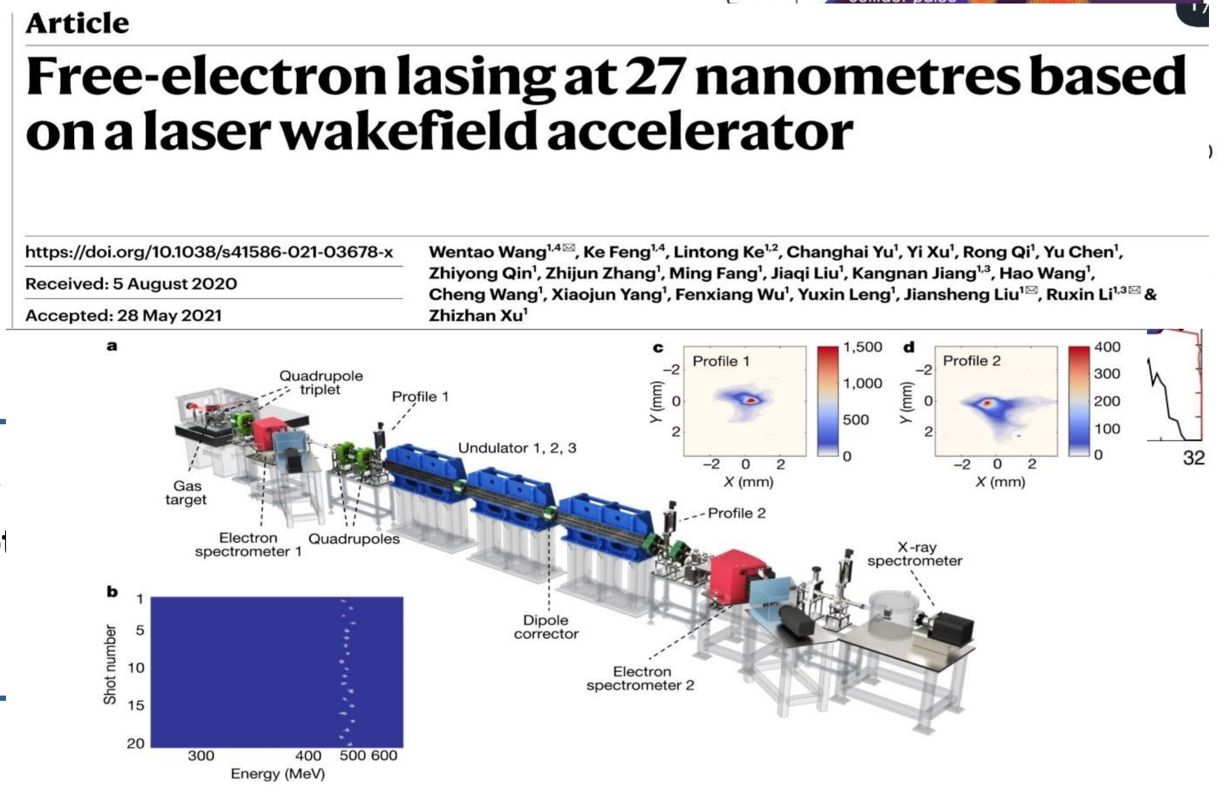
  E. Esarey et al., Phys. Rev. Lett. **79**, 2682 (1997)
  M. Chen et al., Phys. Rev. ST Accel. Beams 17, 051303 (2014)
  Malka et al., PoP, DOI:10.1063/1.3079486 (2014)



- **Density downramp** injection (**S. Bulanov et al., 1998**). Inj
  decrease of the wave speed and this is obtained with a su
  plasma density. **First 2D PIC simulations (P. Tomassini et**
  very low emittances (eps_n=0.2 mm mrad) can be obtair



New
obt


Article

Free-electron lasing at 27 nanometres based on a laser wakefield accelerator

https://doi.org/10.1038/s41586-021-03678-x
Received: 5 August 2020
Accepted: 28 May 2021

Wentao Wang[1,4], Ke Feng[1,4], Lintong Ke[1,2], Changhai Yu[1], Yi Xu[1], Rong Qi[1], Yu Chen[1], Zhiyong Qin[1], Zhijun Zhang[1], Ming Fang[1], Jiaqi Liu[1], Kangnan Jiang[1,3], Hao Wang[1], Cheng Wang[1], Xiaojun Yang[1], Fenxiang Wu[1], Yuxin Leng[1], Jiansheng Liu[1], Ruxin Li[1,3] & Zhizhan Xu[1]

- **Two Color and ReMPI injection (L. L. Yu et al., 2014, P. Tomassini,2017) look very promising (more on next slides).**

Trapping in the bubble regime

Normalized emittance

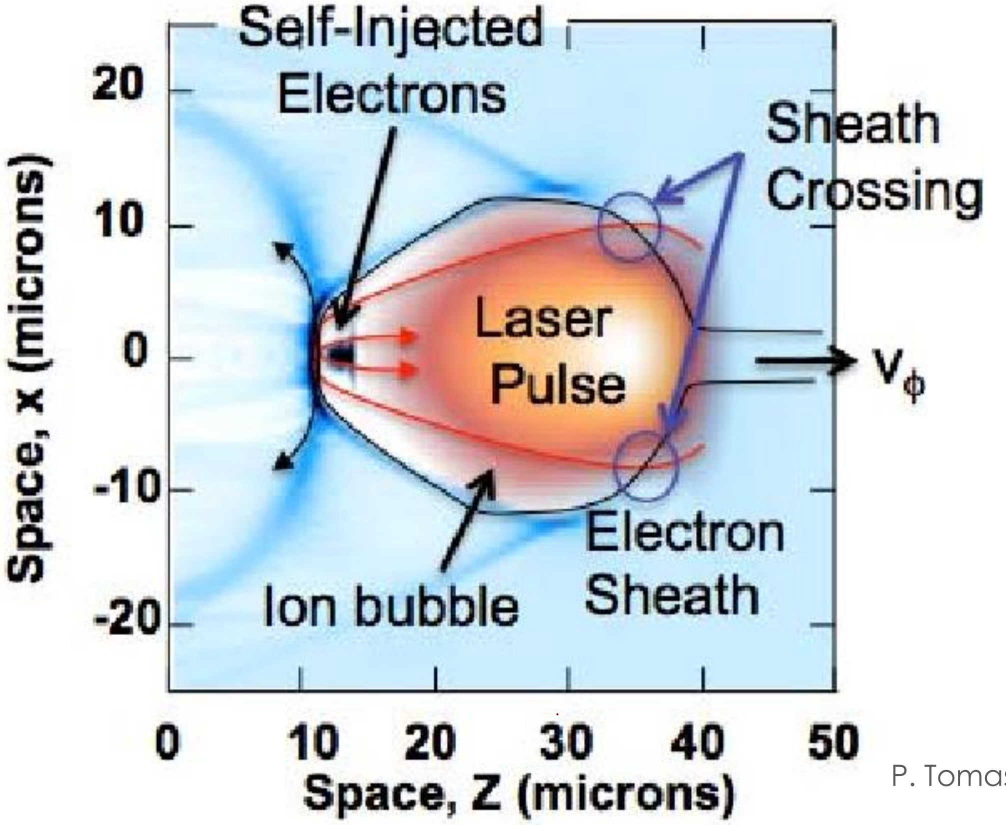
$$\epsilon_n^2 = \langle x^2\rangle\langle u_x^2\rangle - (\langle x\cdot u_x\rangle)^2$$

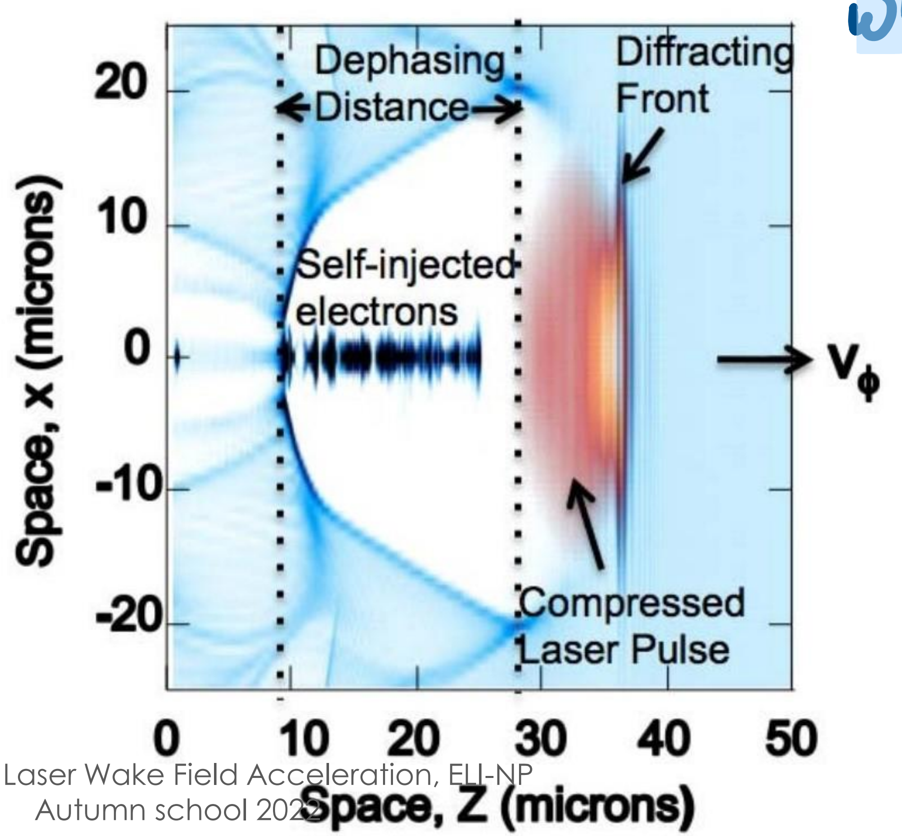
Pros: Automatic process, once the threshold is reached.

Cons: Usually the beam quality (energy spread, normalized emittance...) is poor

Particles are trapped when $v_e \geq v_\phi$!

C. Joshi, IEEE TRANSACTIONS ON PLASMA SCIENCE, VOL. 45, NO. 12, DECEMBER 2017





OUTLOOK

1. Introduction to LWFA
2. Understanding the excitation and the structure of the plasma waves
3. *The wide spot (1D and QSA) limiting case*
4. Limiting factors to high energy gain accelerators
5. 3D effects on
6. **Downramp (or shock) injection**
7. Two-Color injection
8. The Resonant Multi-Pulse Ionisation Injection
9. High-Brilliance e-bunches

Density downramp injection (S. Bulanov et al., 1998).

RAPID COMMUNICATIONS
PHYSICAL REVIEW E VOLUME 58, NUMBER 5 NOVEMBER 1998

Particle injection into the wave acceleration phase due to nonlinear wake wave breaking

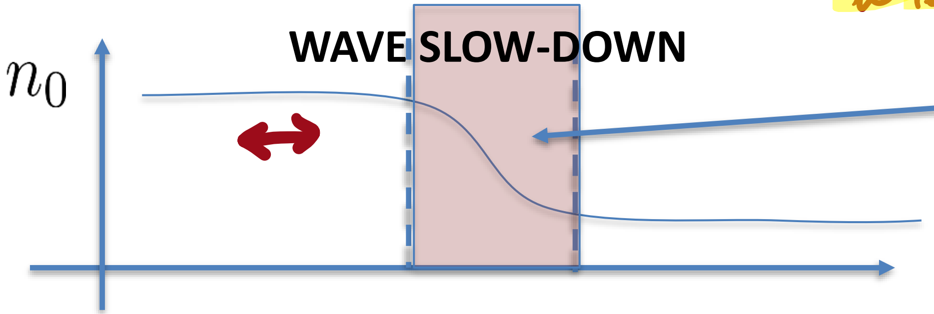
S. Bulanov,[1,2] N. Naumova,[1,3] F. Pegoraro,[4] and J. Sakai[5]
[1]*General Physics Institute RAS, Moscow, Russia*
[2]*Scuola Normale Superiore, Pisa, Italy*
[3]*Forum for Theoretical Physics, INFM, Pisa, Italy*
[4]*Dipartimento di Fisica, Università di Pisa and INFM, Pisa, Italy*
[5]*Laboratory for Plasma Astrophysics, Faculty of Engineering Toyama University, Toyama, Japan*
(Received 4 June 1998)

- **Density downramp** Injection is caused by a local decrease of the wave speed and this is obtained with a sudden decrease of the plasma density.

Reminder: electrons in the plasma wave have an OSCILLATING VELOCITY v

$\omega \simeq \omega_p$

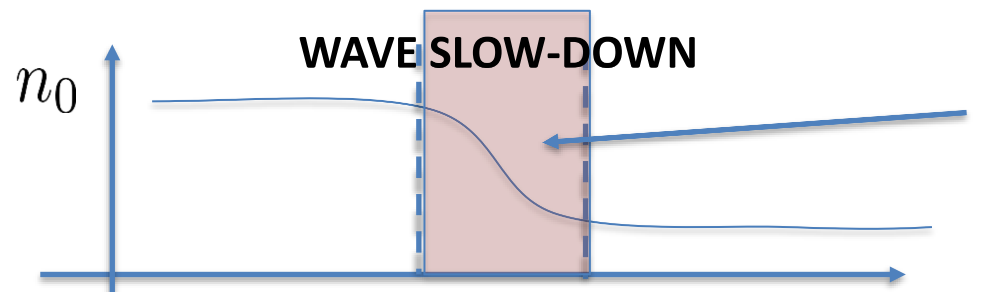
Particles are trapped when they get $v \geq v_\phi$



$$v_{ph}(z,t) = \omega(z,t)/k(z,t)$$

Density downramp injection (S. Bulanov et al., 1998).

G. B. Whitham, Linear and Nonlinear Waves ~Wiley, New York, 1974



- **Slowly varying envelope (but generic dispersion relation)**

$$u(z,t) = A(z,t)e^{i\phi(z,t)}$$

"RAY OPTICS"

$$|\partial A| << |A\partial\phi|$$

$$L \gg \lambda_p$$

Uniform medium:

$$\phi = kz - \omega t = k(z - \Omega(k)t)$$

$$\omega = \Omega(k)$$

DISPERSION RELATION

We can define local ω, k

$$\begin{cases} \omega(z,t) \equiv -\partial_t \phi(z,t) \\ k(z,t) \equiv \partial_z \phi(z,t) \end{cases}$$

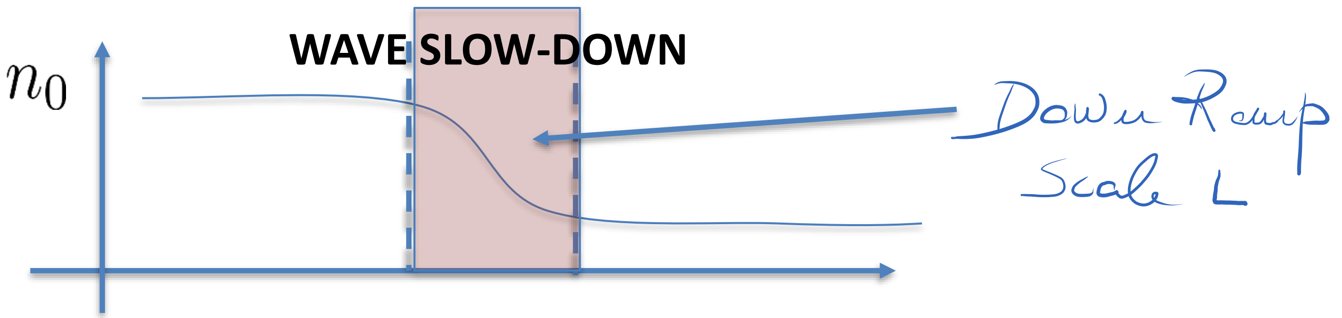
By taking the cross-gradients we get

$$\begin{cases} \partial_z \omega(z,t) \equiv -\partial_z \partial_t \phi(z,t) \\ \partial_t k(z,t) \equiv \partial_t \partial_z \phi(z,t) \end{cases}$$

$$\partial_t k(z,t) = -\partial_z \omega(z,t)$$

This is independent from the current dispersion relation!

Density downramp injection (S. Bulanov et al., 1998).



- **Eikonal approximation** **for the plasma wave**

For $v_{th} << c$

$$L \gg \lambda_p \qquad \partial_t k(z,t) = -\partial_z \omega(z,t)$$

$$\omega = \Omega(k) \simeq \omega_p(z)$$

$$\omega_p = \omega_0 \sqrt{n(z)/n_c}$$

$$\partial_t k(z,t) \simeq -\frac{\omega_0}{2 n_c \omega_p(z)} \partial_z n(z)$$

$$v_{ph}(z,t) \simeq \omega_p(z)/k(z,t)$$

Density downramp injection (S. Bulanov et al., 1998).

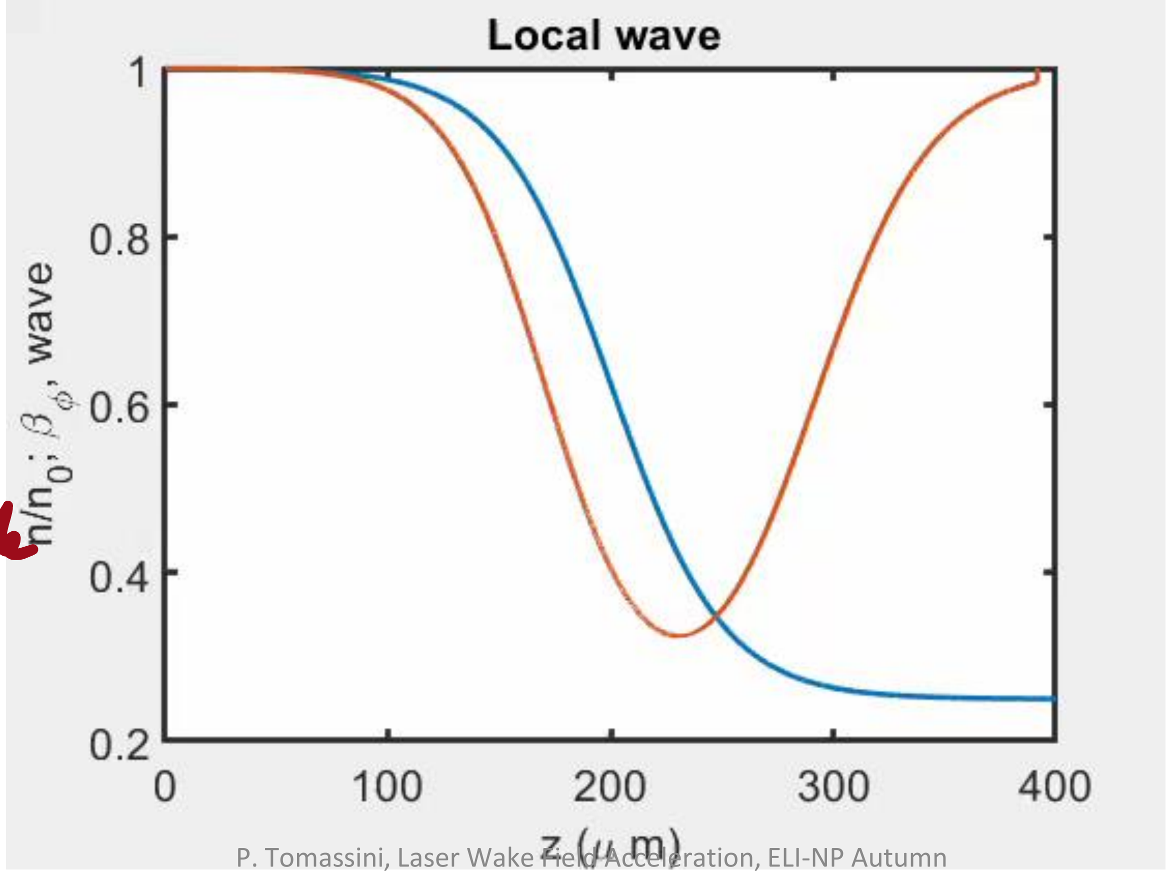
WAVE SLOW-DOWN

$$L \gg \lambda_p \quad \omega_p = \omega_0 \sqrt{n(z)/n_c}$$

$$2n(z) = (n_0 + n_1) + (n_0 - n_1)\tanh(z/L)$$
$$n_0 = 10^{19} cm^{-3}; n_1 = n_0/4; \lambda_p \approx 10 - 20 \mu m$$

And the wave phase oscillating contribution is

NOTE: We solved the equations for the PHASE, also the amplitude will change.



