NUMERICAL STUDY OF LENS FOCUSING METHODS

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OBJECTIVE

- Numerical focusing comparison of four types of optical profiles.

TABLE OF CONTENTS

1. The numerical process of profiles implementation
2. Characterization of Gaussian beam propagation
3. Comparing the focus of several types of profiles
4. Comparison of the beam width of the focus for the four profiles
5. Comparison of the FWHM of the focus

Conclusions

Bibliography

THE BENEFIT OF GOOD FOCUSING

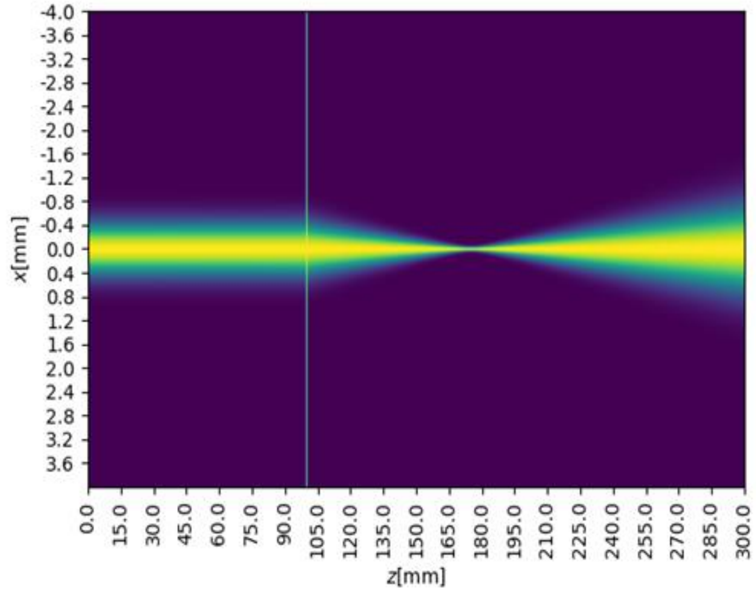
- A good focus means obtaining a maximum intensity in a small a region which can be used for various purposes.

1. Numerical process

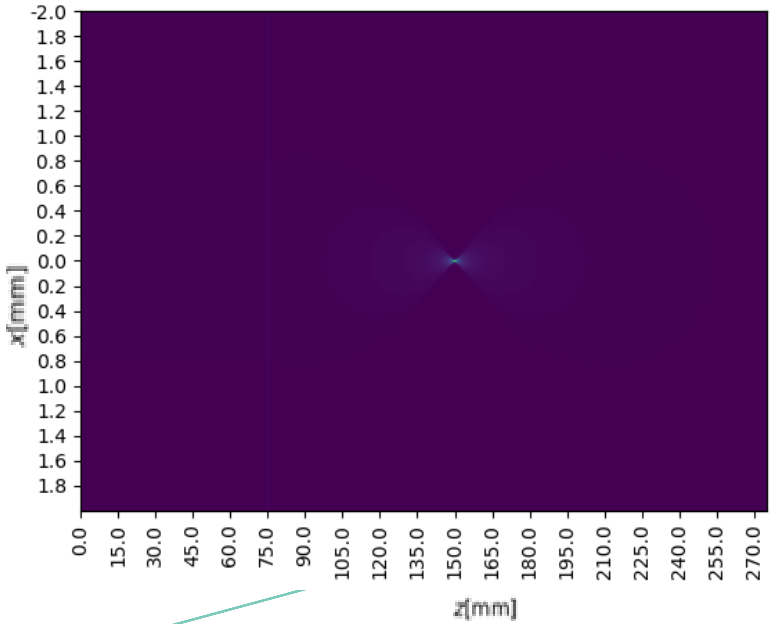
It consists of numerical simulation in the PyParax module of the propagation of various beams. The propagation optical system of the optical profile involves:

- The x-axis is the transverse axis and the axis on which the function is defined.
- Propagation is on the z-axis over a distance of 300 mm.
- The beams are normalised at the maximum value.

Normalised Gaussian Beam irradiance during propagation



Unnormalised Gaussian Beam irradiance during propagation



Optical system of beam propagation:

- f_{input} = initial condition represented by the input profile
- system = [100, ['l', 75, 0, 0], 300]
- f_{output}=the result of the propagation of the initial condition

I implemented three types of profiles in order to compare their focus. The Gaussian beam has already been implemented.