Density (sharp) downramp: first 2D PIC simulations

PHYSICAL REVIEW SPECIAL TOPICS - ACCELERATORS AND BEAMS, VOLUME 6, 121301 (2003)

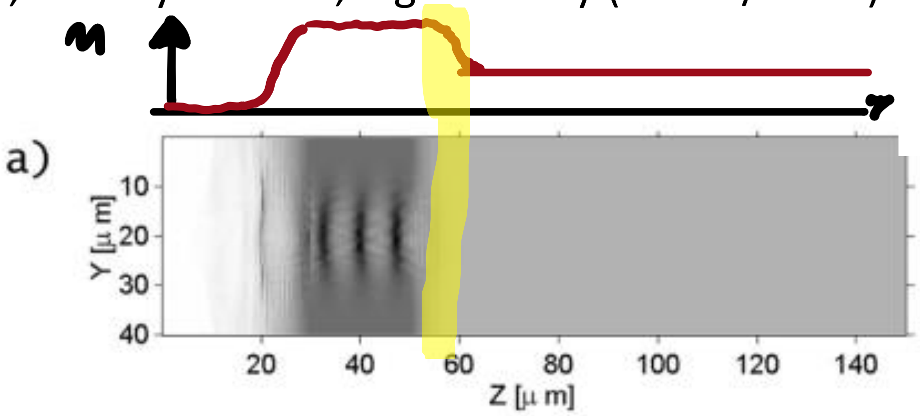
Production of high-quality electron beams in numerical experiments of laser wakefield acceleration with longitudinal wave breaking

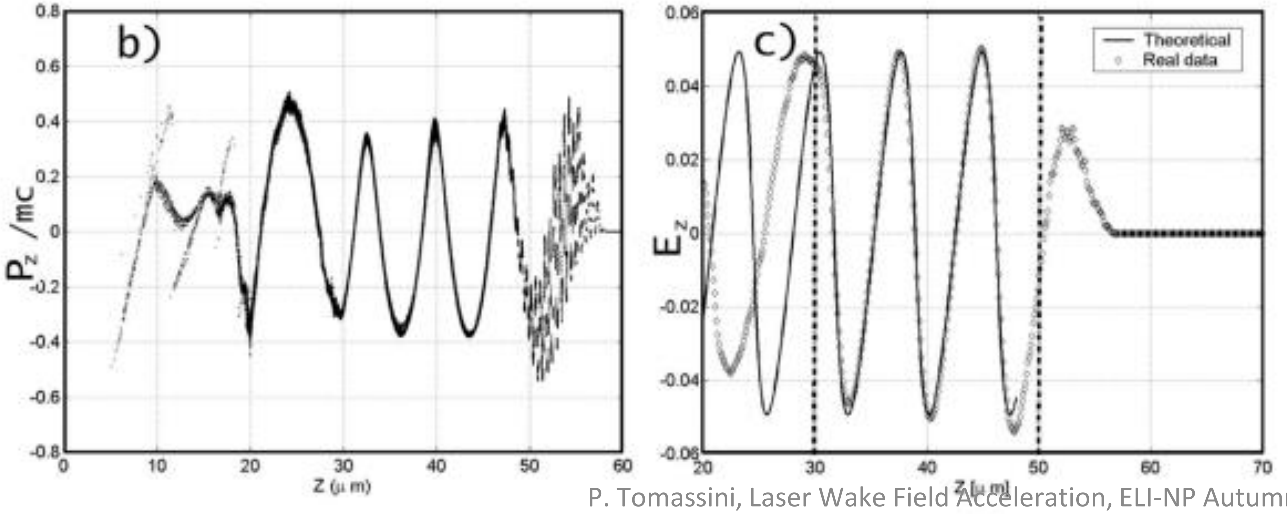
P. Tomassini,* M. Galimberti, A. Giulietti, D. Giulietti,† L. A. Gizzi, and L. Labate‡
Intense Laser Irradiation Laboratory–IPCF, Area della Ricerca CNR, Via Moruzzi 1, 56124 Pisa, Italy

F. Pegoraro
Dipartimento di Fisica, Università di Pisa and I.N.F.M Unita's di Pisa, 56124 Pisa, Italy
(Received 24 July 2003; published 30 December 2003; corrected 30 December 2003)

Moderate pulse intensity (a_0=1.3), sharp transition (scale of about 2 μm) , density ratio: 2 , high density (10^19/cm^3).

QUASI 1D scenario

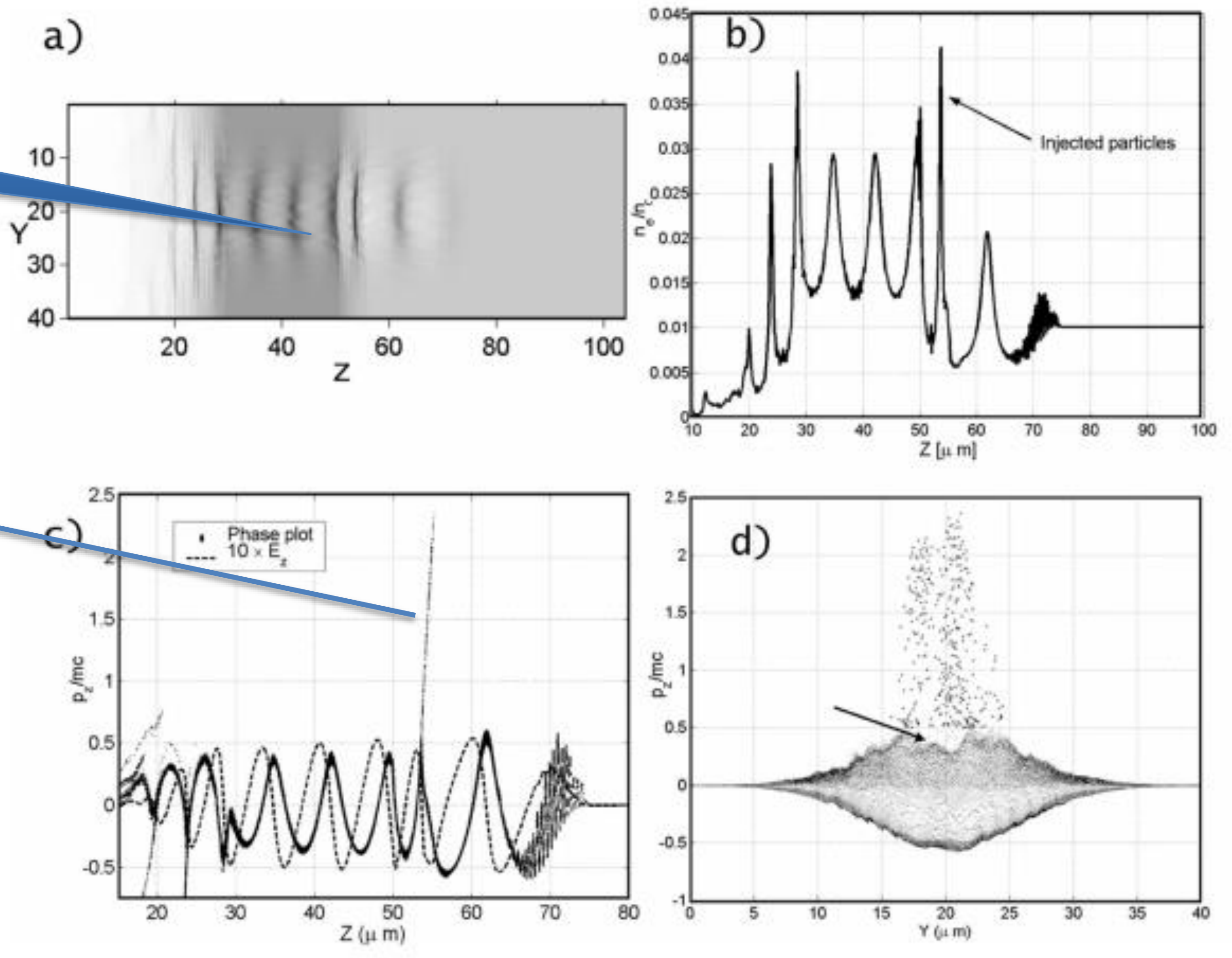




Density downramp First 2D PIC simulations

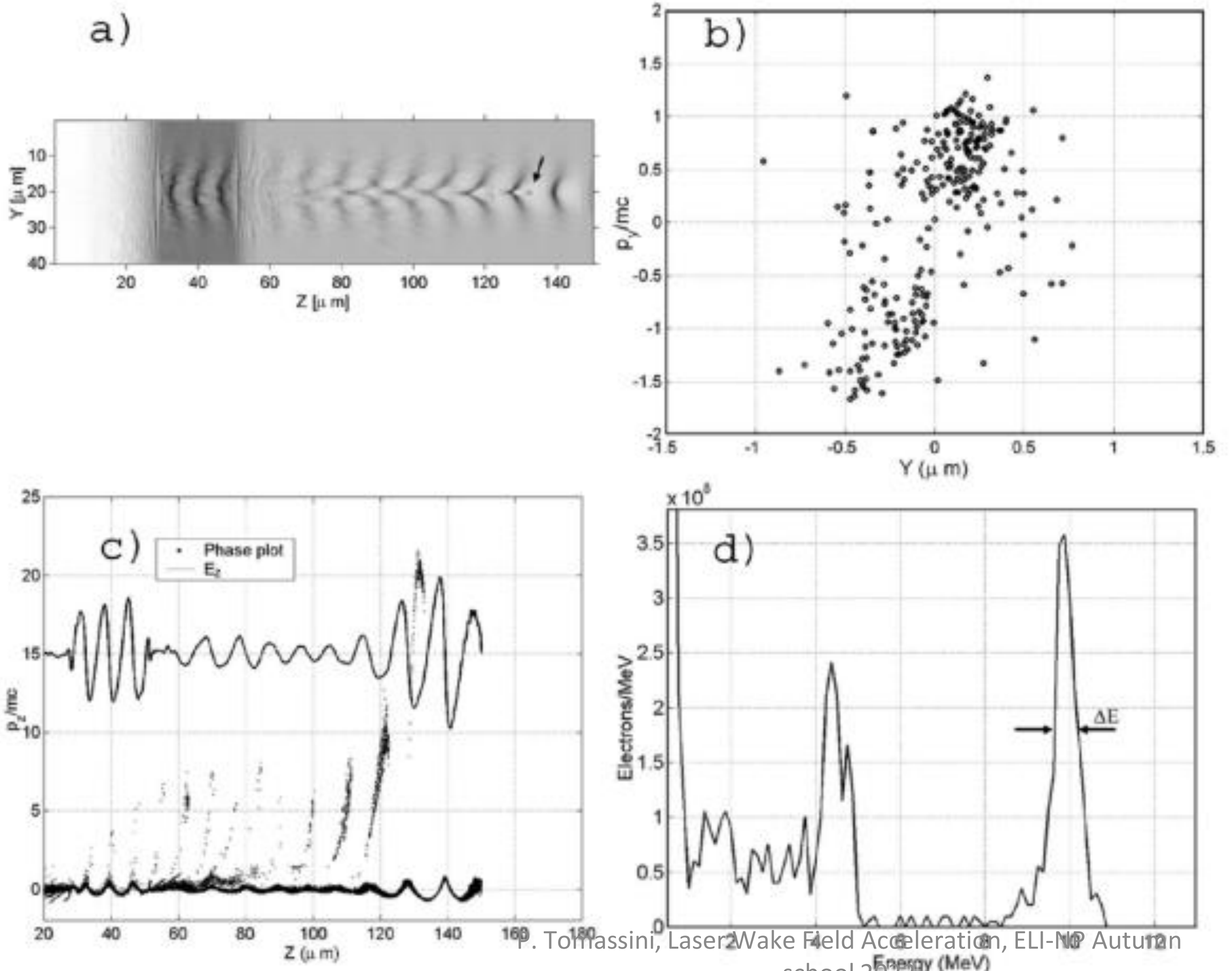
WB

•



Density downramp First 2D PIC simulations

PRST-AB 6 PRODUCTION OF HIGH-QUALITY ELECTRON BEAMS ... 121301 (2003)



$E \simeq 10MeV$

$\sigma(E)/E \simeq 3\%$

$\epsilon_n \simeq 0.2mmmrad$

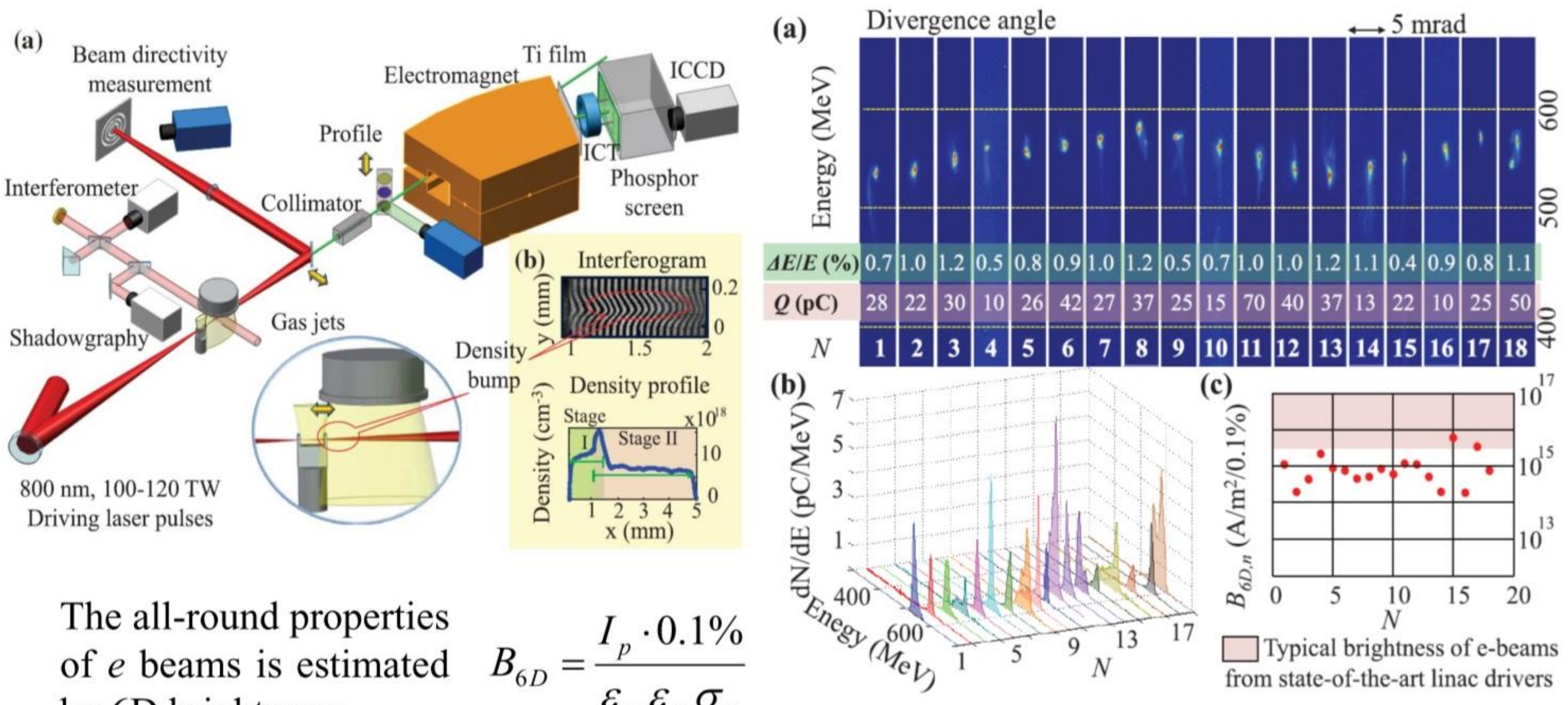
$Q \approx 10pC$

$\sigma(z) \approx 5\mu m$

$\sigma(x) \approx 1\mu m$

High-quality beams with downramp injection

High brightness *e* beam generation



The all-round properties of *e* beams is estimated by 6D brightness:

$$B_{6D} = \frac{I_p \cdot 0.1\%}{\varepsilon_{nx}\varepsilon_{ny}\sigma_\gamma}$$

Phys. Rev. Lett. 117, 124801(2016) .

- **High-quality *e* beam was experimentally generated (peak energy of 200-600 MeV, rms energy spread of 0.4%-1.2%, charge of 10-80 pC and rms divergence of ~0.2 mrad).**
- **The maximum 6D brightness is estimated as ~6.5×10^{15} A/m^2/0.1%, which is close to the typical brightness of *e* beam from state-of-the-art linac drivers.**

Courtesy of Ke Feng, Wentao Wang, and Runxin Li (SIOM)

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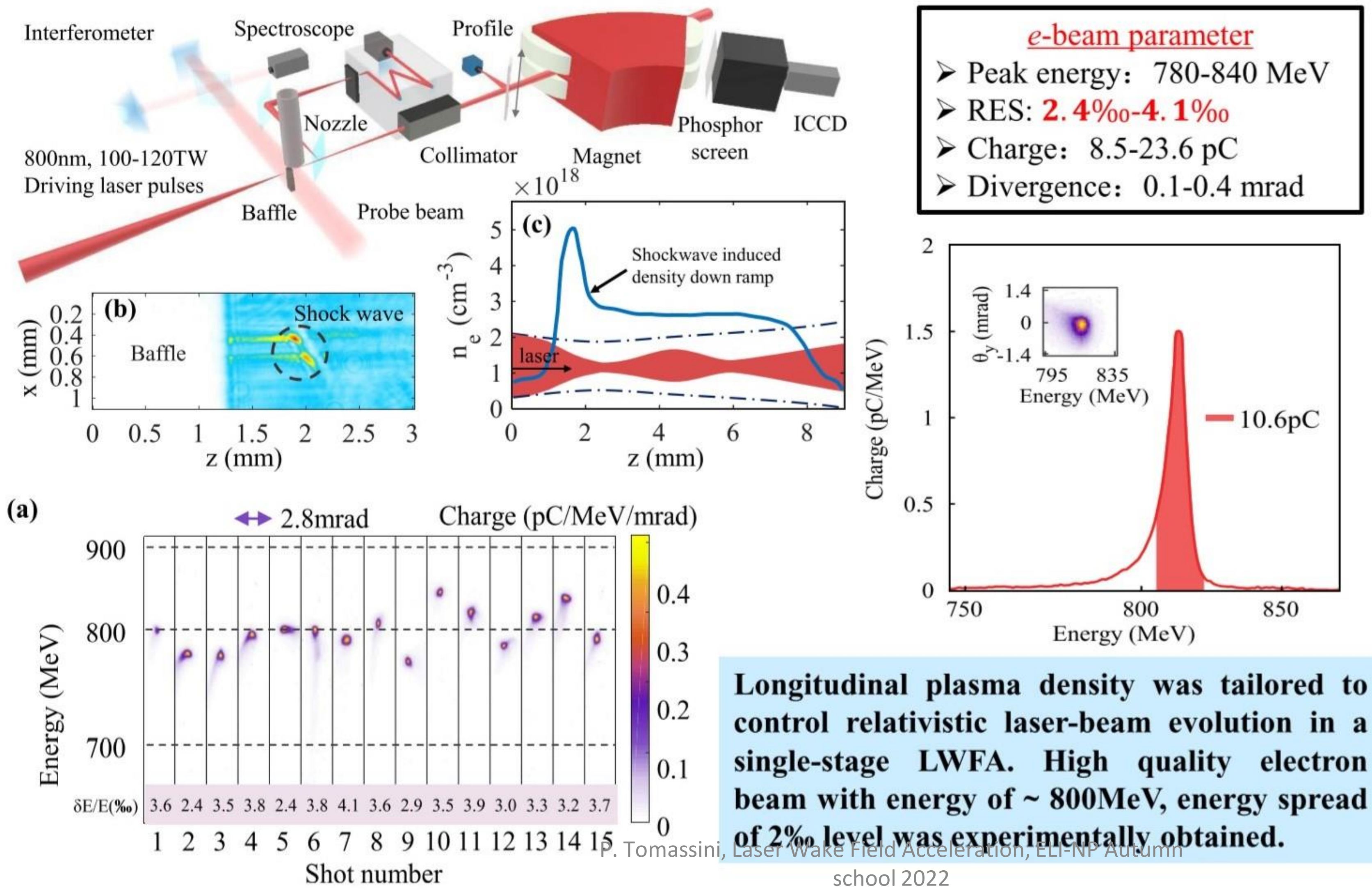
High-quality beams with downramp injection

Near-GeV, 2‰-level *e* beam generation



e-beam parameter
- Peak energy: 780-840 MeV
- RES: **2.4‰-4.1‰**
- Charge: 8.5-23.6 pC
- Divergence: 0.1-0.4 mrad

Longitudinal plasma density was tailored to control relativistic laser-beam evolution in a single-stage LWFA. High quality electron beam with energy of ~ 800MeV, energy spread of 2‰ level was experimentally obtained.

Courtesy of Ke Feng, Wentao Wang, and Runxin Li (SIOM)

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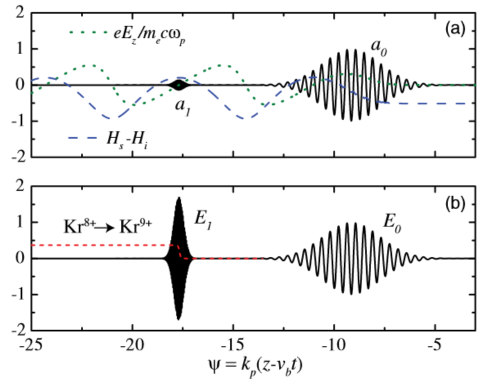
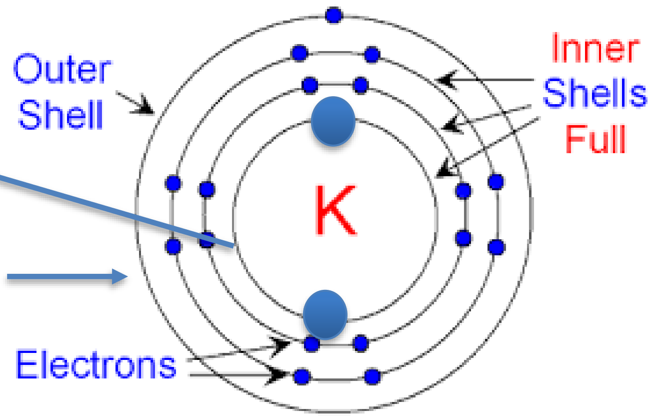


Ionization Injection

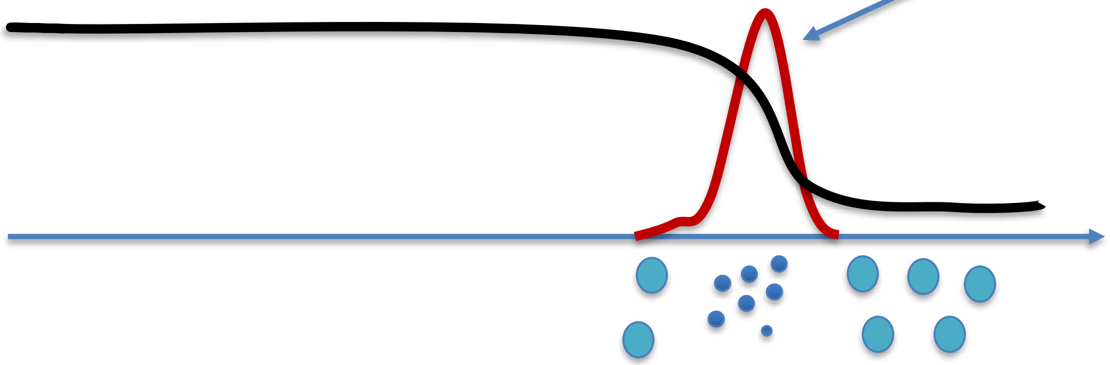
Inner electrons can be extracted
by a very large electric field



Outer electrons need a very
low intensity electric field to be ionized

They constitute the plasma electrons

Ultraintense pulse



New electrons, «born inside the pulse core»

Two Color injection…

PRL 112, 125001 (2014) PHYSICAL REVIEW LETTERS week ending 28 MARCH 2014

Two-Color Laser-Ionization Injection

L.-L. Yu,[1,2,3] E. Esarey,[1] C. B. Schroeder,[1] J.-L. Vay,[1] C. Benedetti,[1] C. G. R. Geddes,[1] M. Chen,[3] and W. P. Leemans[1,2]
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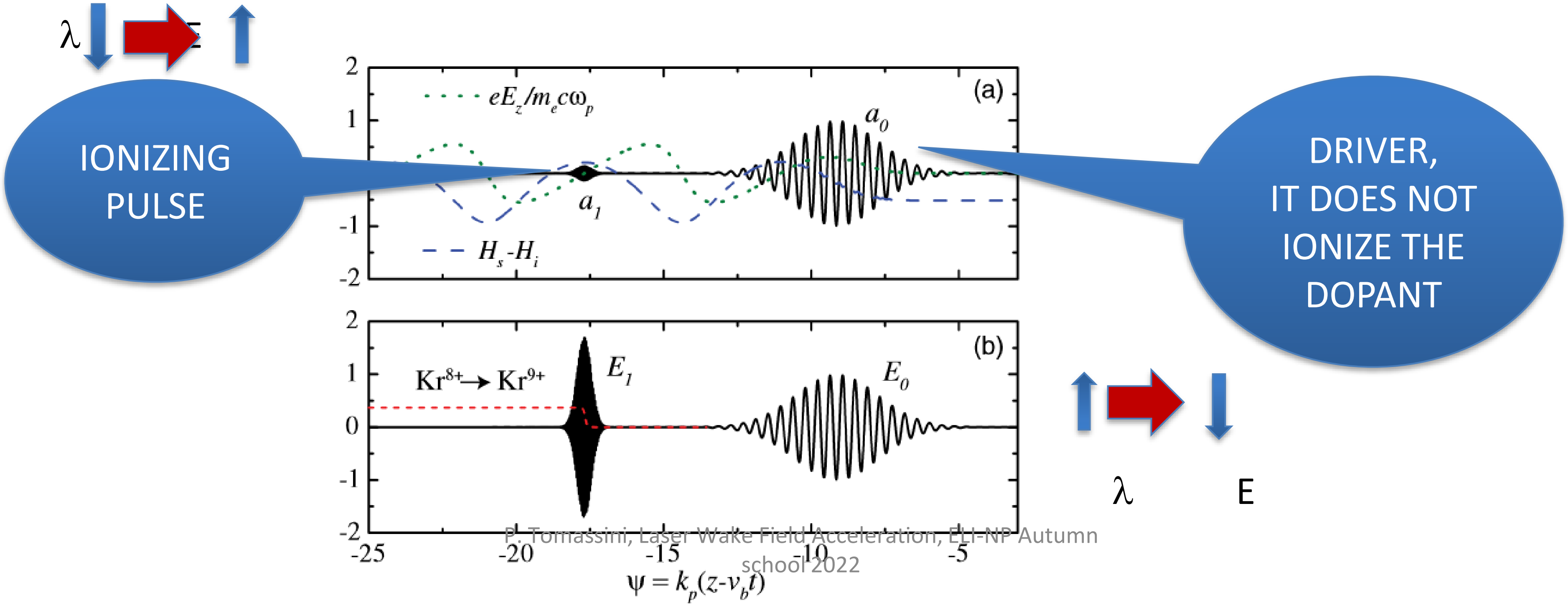
(Received 31 July 2013; published 24 March 2014)

A method is proposed to generate femtosecond, ultralow emittance (~10^{-8} m rad), electron beams in a laser-plasma accelerator using two lasers of different colors. A long-wavelength pump pulse, with a large ponderomotive force and small peak electric field, excites a wake without fully ionizing a high-Z gas. A short-wavelength injection pulse, with a small ponderomotive force and large peak electric field, copropagating and delayed with respect to the pump laser, ionizes a fraction of the remaining bound electrons at a trapping wake phase, generating an electron beam that is accelerated in the wake.

DOI: 10.1103/PhysRevLett.112.125001 PACS numbers: 52.38.Kd, 52.25.Jm

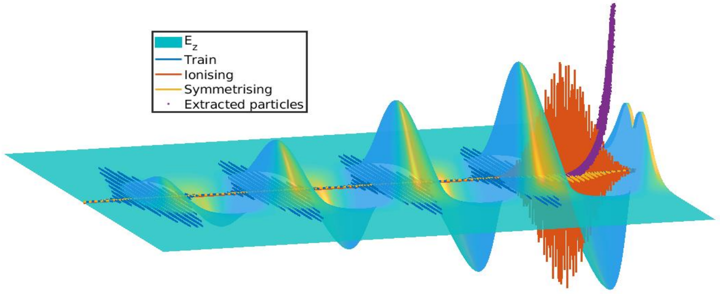
- **It uses two lasers systems**.

A long wavelength (λ>5 μm) pulse excites the plasma wave and the short wavelength one (λ<0.4 μm) extracts electrons by field ionization from an high-Z dopant.
It allows for very low emittance bunches.
However, to date, its experimental feasibility is limited from the lacking of commercial high power, ultrashort and long wavelength laser systems.



OUTLOOK

1. Introduction to LWFA
2. Understanding the excitation and the structure of the plasma waves
3. *The wide spot (1D and QSA) limiting case*
4. Limiting factors to high energy gain accelerators
5. 3D effects on
6. Downramp (or shock) injection
7. Two-Color injection
8. **<u>The Resonant Multi-Pulse Ionisation Injection</u>**
9. High-Brilliance e-bunches



The Resonant Multi-Pulse Ionisation Injection

PHYSICS OF PLASMAS **24**, 103120 (2017)

The resonant multi-pulse ionization injection

Paolo Tomassini,[1,a)] Sergio De Nicola,[2,3] Luca Labate,[1,4] Pasquale Londrillo,[5] Renato Fedele,[2,6] Davide Terzani,[2,6] and Leonida A. Gizzi[1,4]
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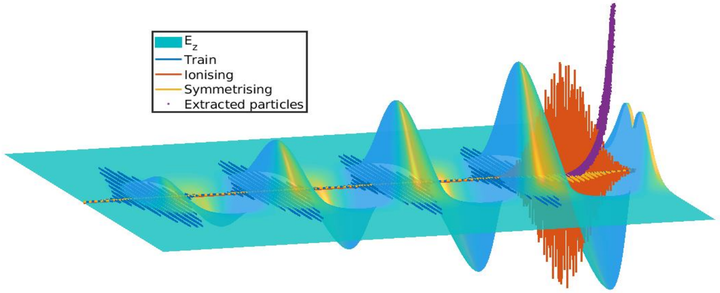
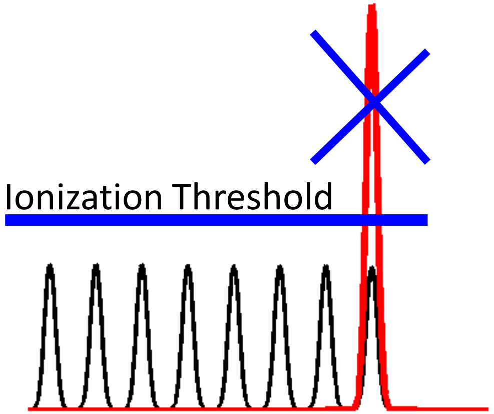
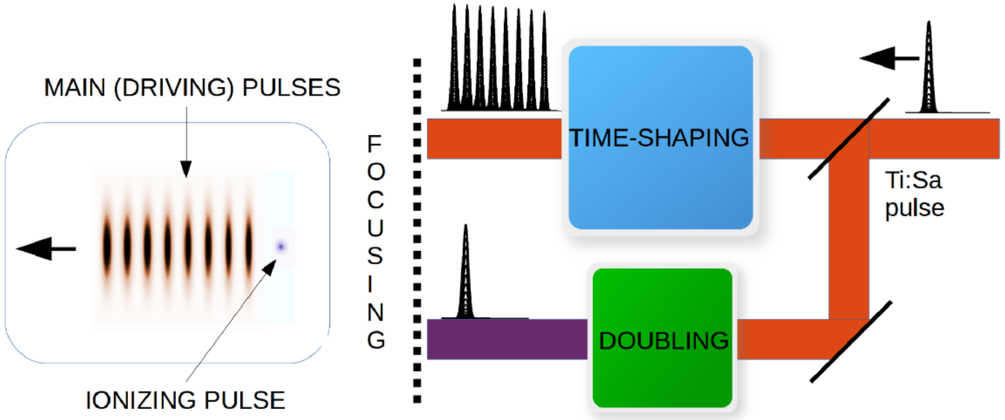
[2]*Dip. Fisica Universita' di Napoli Federico II, 80126 Napoli, Italy*
[3]*CNR-SPIN, Napoli, 80126 Napoli, Italy*



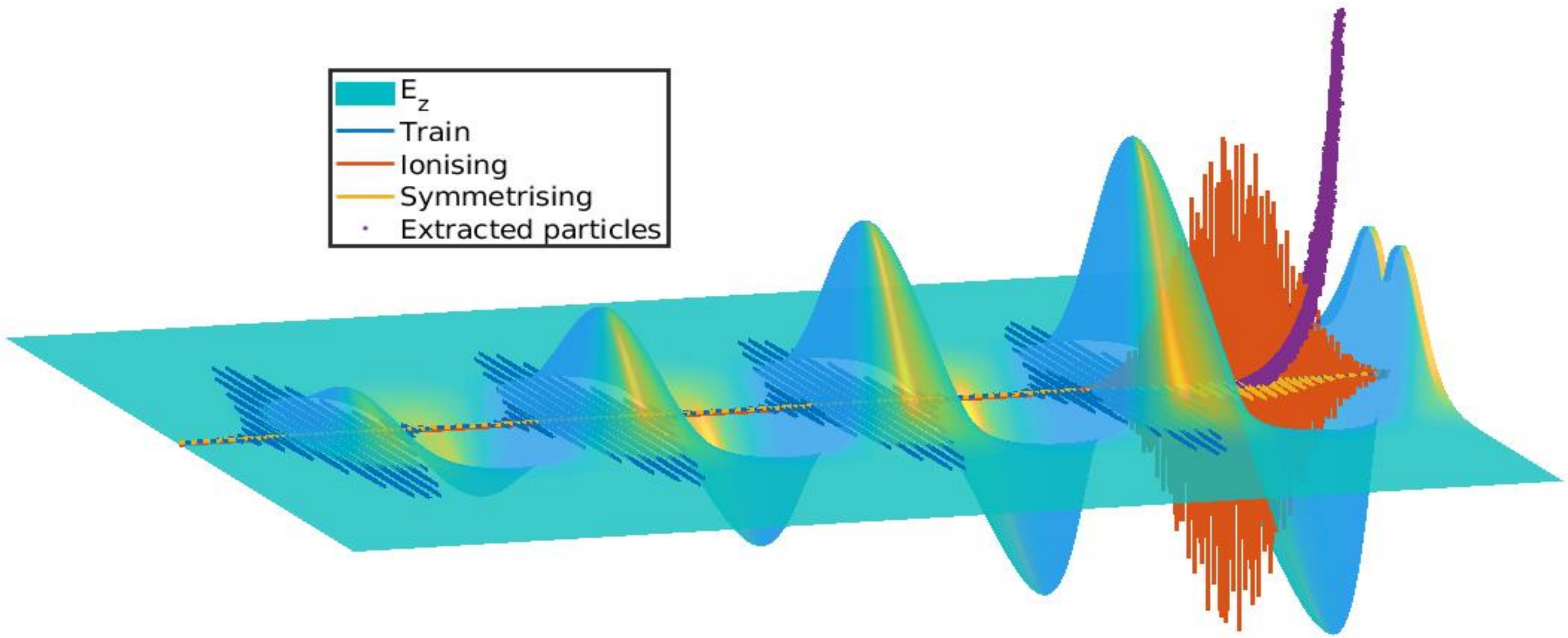
- ReMPI **requires one** short-pulse (e.g Ti:Sa) laser system. It works also with multiple lases systems
- Since a unique very large-amplitude Ti:Sa pulse would fully ionize the atoms (Ar8+ in our selected example), **the pulse is shaped as a resonant sequence** of sub-threshold amplitude pulses.





Intense Laser Irradiation Laboratory-Pisa

ReMPI at a glance



P. Tomassini et al., [*Physics of Plasmas 24, 103120 (2017)*]

ReMPI is FLEXIBLE.
Examples of possible application

Injector for multistage LWFA

30pC, 150MeV, 1.6% , 0.23 μmrad

PHYSICAL REVIEW ACCELERATORS AND BEAMS **22,** 111302 (2019)

High quality electron bunches for a multistage GeV accelerator with resonant multipulse ionization injection

Paolo Tomassini,[1,*] Davide Terzani,[1] Luca Labate,[1,2] Guido Toci,[3] Antoine Chance,[4] Phu Anh Phi Nghiem,[4] and Leonida A. Gizzi[1,2]
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[4]*CEA-Irfu, Centre de Saclay, Université Paris-Saclay, 91191 Gif sur Yvette, France*

Single stage 5GeV accelerator

30pC, 5GeV, 1% (proj) , 0.04% (slice) 0.08 μmrad

High-quality 5 GeV electron bunches with resonant multi-pulse ionization injection

P Tomassini[3,1], D Terzani[1], F Baffigi[1], F Brandi[1], L Fulgentini[1], P Koester[1], L Labate[2,1], D Palla[1] and L A Gizzi[2,1]

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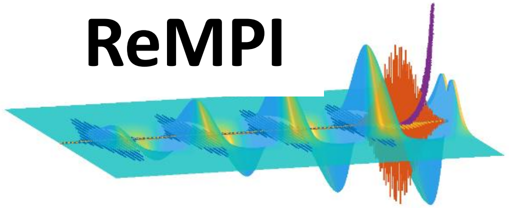
Plasma Physics and Controlled Fusion, Volume 62, Number 1

Special Issue Featuring the Invited Talks from the 46th EPS Conference on Plasma Physics, Milan, 8-12 July 2019

ısma Phys. Control. Fusion **62** 014010

ReMPI



Beams for FEL or Thomson backscattering X/gamma

30pC, 4.5GeV, 0.9% (proj) , 0.03% (slice) 0.08 μmrad

Brilliant X-ray Free Electron Laser driven by Resonant Multi Pulse Ionization Injection accelerator, P. Tomassini et al. Proc. FEL conference 2022

Attosecond high-brightness beams

1-10pC, 0.2-5GeV, 1-2%, 0.08 μmrad, 200as

Sub femtosecond high-brigthness electron bunches with the Resonant Multi Pulse Ionisation injection. P. Tomassini, ELI-NP report 2022.

*High-brightness attosecond electron bunches with the **Re**sonant Multi-Pulse Ionisation injection*. P.Tomassini et al, **in preparation**

High-Brightness 0.5 GeV ultrashort and 5 GeV FEL oriented beam.

Examples of (projected) Beam Brightness performances

https://pwfa-fel.phys.strath.ac.uk

$$B_{n,5D} = \frac{I}{\epsilon_{n,x}\epsilon_{n,y}} \qquad B_{n,6D} = \frac{B_{n,5D}}{\delta E/E/0.1\%}$$

The final beam brightness is

$$B_{n,5D} \approx 2\times 10^{18} A/m^2$$

$$B_{n,6D} \approx 2\times 10^{17} A/m^2/0.1\%$$

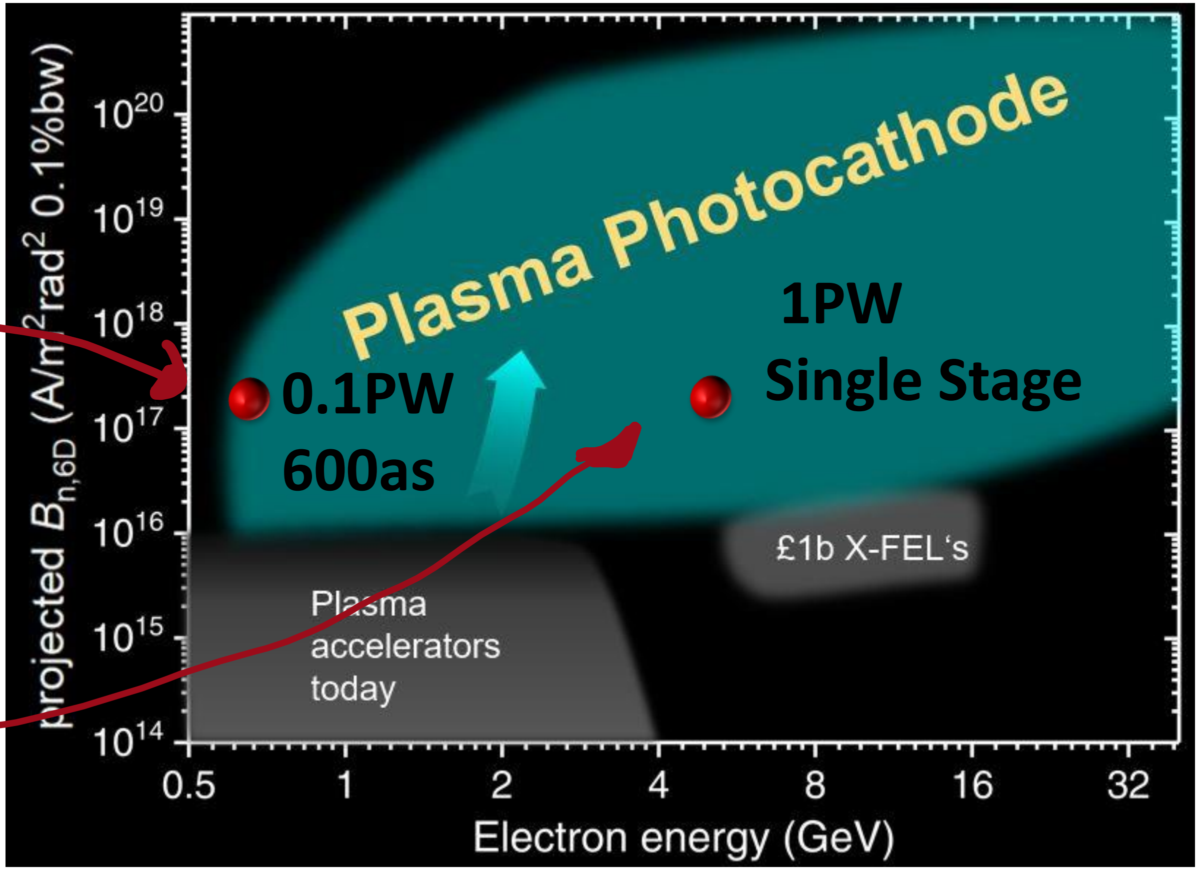
600as high-brigthness 0.6GeV beam with 100 TW Ti:Sa laser

The final beam brightness is

$$B_{n,5D} \approx 2\times 10^{18} A/m^2$$

$$B_{n,6D} \approx 2\times 10^{17} A/m^2/0.1\%$$

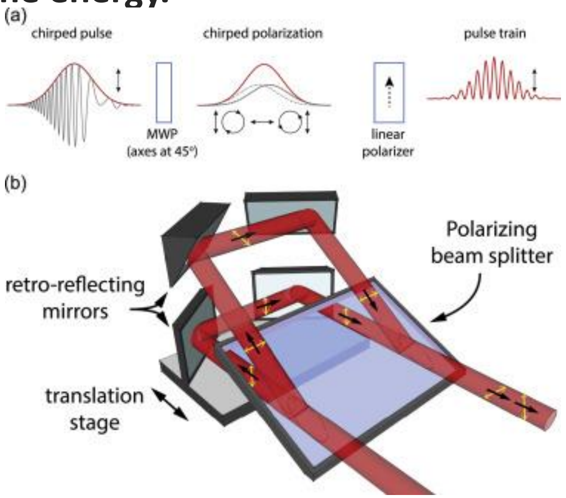
3fs high-brigthness 5GeV beam with 1PW TW Ti:Sa laser



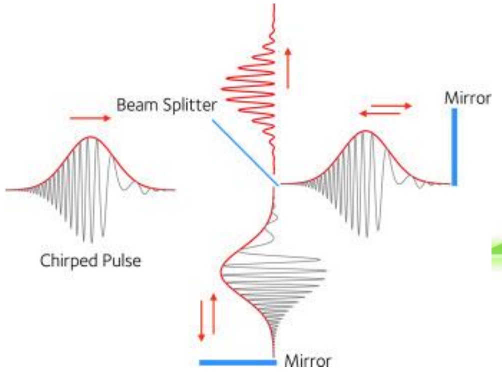
Multi-Pulse LWFA

- **The multi-pulse approach to LWFA has been proposed so far [D. Umstadter et al, PRL 72, (1994)]. A multi-pulse train can generate plasma waves with larger amplitude than those driven by a single pulse with the same energy.**

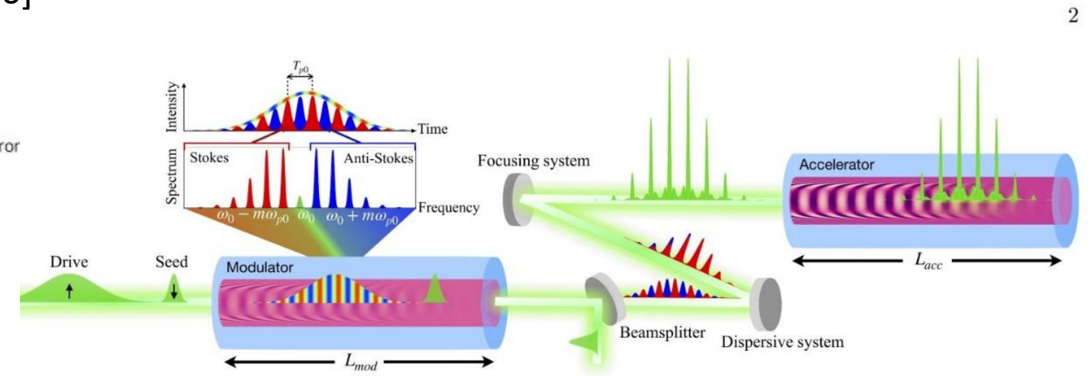
Three pssible pulse-shaping schemes have been proposed very recently



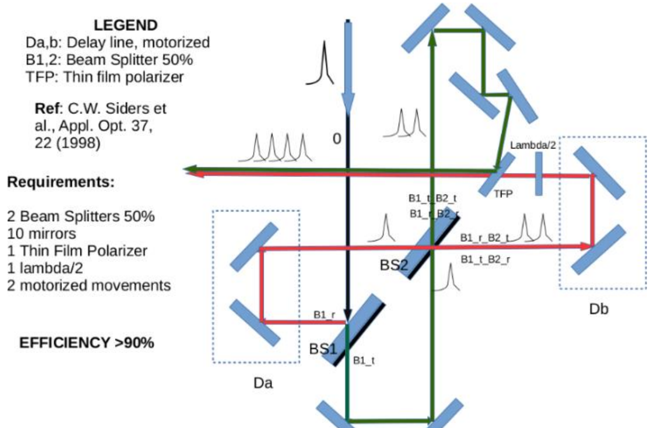
[J. Cawley et al, PRL 2017]
[R.J. Shallow et al, NIM A 2016]



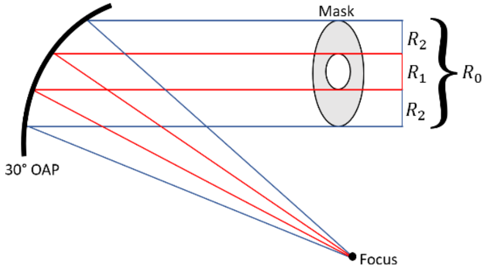
[O. Jakobsson, et al. PRL **127** (2021)]

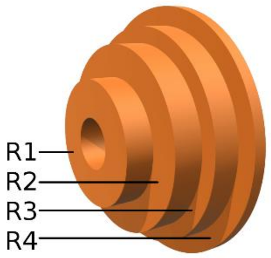


Older methods include either multiple beam-splitters setup or phase masks



[W. Siders et al., Appl. Opt, 37 (22) 1998]





[G. Vantaggiato et al., NIMA 2018]
[A. Marasciulli et al., CLEO conf. 2021]

Extracted beam by tunnel ionisation
Current model based on ADK rate

Theory of beam emittance from field ionisation has been cast by C. Schroeder et al.
[Thermal emittance from ionization-induced trapping in plasma accelerators, PRAB 10130 (2014)]

There, the key parameter is $\Delta_0 = \left(\frac{2E_{0,x}}{3E_a}\right)^{1/2}\left(\frac{U_H}{U_I}\right)^{3/4} \ll 1$ and **far from ionisation saturation**, the leading order estimation of the *rms* transverse size and momentum i

$$\sigma_{u_x} \approx \Delta_0 a_0 \qquad \sigma_{u_x} \approx 0 \qquad \sigma_x \simeq \sigma_y \approx \frac{1}{\sqrt{2}}\Delta_0 w_0$$

Notably, ponderomotive force affects the single size and momentum values, but being the ponderomotive force (mostly) linear, normalised emittance is still evaluted as

$$\epsilon_{nx} \simeq \sigma_{u_x} \cdot \sigma_x \approx \frac{1}{\sqrt{2}}\Delta_0^2 a_0 w_0$$

$$\epsilon_{nx} \simeq \sigma_{u_y} \cdot \sigma_y \approx 0$$

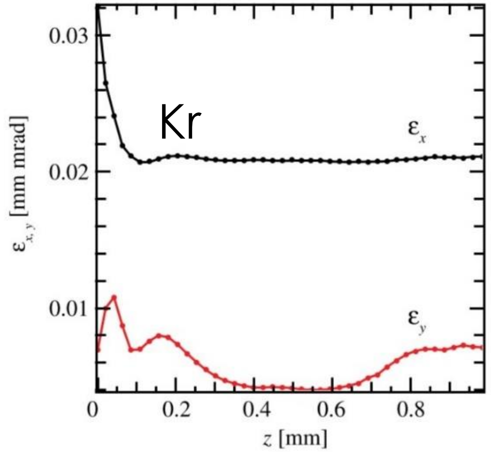


FIG. 9. The normalized emittance evolution in the laser polarization plane ϵ_x (black curve) and orthogonal to the laser polarization plane ϵ_y (red curve). (Two-color ionization injection parameters are the same as Fig. 1.)

Though σ_{ux} contains Δ_0^2 corrections, theory for the beam emittance is only first order in Δ_0 and **doesn't take into account saturation effects**

Extracted beam by tunnel ionisation
New model based on ADK rate

High Power Laser Science and Engineering, (2022), Vol. 10, e15, 16 pages.
doi: 10.1017/hpl.2021.56

HIGH POWER LASER SCIENCE AND ENGINEERING

RESEARCH ARTICLE

Accurate electron beam phase-space theory for ionization-injection schemes driven by laser pulses

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[2] *ELI-NP, Magurele, Ilfov, Romania*
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[4] *INFN, Sect. of Pisa, Pisa, Italy*

(Received 31 August 2021; revised 25 October 2021; accepted 26 November 2021)

Highly accurate modeling of the 6D phase-space for tunnel-ionisation extracted electrons (ADK theory).

Single-cycle detailed description of the phase-space and whole bunch emittance

Also valid in the deep-saturation regime

$$\rho \equiv \frac{3E}{2E_a}\left(\frac{U_H}{U_I}\right)^{3/2} = a/a_c = \Delta^2$$

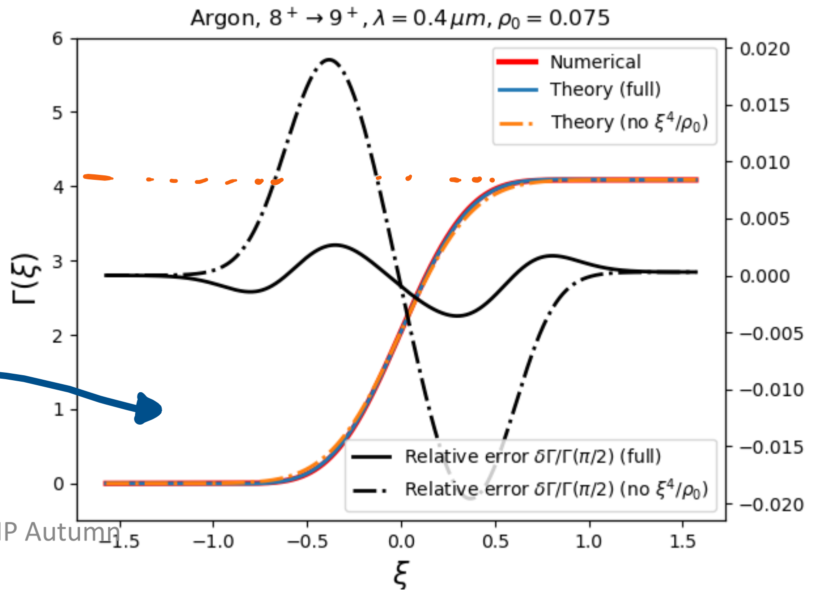
$$W = C \times \rho^{\mu} e^{-\frac{1}{\rho}} \qquad \rho = \rho_0 |\cos(\xi)|$$

ADK rate rewritten as in [P. Tomassini et al., PoP 24, 103120 (2017)]

$$\frac{1}{n_{0,i}}\frac{dn_e}{d\xi} = -\frac{\partial}{\partial_\xi} e^{-\Gamma(\xi)};$$

$$\Gamma(\xi) \equiv \frac{1}{k_{0,x}}\int_{-\pi/2}^{\xi} dx W(x)$$

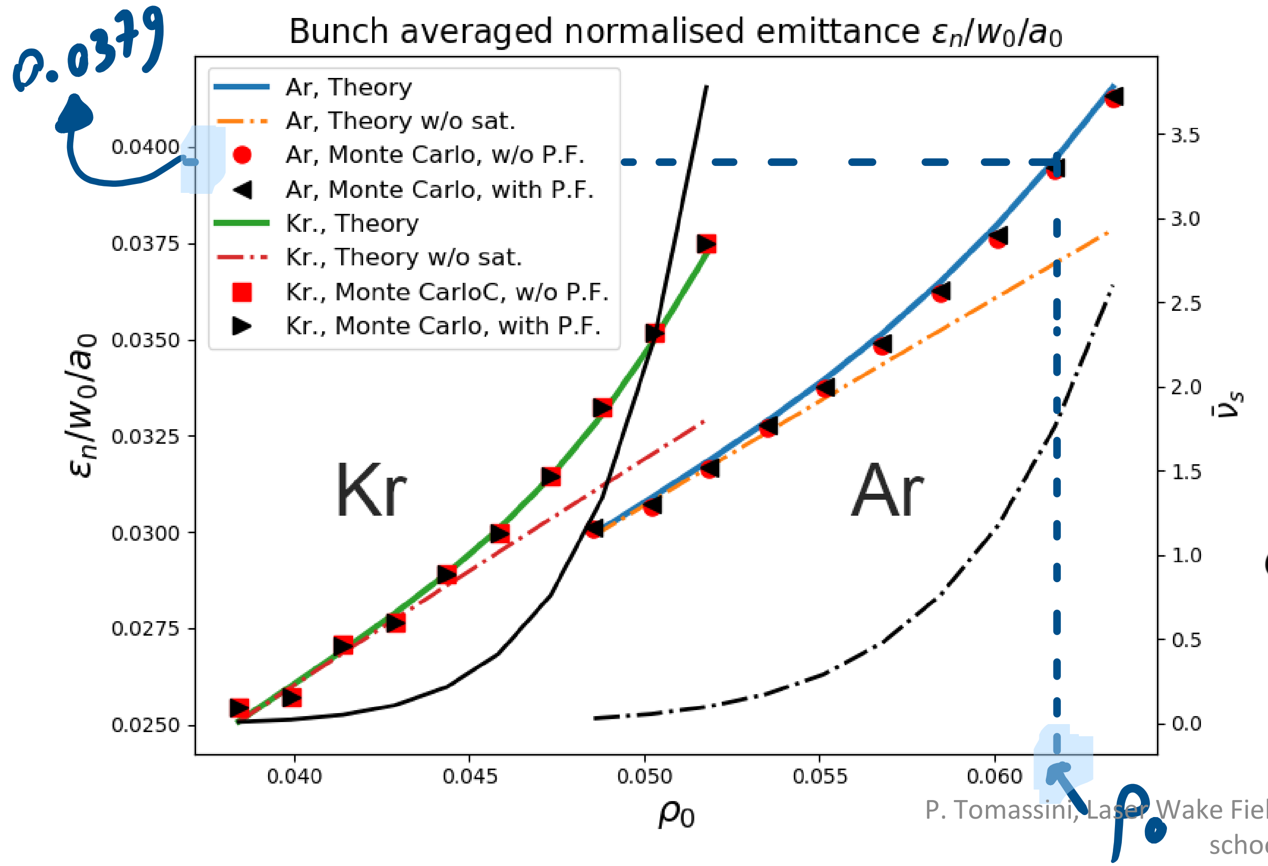
$$= \frac{k_{ADK}}{k_0}\rho_0^{\mu}\int_{-\pi/2}^{\xi} dx (\cos x)^{\mu} e^{-\frac{1}{\rho_0 \cos x}} \simeq \nu_s(\rho_0)\mathcal{G}\left(\frac{\xi}{\sqrt{2\rho_0}}\right)$$



Extracted beam by tunnel ionisation
New model based on ADK rate

$$\rho \equiv \frac{3E}{2E_a}\left(\frac{U_H}{U_I}\right)^{3/2} = a/a_c = \Delta_0^2$$

Minimum obtainable «thermal» normalised emittance



$Ar^{8^+ \to 9^+}$ $\rho_0 = a_{0,i}/a_c = 0.062$

$\lambda_i = 0.4\mu m$
$a_c = 7.41$
$a_{0,i} = 0.46$
$w_0 = 4.0\mu m$
$\epsilon_n|_{th} = 72 nm\, rad$

$\lambda_i = 0.2\mu m$
$a_c = 3.71$
$a_{0,i} = 0.23$
$w_0 = 4.0\mu m$
$\epsilon_n(min) = 37 nm\, rad$

IV harm   IV harm

ReMPI (and Two Color) trapping thresholds

Conserved Hamiltonian in 1D+QSA

$$\mathcal{H}(\psi,\gamma) = \gamma(1-\beta\beta_{ph}) - \phi(\psi)$$

$$\psi \equiv k_p(z-\beta_\phi ct)$$

[E.Esarey & M. Pillov, PoP **2** 1432 (1995)]

$$E_{norm} = E_z/E_0;\ E_0 = mc\omega_p/e$$

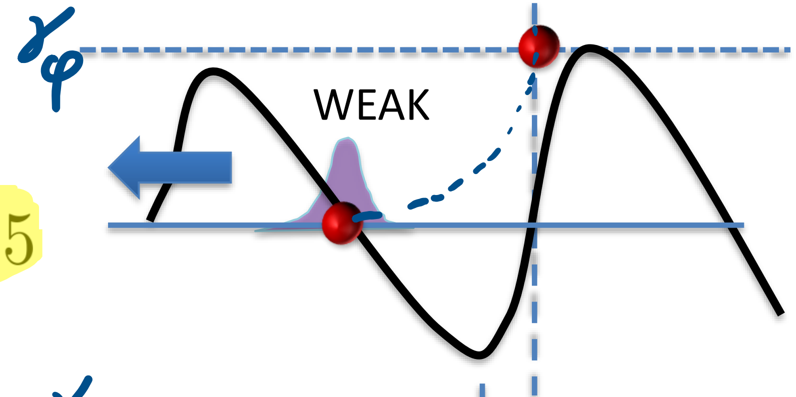
$$\mathcal{F} \equiv \frac{1}{2} E_{norm}|^2_{max}$$

$$(1+\phi_{Max,min}) = \mathcal{F} \pm \beta_{ph}\sqrt{(1+\mathcal{F})^2-1}$$

$$2|\beta_{ph}|\sqrt{(1+\mathcal{F})^2-1} \geq 1 - 1/\gamma_{ph}$$

Weak (standard) trapping threshold

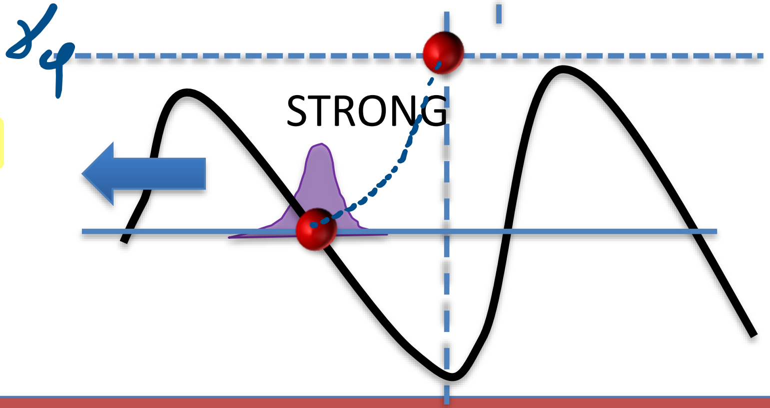
$$E_{norm} \simeq 0.5$$



Strong trapping threshold

$$\mathcal{F} + |\beta_{ph}|\sqrt{(1+\mathcal{F})^2-1} \geq 1 - 1/\gamma_{ph}$$

$$E_{norm} \simeq 0.7$$



[Tomassini et al.; Phys. Plasmas 24 (2017)]

Strong trapping condition is more demanding than the standard «weaker» one

OUTLOOK

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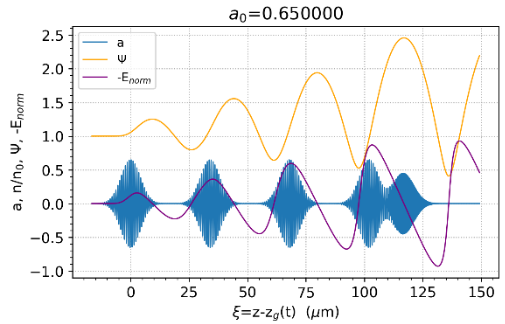
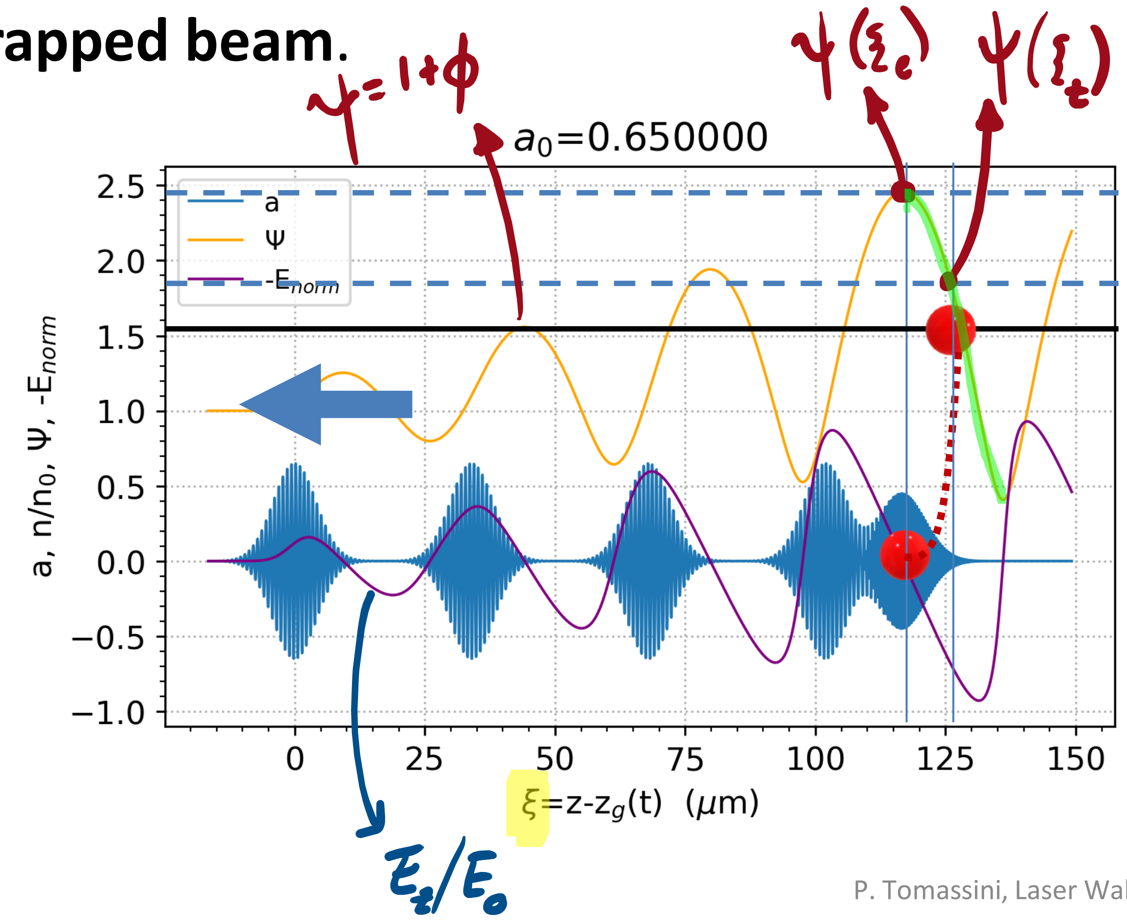
6. Downramp (or shock) injection
7. Two-Color injection
8. The Resonant Multi-Pulse Ionisation Injection
9. **High-Brilliance e-bunches**



ReMPI (and Two Color) attosecond sources

In Two Color and ReMPI ionization injection schemes the extraction of the electrons from the dopant occurs in a controlled way. **By tuning the distance between the node of the accelerating gradient and the peak of the ionization pulse we can vary the length of the trapped beam**.



$$\mathcal{H}(\eta,\gamma) = \gamma(1-\beta\beta_{ph}) - \phi(\eta)$$ 1D QSA

$$\eta = k_p(z-z_g) = k_p \cdot \xi$$ (phase)

$$\gamma_\perp^2 = 1 + u_\perp^2 = 1 + (a - a_{ex})^2$$ @ extr.

$$\phi(\xi_{tr}) = \phi(\xi_{ex}) - 1 + \gamma_\perp(\xi_{ex})/\gamma_\phi$$

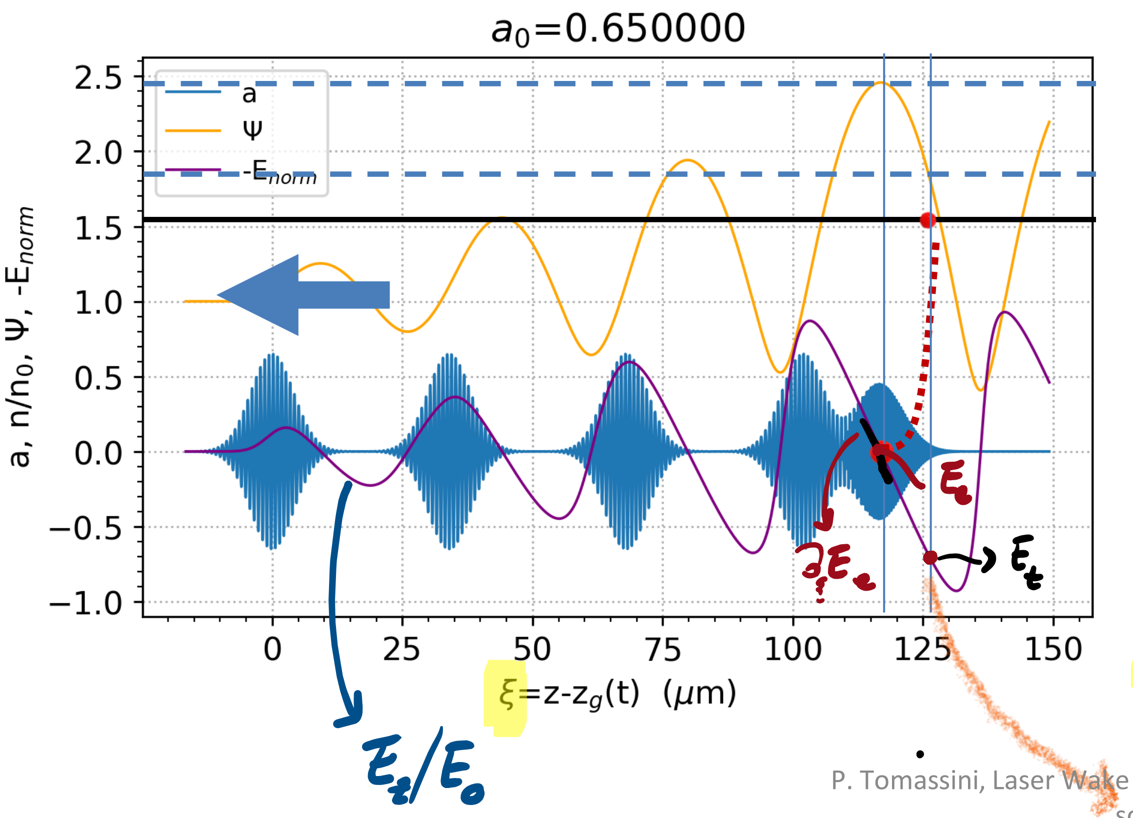
↳ @ trapping ↷ @ extraction

$$\xi_{tr} = \phi^{-1}(\phi(\xi_{ex}) - 1 + \gamma_\perp(\xi_{ex})/\gamma_\phi)$$

ReMPI (and Two Color) attosecond sources

In Two Color and ReMPI ionization injection schemes the extraction of the electrons from the dopant occurs in a controlled way. **By tuning the distance between the node of the accelerating gradient and the peak of the ionization pulse we can vary the length of the trapped beam**.

$$E_{norm} = E_z/E_0 = -\frac{\partial \phi}{\partial \eta}$$



We consider the electrons extracted by field ionization by the pulse of FWHM length cT. In the case of unsaturated regime the extraction coordinate is a random gaussian variable of variance

$$\sigma_{\xi_e} \simeq cT\sqrt{\frac{\rho_0}{4\log 2}}$$

The rms trapping positions can be evaluated in the limit $k_p\sigma_{\xi_e} \ll 1$ as

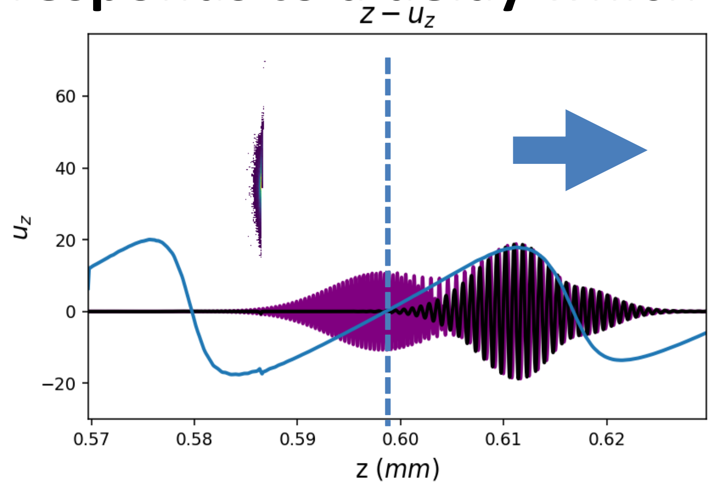
$$\sigma_{\xi_t} \simeq \sigma_{\xi_e}\frac{1}{E_t}\sqrt{E_e^2 + \frac{1}{2}(\partial_\xi E|_e)^2\sigma_{\xi_e}^2}$$

average field strongly depends on ξ_e

depends on the regime

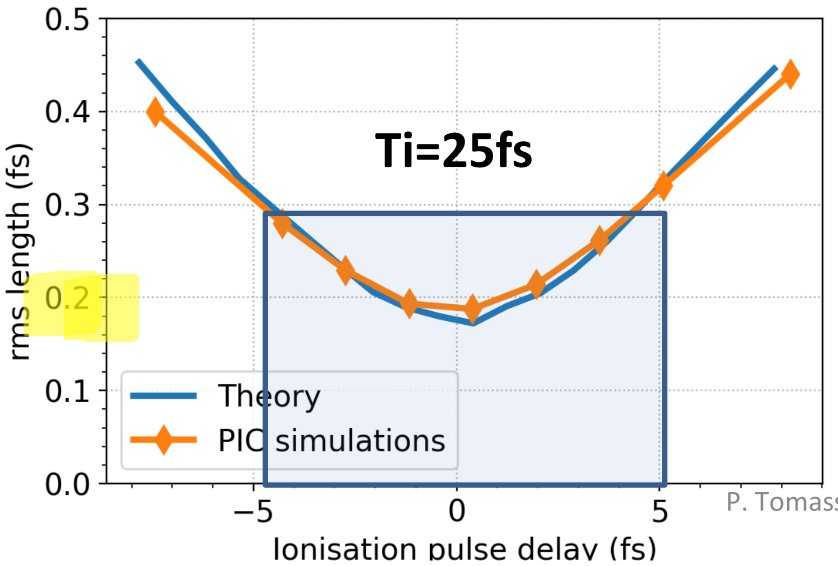
ReMPI (and Two Color) attosecond sources

Scan of the beam length as a function of the ionization pulse position. **The "zero position" corresponds to a delay which places the pulse on the node of the accelerating gradient**

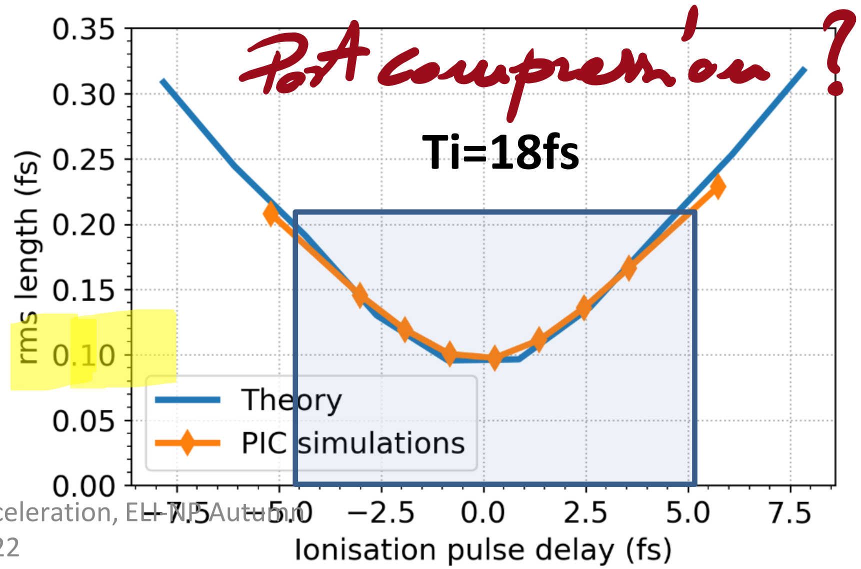


- **SINGLE Ti:Sa 200TW laser system, Circularly Polarised pulses**
- 4x 23fs FWHM pulses, w0=45 μm , total 5J on TEM00,
- 1x 25fs or 18fs FWHM ionization pulse **in II** harmonics, w0=3.5 μm
- 100%Ar (8+) plasma, n0=5e17 1/cm^3

- **PIC simulations with the quasi-3D FB-PIC code. Resolution λi/24 and λi/8 in the longitudinal and transverse directions**



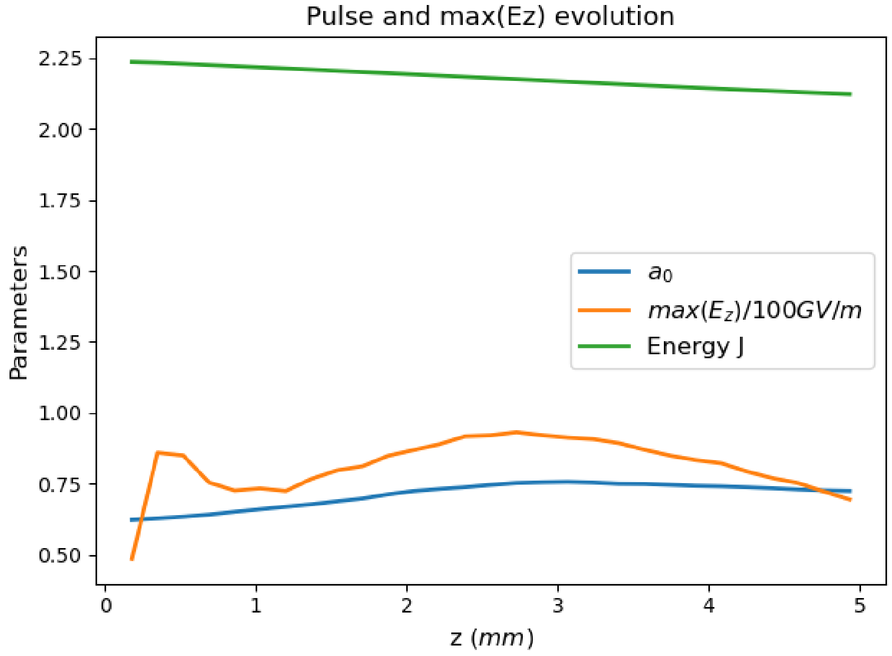
Timing jitter below 3fs *rms*

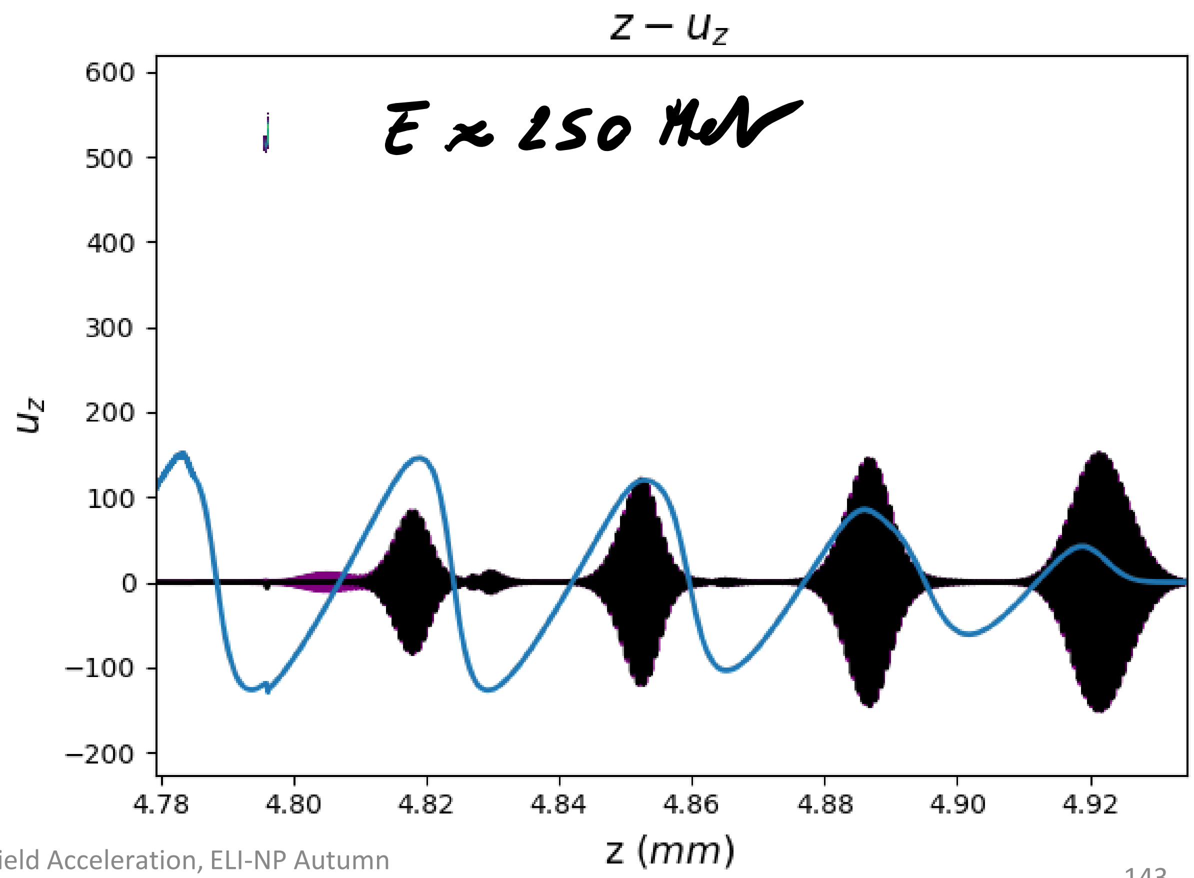


Flexible attosecond bunches source 1

- **SINGLE Ti:Sa 100TW laser system, Circularly Polarised pulses**
- 4x 23fs FWHM pulses, w0=30 μm , total 2.3J on TEM00,
- 1x 25fs FWHM ionization pulse **in II** harmonics, w0=3.5 μm, on TEM00

- 100%Ar (8+) plasma, n0=1e18 1/cm^3, 5 mm plate
- NO guiding needed





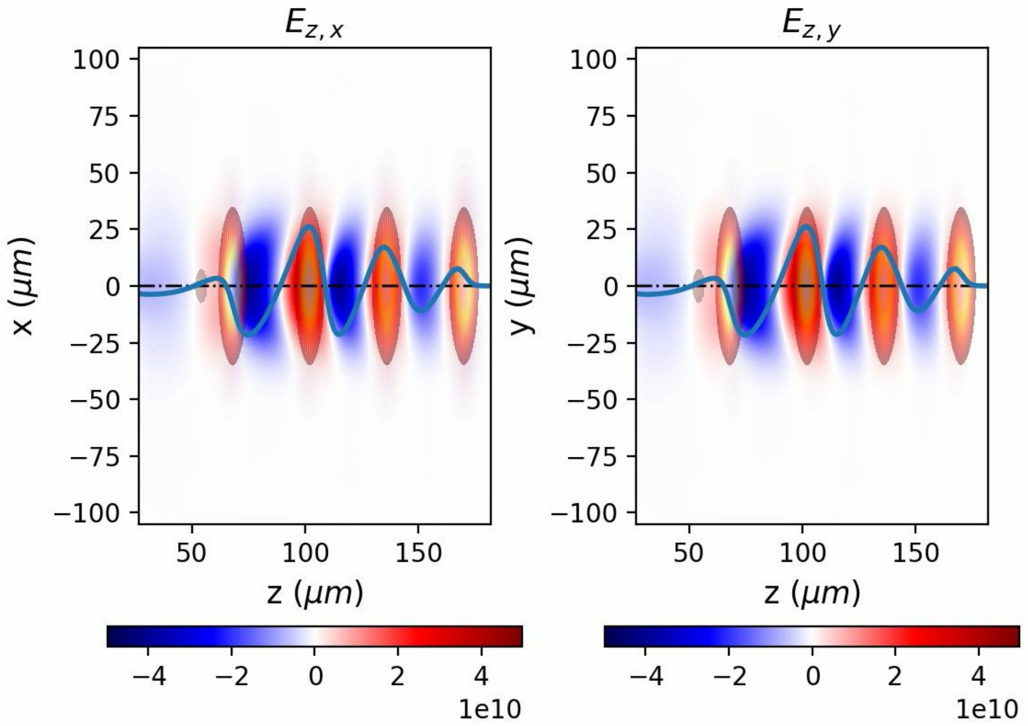
Flexible attosecond bright bunches sources 1

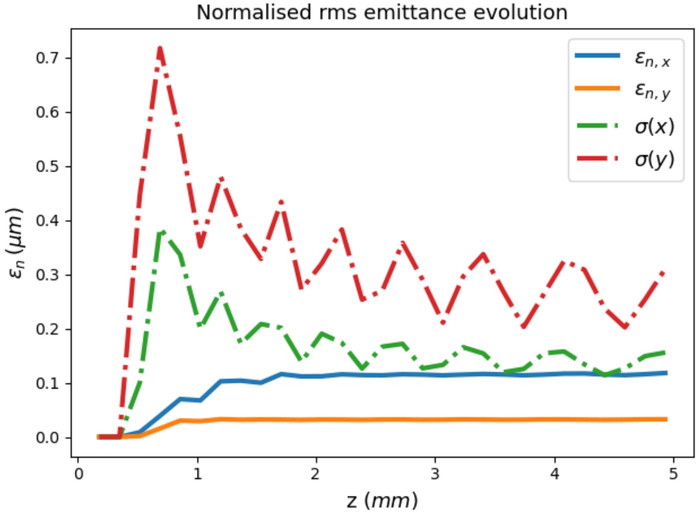
Charge	Energy	Energy spread	Duration	Norm emittance	Current
5 pC	250 MeV	1.6% rms	400as	30 nm rad 110 nm rad	8kA

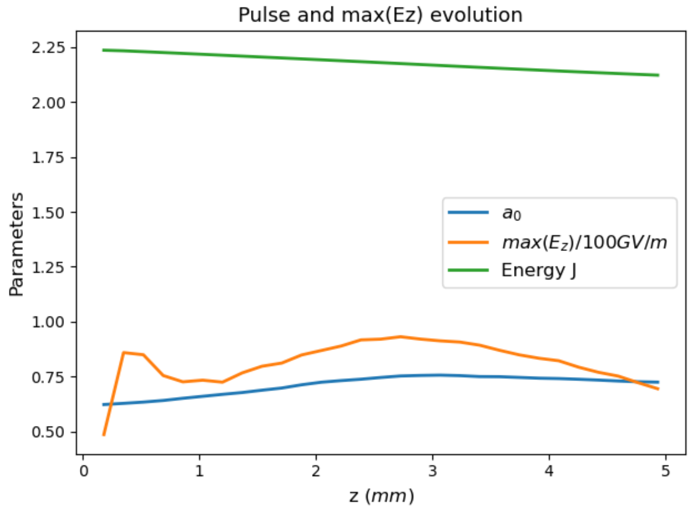
The final beam brightness is

$$B_{n,5D} \approx 3 \times 10^{18} A/m^2$$

$$B_{n,6D} \approx 1 \times 10^{17} A/m^2/0.1\%$$

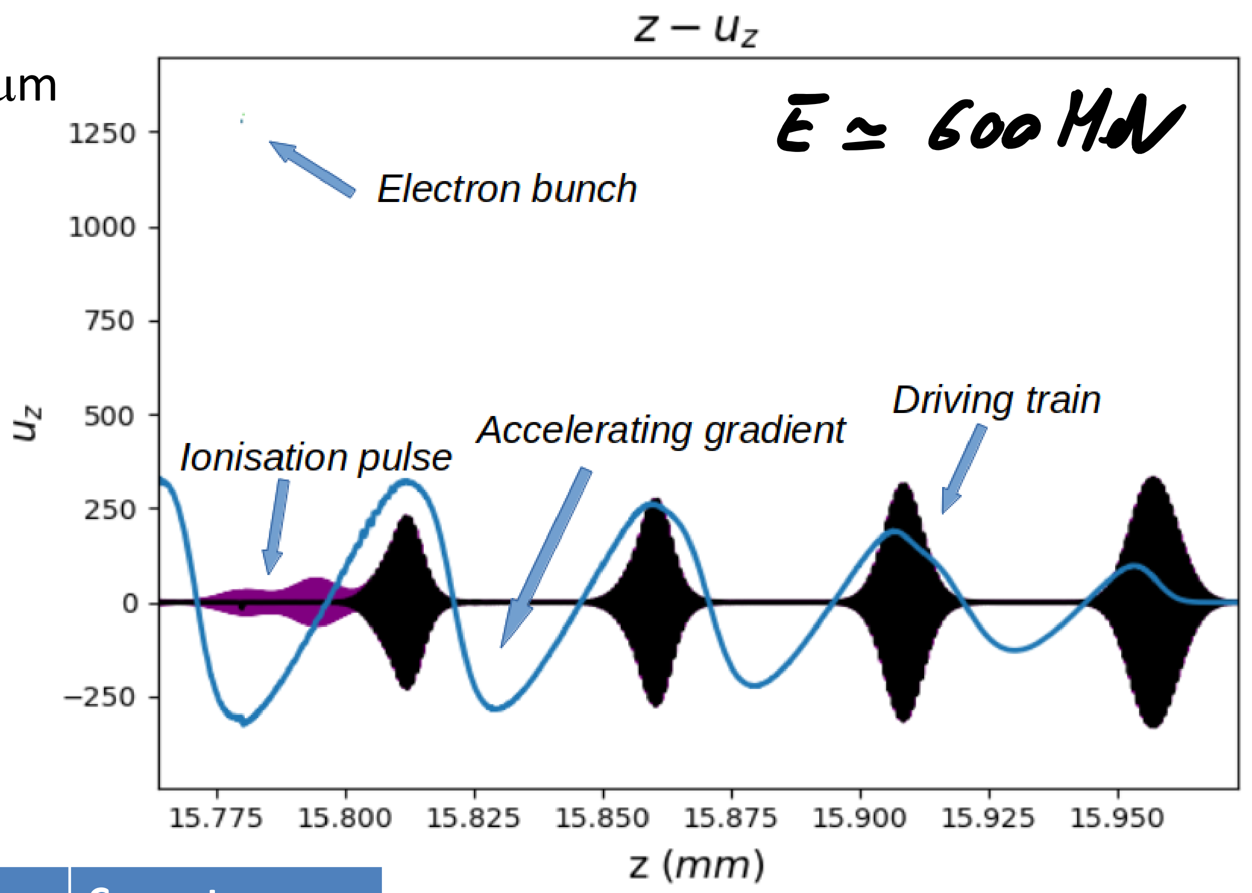






Flexible attosecond bunches source 2

- **SINGLE Ti:Sa 100TW laser system, Circularly Polarised pulses**
- 4x 23fs FWHM pulses, w0=30 μm , total 2.3J
- 1x 25fs FWHM ionization pulse **in II H**, w0=3.5 μm
- 100%Ar (8+) plasma, n0=1e18 1/cm^3 16 mm
- Parabolic Guiding



The final beam brightness is

$$B_{n,5D} \approx 2 \times 10^{18} A/m^2$$

$$B_{n,6D} \approx 2 \times 10^{17} A/m^2/0.1\%$$

Charge	Energy	Energy spread	Duration	Norm emittance	Current
5 pC	600 MeV	0.8% rms	600as	35 nm rad 120 nm rad	6kA

This Conceptual Design Report (CDR) presents the worldwide first high energy plasma-based accelerator that can provide industrial beam quality and user areas. It is the important intermediate step between proof-of-principle experiments and ground-breaking, ultra-compact accelerators for science, industry, medicine or the energy frontier.

The EuPRAXIA CDR is the result of a 4-year-long design study, funded by the European Union within the Horizon 2020 programme.

CNR INO CONSIGLIO NAZIONALE DELLE RICERCHE ISTITUTO NAZIONALE DI OTTICA

High-Brightness 5 GeV FEL-quality beam for EuPRAXIA single stage line

EuPRAXIA — eli nuclear physics

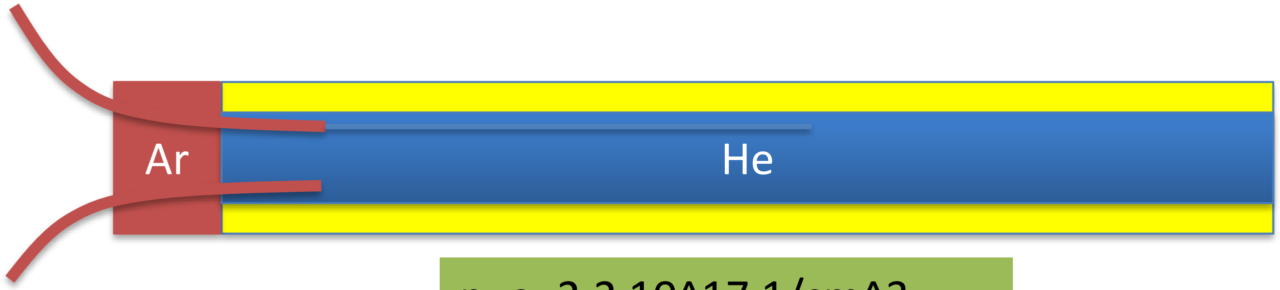
Motivation: Within the EuPRAXIA project we aimed at generating 5GeV bunches with FEL quality

GOAL:

δE/E SLICE	ε_n SLICE	Q	I peak
<0.1%	<0.1 **mmmrad**	>30 pC	>2kA

- As in any high-energy bunch (>>1GeV) setup a PW laser system is required.

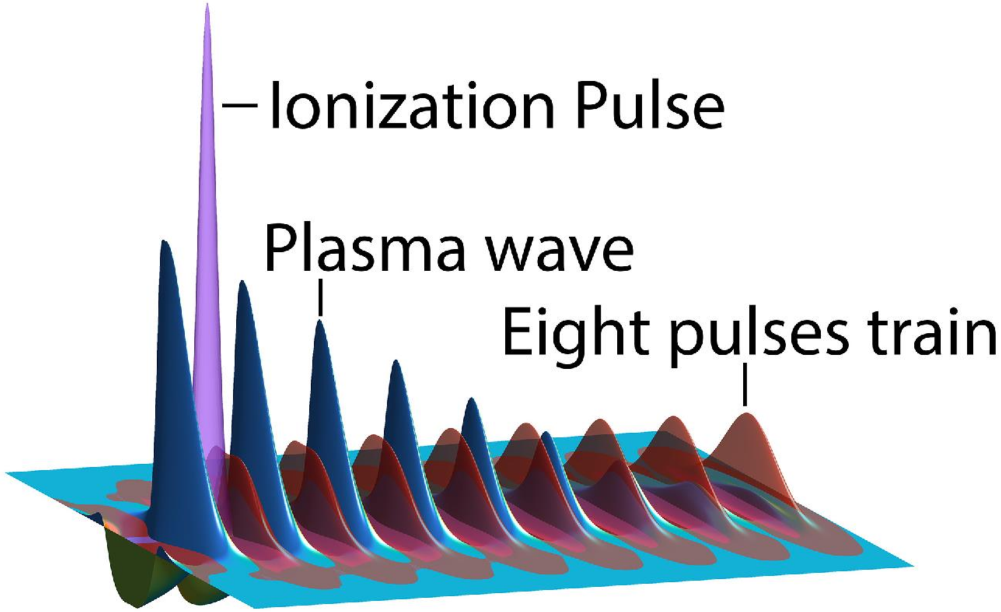
- The train of **LP 8** pulses, **55 fs** long, delivers about **900 TW** of power and are focused with a waist of **90 μm**.
- The target is a sequence gas-cell+capillary, being the gas-cell filled with a mixture Argon+Helium for injection. The capillary guides the pulse for about 20cm



n_e=2.2 10^17 1/cm^3

High-Brightness 5 GeV FEL-quality beam for EuPRAXIA single stage line

P. Tomassini et al., High-quality 5GeV electron bunches with the resonant multi-pulse ionization injection, PPCF P 62 (2020) 014010



Ionization pulse, 4^th harmonics of the Ti:Sa pulse

Wavelength	Energy	Duration	Waist
0.2 μm	0.06J	45 fs	4 μm

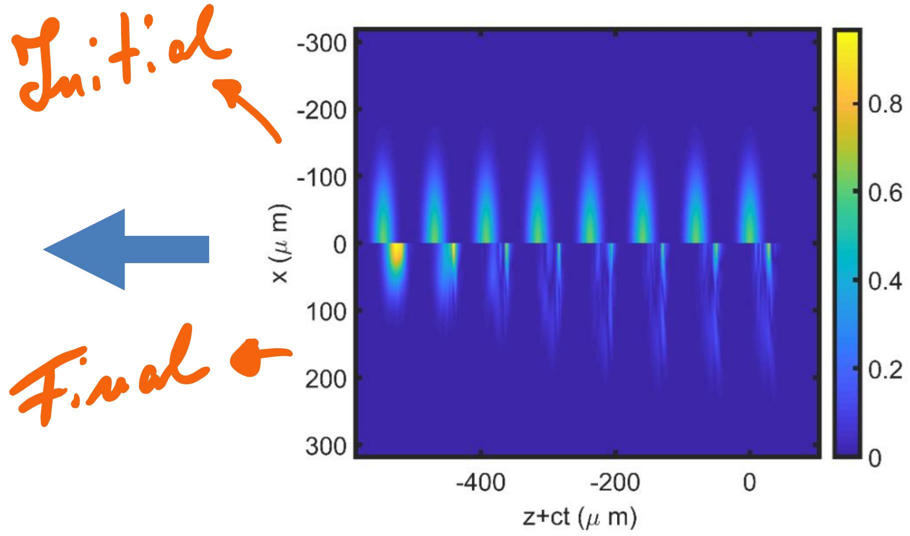
«Injector» gas jet (50%He,50%Ar)

Up/Down ramps	Plateau	Background plasma density
2mm	5mm	2.2 10^17 1/cm^3

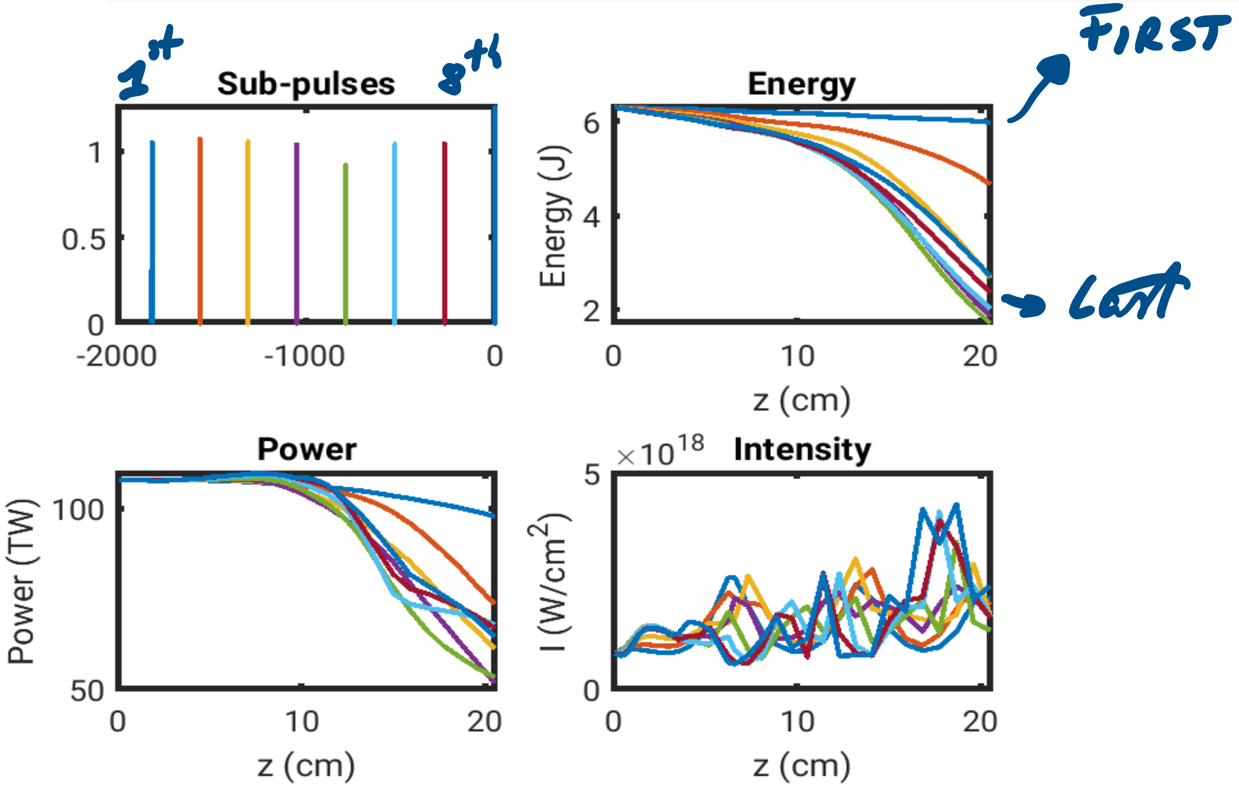
«booster» capillary (100% He)

Up/Down ramps	Plateau	Background plasma density	Channel depth for guiding
2mm	22 cm	2.2 10^17 1/cm^3	90% of the matched value

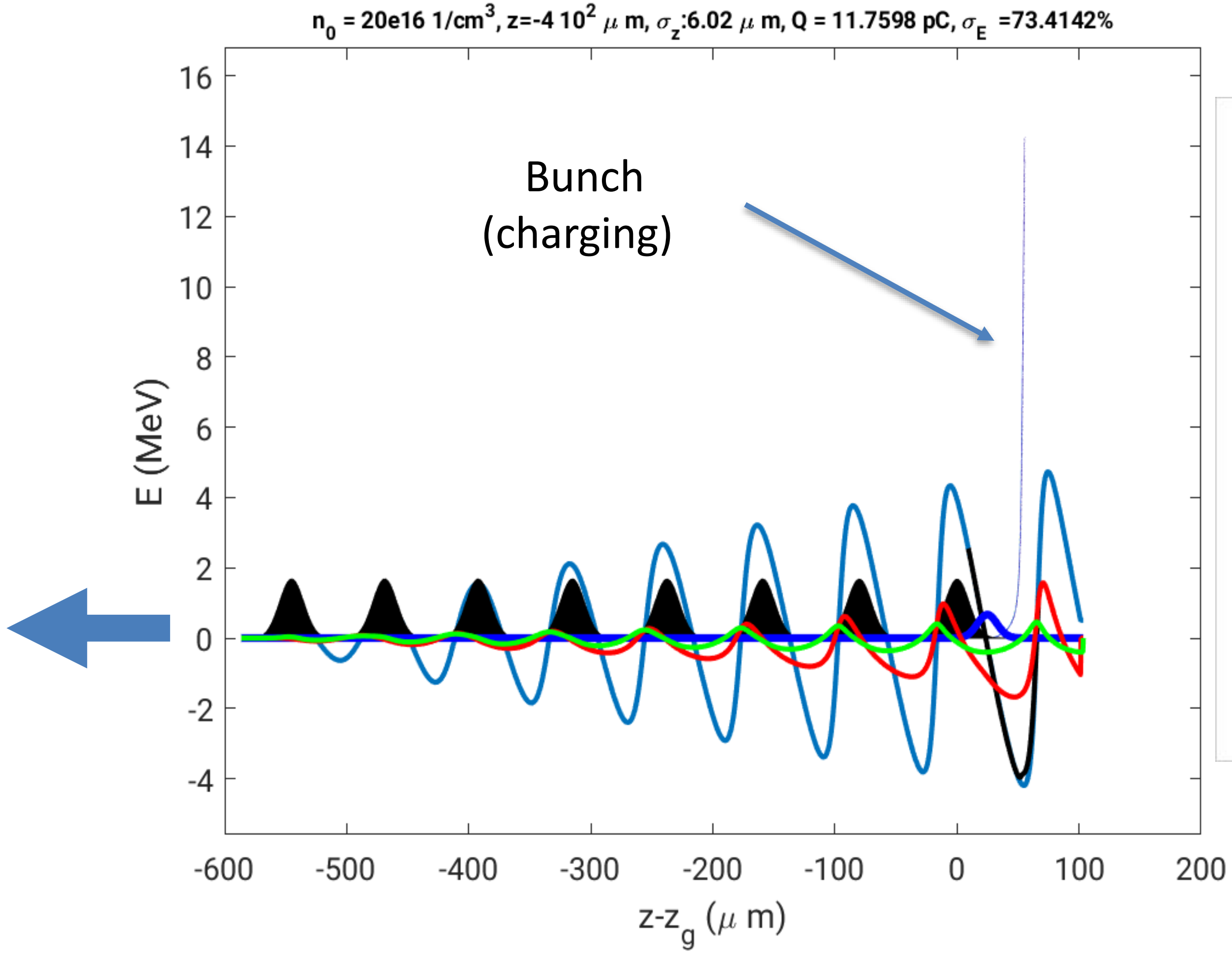
High-Brightness 5 GeV FEL-quality beam for EuPRAXIA single stage line

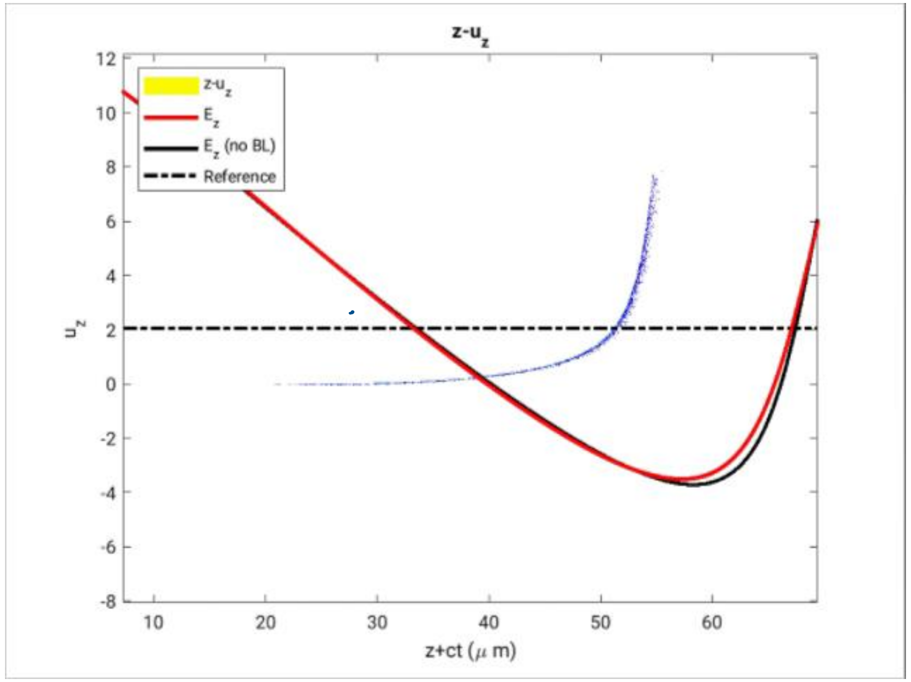


- Complex evolution of the pulses on the tail due to their propagation in a **perturbed background** plasma.
- An overall pump depletion of 70% is observed

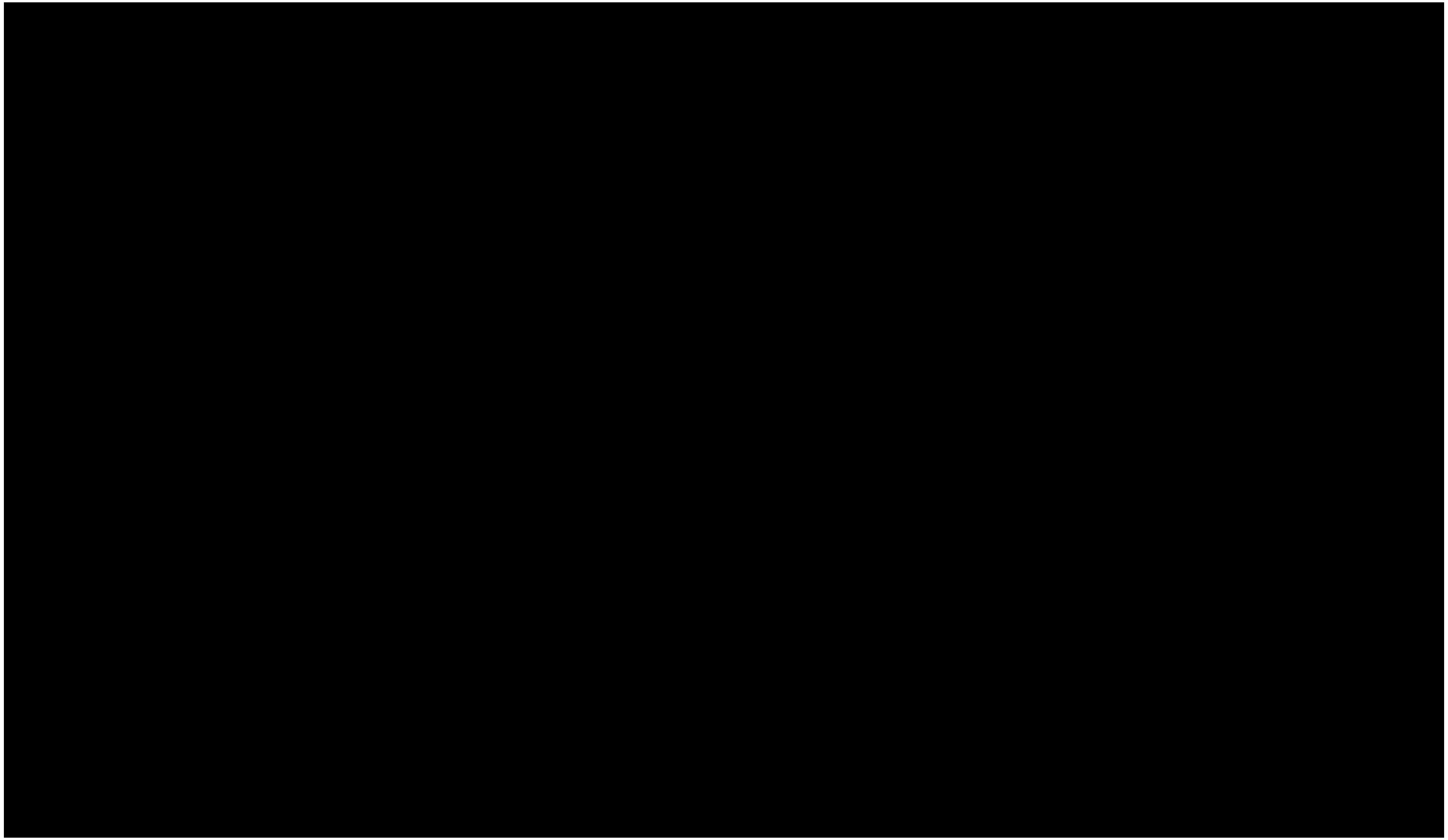


High-Brightness 5 GeV FEL-quality beam for EuPRAXIA single stage line



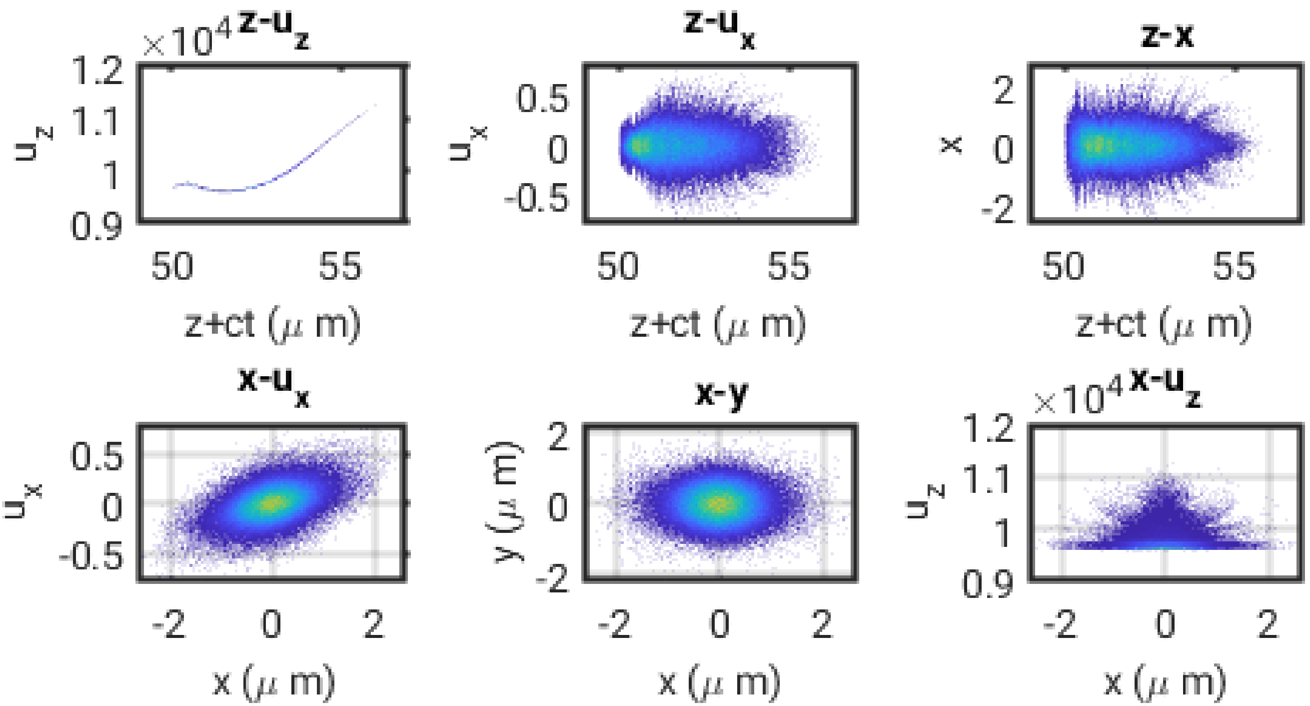


High-Brightness 5 GeV FEL-quality beam for EuPRAXIA single stage line



High-Brightness 5 GeV FEL-quality beam for EuPRAXIA single stage line

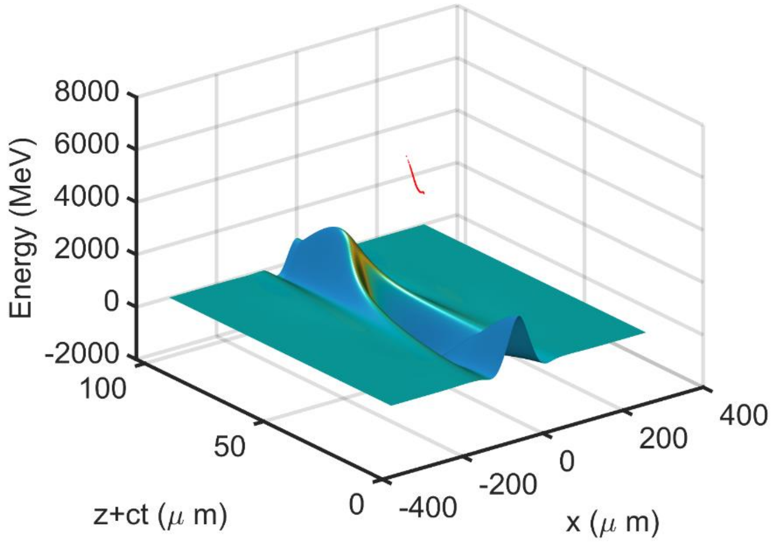
Projected beam quality



The accelerator fully complies with the requirements.
The final beam 6D brightness is

$$B_{n,6D} \approx 4 \times 10^{19} A/m^2$$

$$B_{n,6D} \approx 4 \times 10^{16} A/m^2/0.1\%$$



dE/E	ε_n	Q	I peak
0.9% **(92% of the charge)** 1.8 % (tail included)	0.085 mm mrad	32 pC	3.5 kA

High-Brightness 5 GeV FEL-quality beam for EuPRAXIA single stage line

Slice analysis

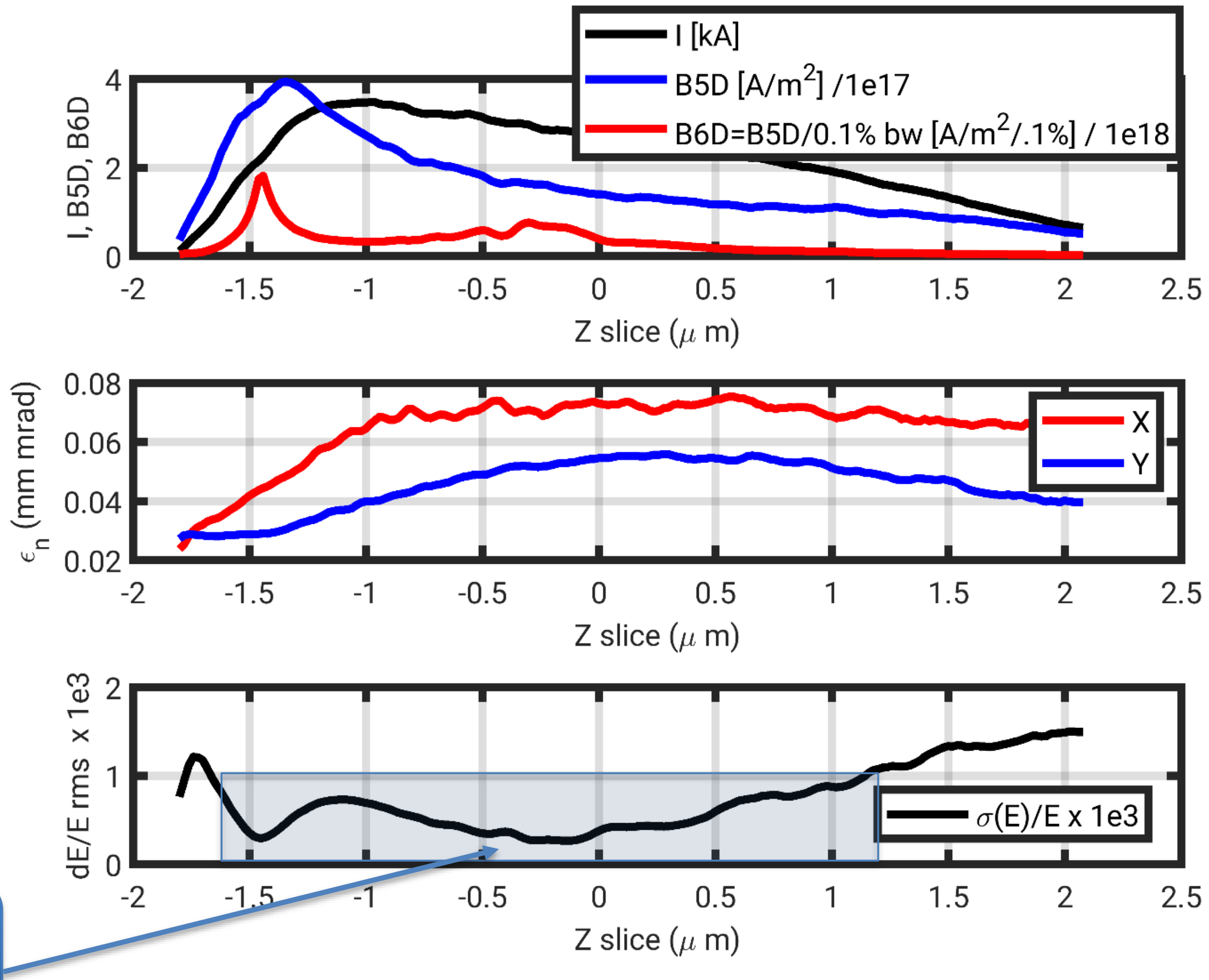
dE/E SLICE	ε_n SLICE	Q	I peak
0.05% **(min)**	**<0.08 mmmrad**	30 pC	3.5 kA

The slice analysis is FEL oriented
and reveals outstanding peak values

$$B_{n,5D} = \frac{2}{\pi^2}\frac{I}{\epsilon_{n,x}\epsilon_{n,y}}$$

$$B_{n,6D} = \frac{B_{n,5D}}{\delta E/E} \qquad B_{n,6D}|_{peak} \approx 2 \times 10^{21} A/m^2$$

$$B_{n,6D} = \frac{B_{n,5D}}{\delta E/E/0.1\%} \quad B_{n,6D} \approx 2 \times 10^{18} A/m^2/0.1\%$$



About **90%** of the slices have σ(E)/E<0.1%

High-Brightness 5 GeV FEL-quality beam for EuPRAXIA single stage line

PROJECTED 6D brightness

$$B_{n,6D} \approx 4 \times 10^{19} A/m^2$$

$$B_{n,6D} \approx 4 \times 10^{16} A/m^2/0.1\%$$

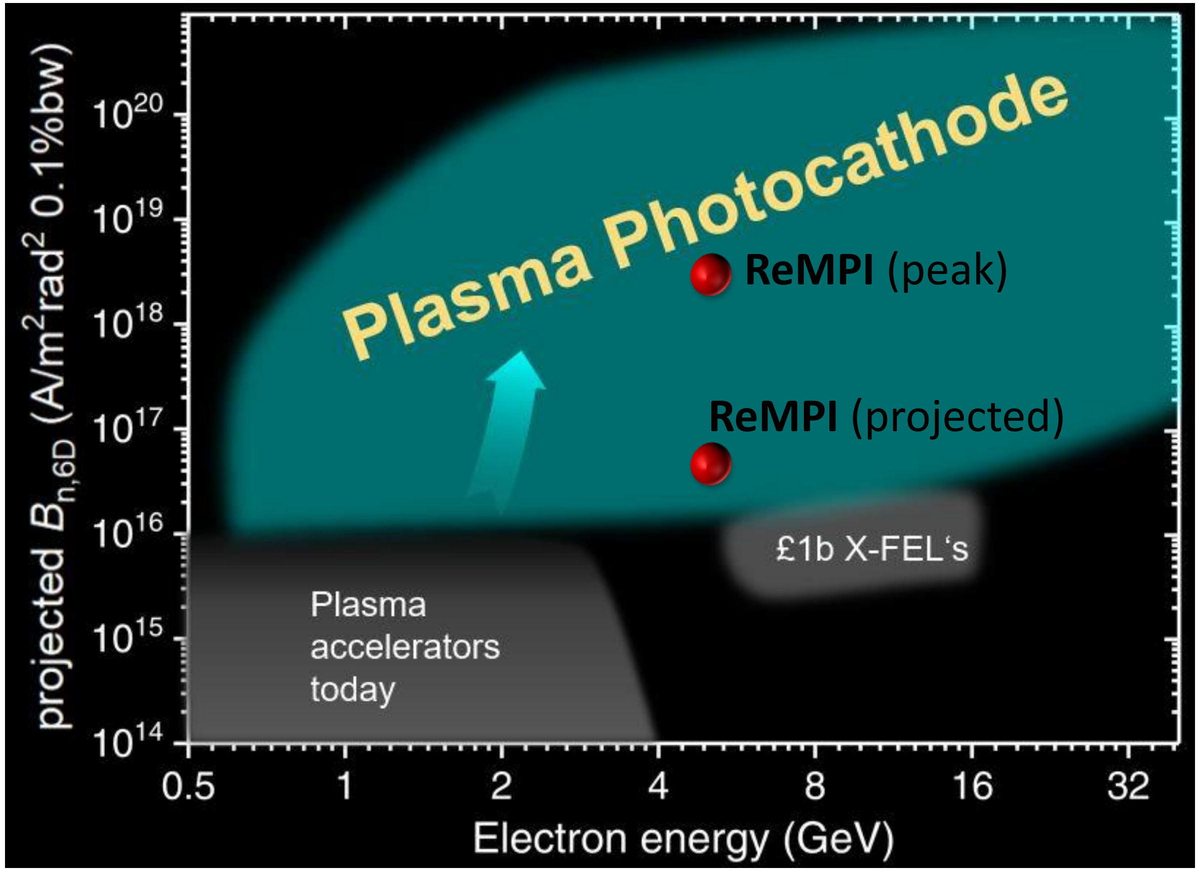
SLICE 6D PEAK Brightness

$$B_{n,5D} = \frac{2}{\pi^2} \frac{I}{\epsilon_{n,x}\epsilon_{n,y}}$$

$$B_{n,6D} = \frac{B_{n,5D}}{\delta E/E} \qquad B_{n,6D}|_{peak} \approx 2 \times 10^{21} A/m^2$$

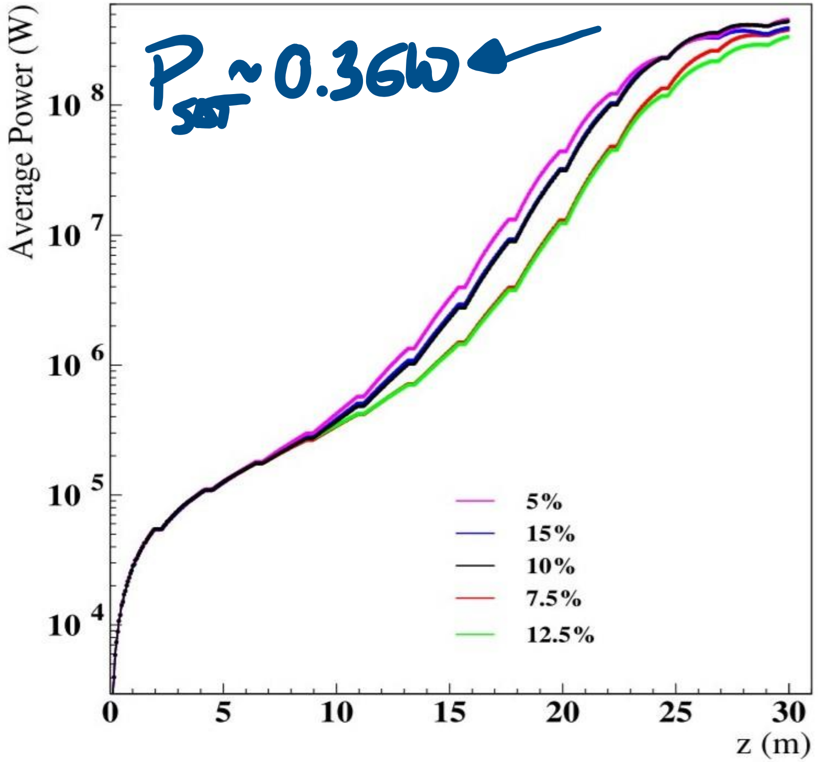
$$B_{n,6D} = \frac{B_{n,5D}}{\delta E/E/0.1\%} \quad B_{n,6D} \approx 2 \times 10^{18} A/m^2/0.1\%$$

https://pwfa-fel.phys.strath.ac.uk



FEL results @0.15nm

Start-to-end simulations from the ReMPI accelerator, the beam transport (**A. Giribono**) and the undulator (**F. Nguyen and L. Giannessi**). **Density-jitter** (in the downramp) **study**

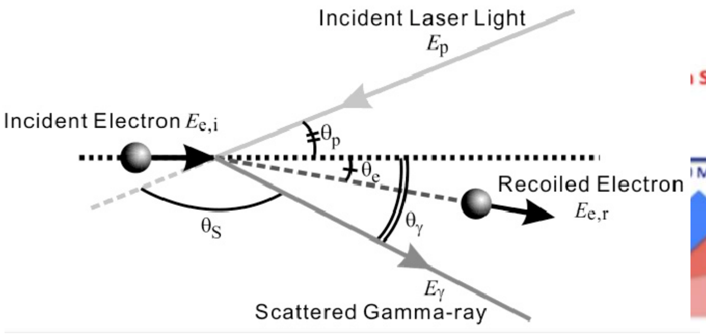


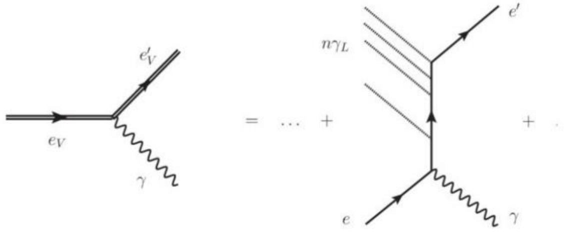
Genesis 1.3 simulations.

e-beam	L_G [m]	$E_p(z_{exit})$ [μJ]	λ_{exit} [nm]	N_γ/pulse [10^{10}]	N_γ/sec [10^{23}]
7.5%	1.753	9.28	0.152619	0.714	2.92
15%	1.781	9.60	0.152533	0.739	3.02
5%	1.912	11.15	0.152546	0.858	3.50
12.5%	1.756	8.22	0.152574	0.632	2.58
10%	1.791	10.78	0.152568	0.829	3.39

Compton vs Thomson scattering

COMPTON SCATTERING





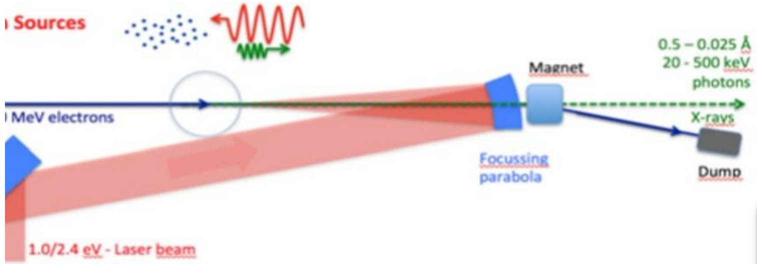
$$E_e = \gamma m c^2$$

$$\lambda_c = h/mc \simeq 2.4 \cdot 10^{-6} \mu m$$

Quantum recoil

$$\lambda_\gamma = \lambda_0 \frac{1-\beta \cos\theta_S}{1+\beta} + \frac{h}{mc\gamma} \frac{1+\cos\theta_S}{1+\beta}$$

Thomson backscattering X ray source



Courtesy of Luca Serafini

THOMSON BACK-SCATTERING

If **recoil is negligible**, quantum effects can be neglected and a classical description of the fields can be done

$$\lambda_c/\gamma \ll \lambda_0/4\gamma^2 \Rightarrow \lambda_0 \gg \gamma\lambda_c$$

$$\lambda_\gamma \simeq \lambda_0 \frac{1}{4\gamma^2} + \frac{h}{mc\gamma}$$

For a backscattered radiation

Example: Ti:Sa laser $\gamma \ll \lambda_0/\lambda_c \simeq 10^5$

$$\lambda_0 = 0.8 \mu m$$

$$E_{electron} \leq 10^s \text{ of GeV}$$

Thomson scattering theory (very early history)

Linear Thomson/Compton processes have been proposed as X/γ sources in the early 60's
[F.R. Arutyunian, V.A. Tumanian, Phys. Lett. 4, 176 (1963)] [R.H. Milburn, Phys. Rev. Lett. 10, 75 (1963)]
[C. Bemporad, R.H. Milburn, N. Tanaka, Phys. Rev. 138, 1546 (1965)]

First work on nonlinear Thomson backscattering from a single electron and a counterpropagating flat-top pulse
[E. Esarey, S.K. Ride, P. Sprangle, Phys. Rev. E 48(4), 3003 (1993)]

Idea of using PWF e-beams for Thomson sources [P. Catravas, E. Esarey, W.P. Leemans, Meas. Sci. Technol. 12, 1828 (2001)]

Idea of using TS as attosecond source [K. Lee, Y.H. Cha, M.S. Shin, B.H. Kim, D. Kim, Phys. Rev. E 67, 026502 (2003)]

First proposals for medical imaging [E.G. Bessonov, A.V. Vinogradov, A.G. Tourianskii, Instrum. Exp. Tech. 45(5), 718 (2002)]

Initial phase effect firstly envisaged in [F. He, Y.Y. Lau, D.P. Umstadter, R. Kowalczyk, Phys. Rev. Lett. 90(5), 055002 (2003)]

First work on nonlinear Thomson backscattering for a whole electron bunch, including initial phase effects and a comprehensive analysis of the varius working regimes [P. Tomassini, A. Giulietti, D. Giulietti and L.A.Gizzi, Appl. Phys. B 80, 419–436 (2005)]

Example of TS nonlinear effects

TOMASSINI et al. Thomson backscattering X-rays from ultra-relativistic electron bunches and temporally shaped laser pulses 425

Nonlinear (flat-top) regime

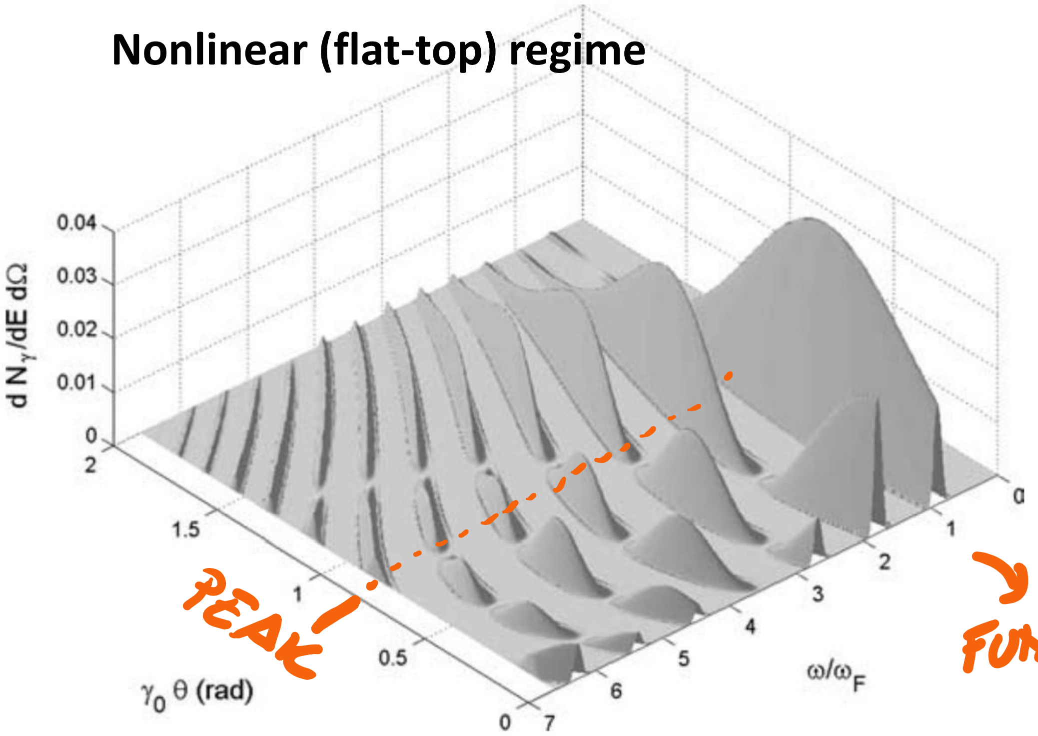


FIGURE 5 Spectral and angular distribution of the photons emitted by the head-on collision between a 5-MeV electron ($\gamma_0 = 10$) with an *off-axis* $\vec{v}$ vector ($\theta_e = 50$ mrad, $\phi_e = \pi/2$) and a laser pulse of amplitude $a_0 = 1.5$, wavelength $\lambda_0 = 1$ μm and duration $T_2 = 20$ fs. The radiation is collected at the azimuthal angle $\phi = \pi/2$ and the scattered-photon pulsation is shown in units of the fundamental frequency $\omega_F = \omega_0/\rho_0$ at ($\theta = \theta_e, \phi = \phi_e$)

K = a_0 = 1.5

γ = 10

$$\omega_{Res,n} \simeq 4\gamma^2 n\omega_0/(1+\gamma^2\tilde{\theta}^2 + a_0^2/2)$$

$$\tilde{\theta}^2 = \theta^2 + \theta_e^2 - 2\theta\theta_e\cos(\phi-\phi_e)$$

↳ Angle between the emission direction and particle speed

The bandwidth of each harmonic is very low and they **are well separated.**

Several (well separated) harmonics are produced.

An increasing complexity with ***n*** of the angular distribution is found

Codes we are using for TS simulations

Thomson Scattering Simulations Tools (TSST) [P. Tomassini, 2005-]

Semi-analytical tool, particularly suitable for very long pulses. Uses analytical results from P. Tomassini et al., APB 80 (2005). TEM00 modes allowed with arbitrary longitudinal profiles. Perfect (or quasi-perfect) backscattering with the beam is supported.

* Thanks to D. Dreghici for the nice logo

Relativistic Nonlinear Thomson Scattering (RelNTS) [P. Tomassini, 2021-]

(Brand new) Fully numerical code, suitable for not-so-long pulses. Structured laser pulses and arbitrary incidence angles allowed. Fully parallelised

NOTE: the time distribution of the collected radiationis linked to the time distribution of the electron bunch by the simple kinematic relation (backscattering)

$$\sigma(t_X) \simeq \sigma(t_b) + T/(4\gamma^2)$$

x/γ Beam Pulse

Useful relations to start dimensioning a source

«VERY Good bunches»
The photons energy spread is dominated by boost variation at different angles, **NOT** by energy spread and divergence of the bunch

$$\Psi \equiv \gamma \cdot \theta_c$$

Normalized acceptance
EXTREMELY USEFUL parameter

Angularly integrated spectrum

$$S(\omega) = (\mathcal{F}N_e)\begin{cases} \frac{\alpha}{64}\frac{Ta_0^2}{\omega_0^2\gamma^6}(\omega^2 - 4\omega\omega_0\gamma^2 + 8\omega_0^2\gamma^2) \\ \text{if} \quad \frac{4\gamma^2\omega_0}{1+\gamma^2\theta_{\text{Max}}^2} \leq \omega \leq 4\gamma^2\omega_0, \\ 0, \quad \text{otherwise,} \end{cases}$$

$$\delta\omega/\omega \simeq \Psi^2(1+\Psi^2/2); \text{ if } \Psi \ll 1 \Rightarrow \delta\omega/\omega \simeq \Psi^2$$

$$N_{\text{Acc}}(\psi) = (\mathcal{F}N_e)\frac{1}{2}\alpha\omega_0 T a_0^2 \psi^2 \frac{(1+\psi^2+2\psi^4/3)}{(1+\psi^2)^3}.$$

OVERLAP factor

$$\propto Q \qquad N_\gamma \propto \mathcal{E}/w_0^2$$

$$\text{if } \Psi \ll 1 \Rightarrow N_{Acc}(\Psi) \simeq \delta\omega/\omega \simeq \Psi^2$$

P. Tomassini et al., APB 80 (2005).

USEFUL RELATIONS for a «DECENT» BUNCH

«Decent bunches»
The bunch has **energy spread well below 100%**
Also, transverse momentum **u** does not exceed unity

$$\Psi \equiv \gamma \cdot \theta_c$$

Normalized acceptance
EXTREMELY USEFUL parameter

$$\psi_e = \sigma(|\vec{u}_\perp|) \approx \gamma\sigma(\theta_e) \quad \Rightarrow \quad \sigma(\psi_e) < 1, \quad \sigma(\gamma)/\gamma \ll 1$$

Estimation of the final collected photons energy spread

$$\delta\omega/\omega \approx \Psi^2 + \sigma(u_\perp)^2 + 2\frac{\delta\gamma}{\gamma} + a_0^2/2$$

NOTE: some authors quote **WRONG** expressions as $(\epsilon_n/r)^2$ as for a bunch with **correlated** (x, u_x)

$$\epsilon_n/r < \sigma(u_\perp)$$

NOTE: IT IS **USELESS** TO REDUCE Ψ BELOW:

$$\Psi^2_{min} \approx \sigma(u_\perp)^2 + 2\frac{\delta\gamma}{\gamma} + a_0^2/2$$

Energy Spread

$$(\delta\omega/\omega)_{min} \simeq \sigma(u_\perp)^2 + 2\frac{\delta\gamma}{\gamma} + a_0^2/2$$

This means that it is useless to reduce the acceptance Ψ below the value dictated by beam quality

P. Tomassini et al., APB 80 (2005).

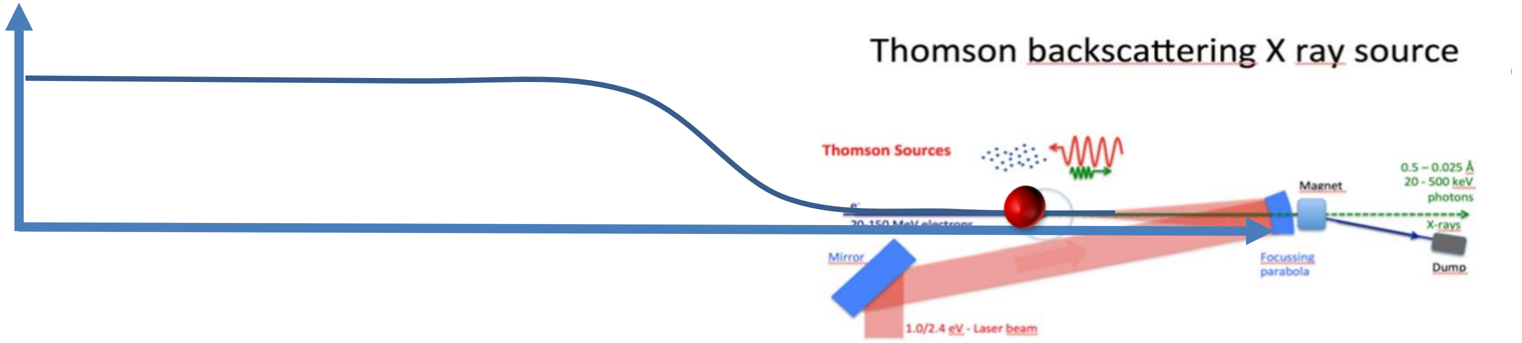
TS results for the high-brightness **5GeV** beam

Projected beam quality @ downramp

dE/E	Q	σ(u perp.)	σ(x perp.)	σ(x long.)
0.9%	32pC	0.12	0.75 μm	1μm

$$(\delta\omega/\omega)_{min} \simeq \sigma(u_\perp)^2 + 2\frac{\delta\gamma}{\gamma} + a_0^2/2$$

$$(\delta\omega/\omega)_{min} \approx 5\%$$



Counterpropagating pulse Yb:YAG (1.053 μm)

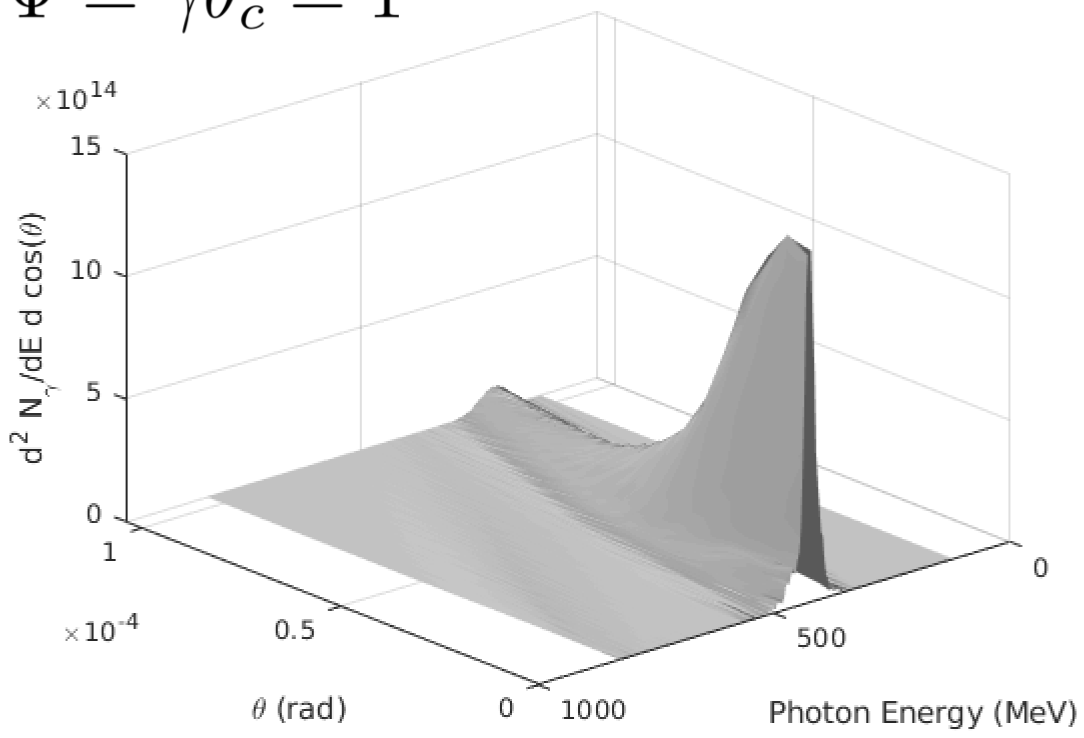
Energy	Duration	w0	a0
1J	2ps	12.5 μm	0.2

- Y. Wang et al. 1.1 J Yb:YAG picosecond laser at 1 kHz repetition rate
- 2020 Dec 15;45(24):6615-6618. doi: 10.1364/OL.413129

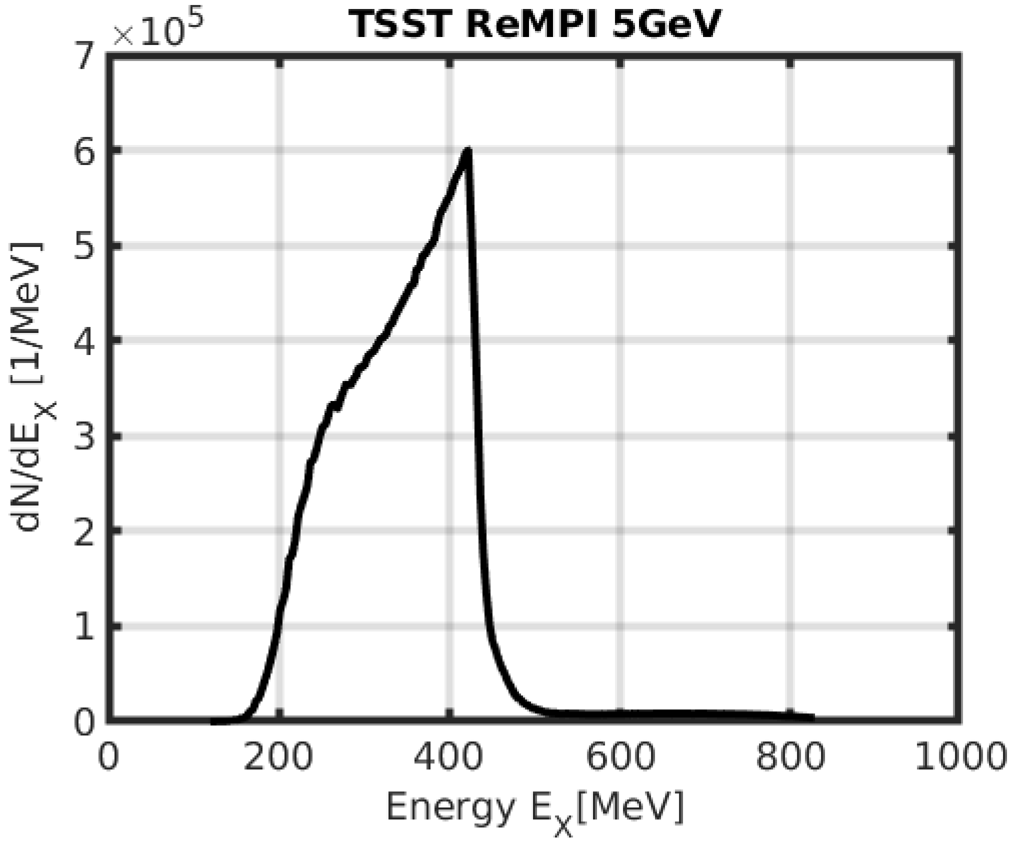
TS results for the high-brightness **5GeV** beam

Large acceptance yield

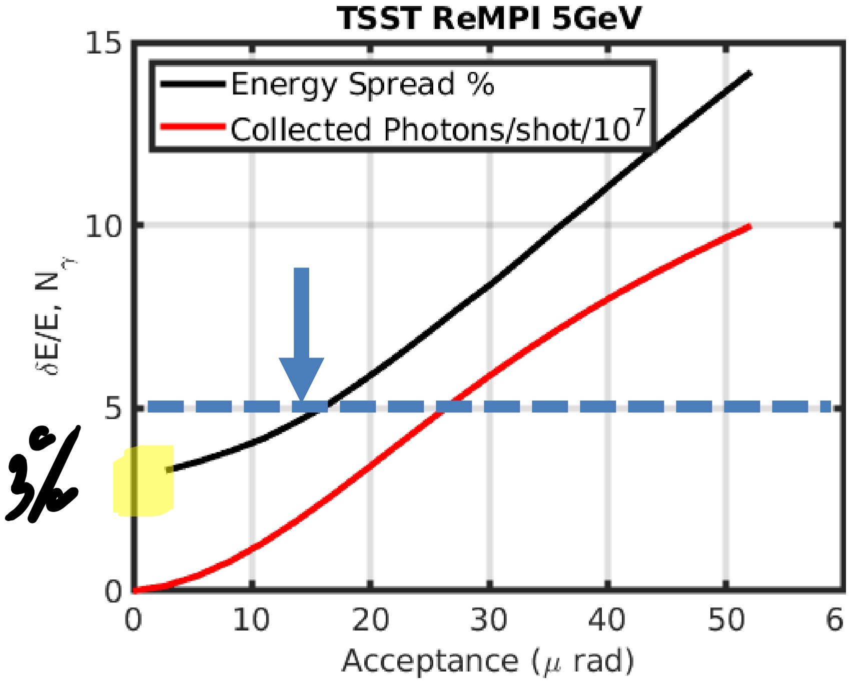
$$\Psi = \gamma\theta_c = 1$$

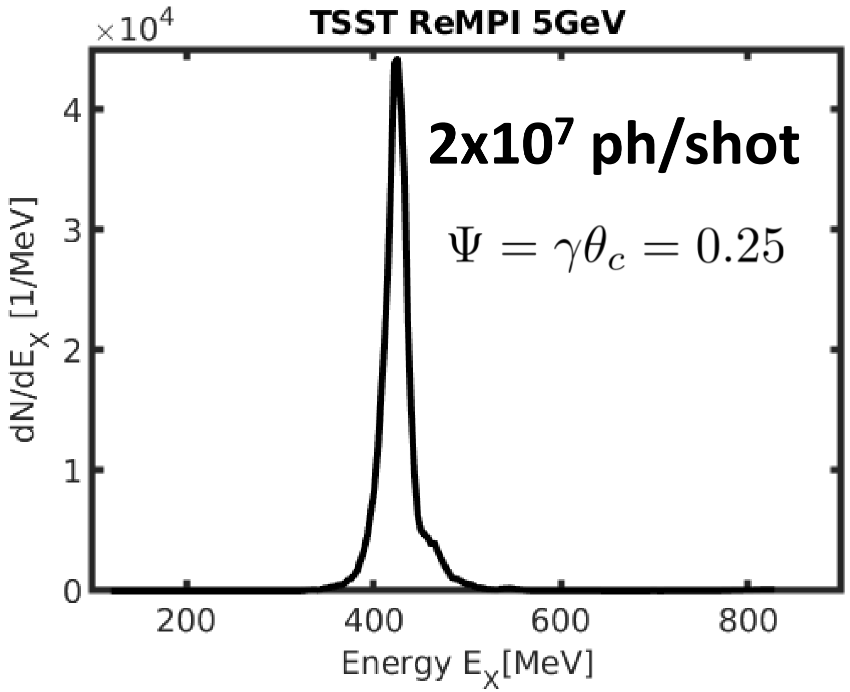


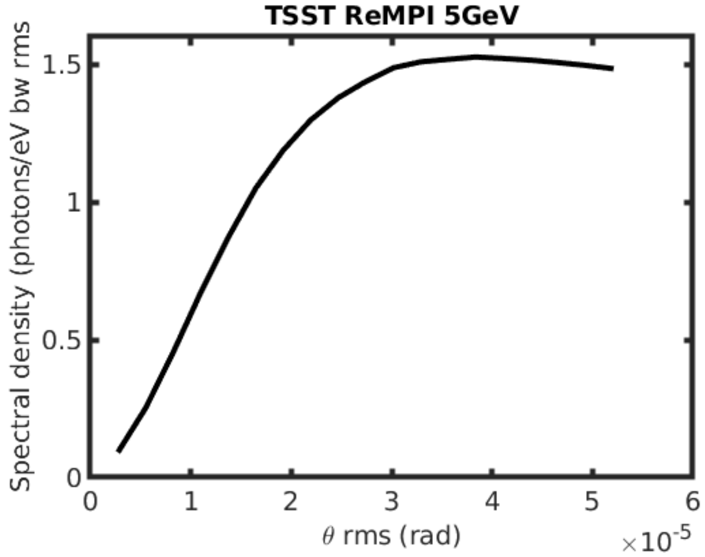
$1.0\text{x}10^8$ ph/shot



TS results for the high-brightness **5GeV** beam





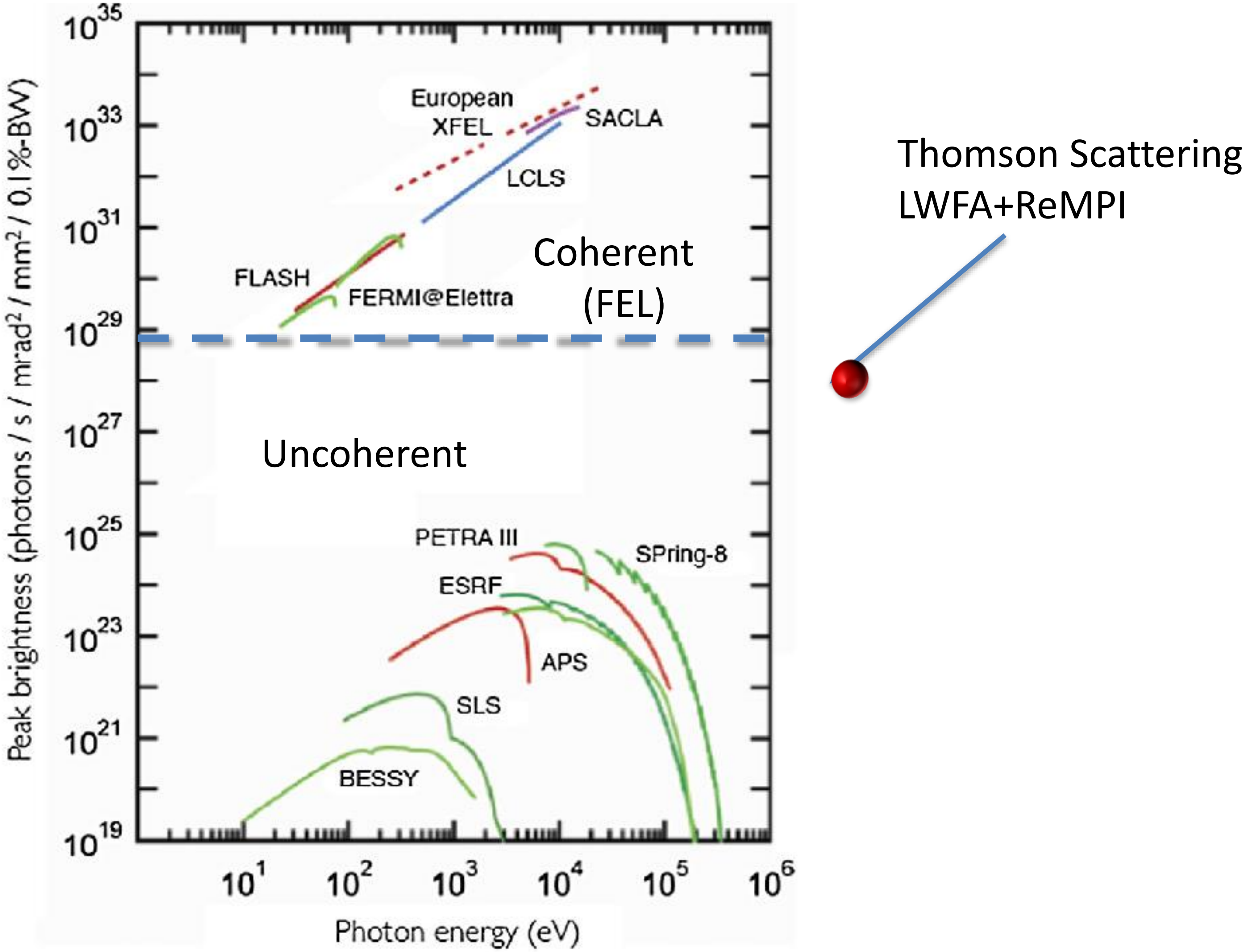


Selecting a normalised acceptance of $\Psi = \gamma\theta_c = 0.25$
a quasi monochromatic source (FWHM energy spread of 5%)
2×10^7 ph/shot can be obtained.

$$B = \frac{N_{ph}\text{‰}bw}{\delta t_\gamma(s)S(mm^2)\theta_{max}^2(mrad^2)} = 2 \cdot 10^{28}\, ph/\left(s \cdot mm^2 \cdot mrad^2 \cdot \text{‰}bw\right)$$

Gamma beam Brilliance with the **5GeV** beam

$$B = \frac{N_{ph}‰bw}{\delta t_{\gamma}(s)S(mm^2)\theta_{max}^2(mrad^2)} = 2\cdot 10^{28}\, ph/\left(s\cdot mm^2\cdot mrad^2\cdot ‰bw\right)$$



Thank you for your attention