a) Ideal Top-hat function

b) Lorenz function

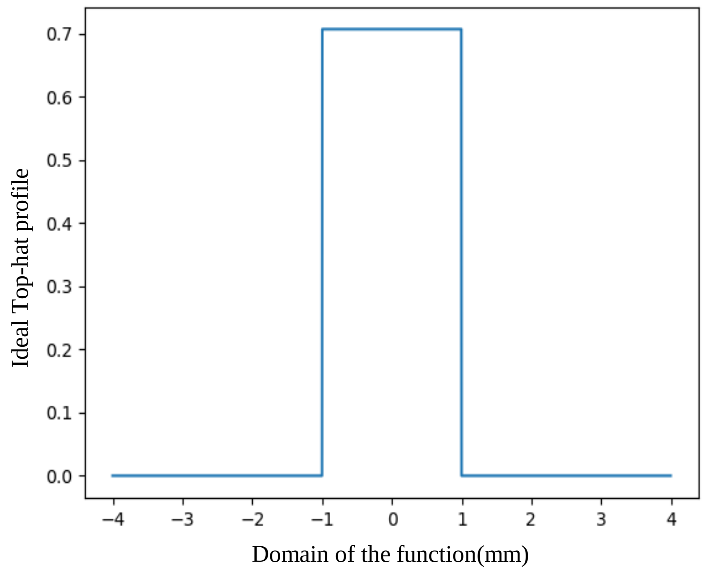
c) Woods-Saxon function

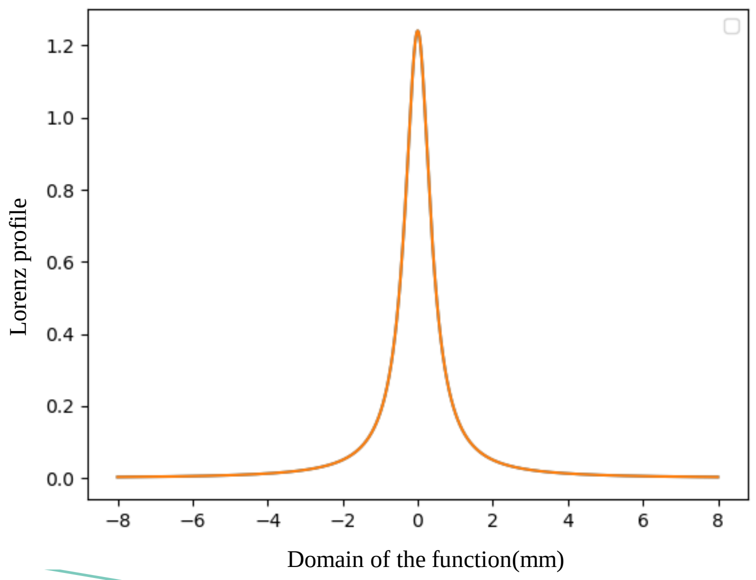
$$f_{top-hat}(x) = \begin{cases} 1, & x_{min} < x < x_{max} \\ 0, & otherwise \end{cases}$$

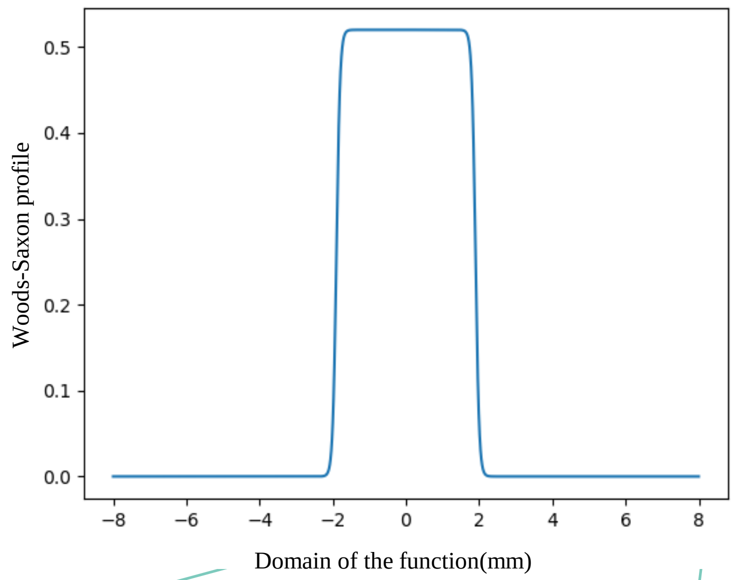
$$f_{Lorenz}(x) = \frac{1}{a+x^2}$$

$$f_{Woods-Saxon}(x) = \frac{1}{1+e^{\left(\frac{|x|-R}{a}\right)}}$$

(Andrea Aiello et. al., Observation of concentrating paraxial beams, 2020)



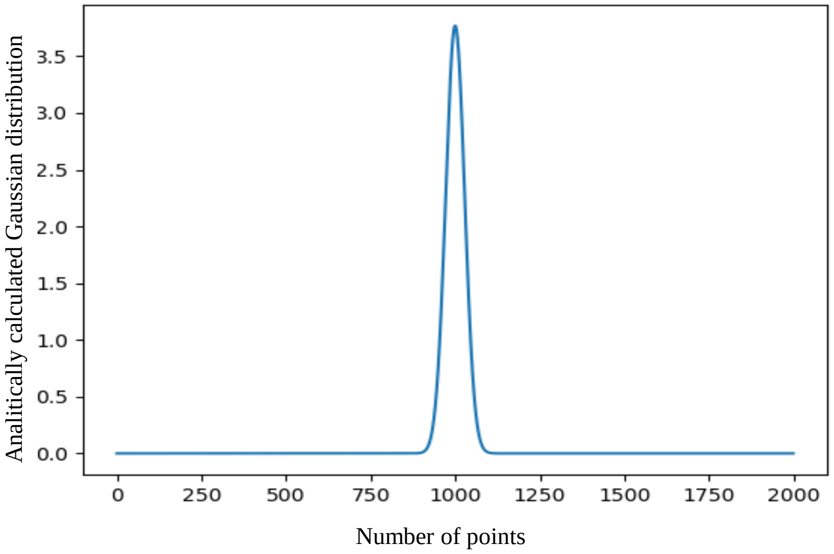


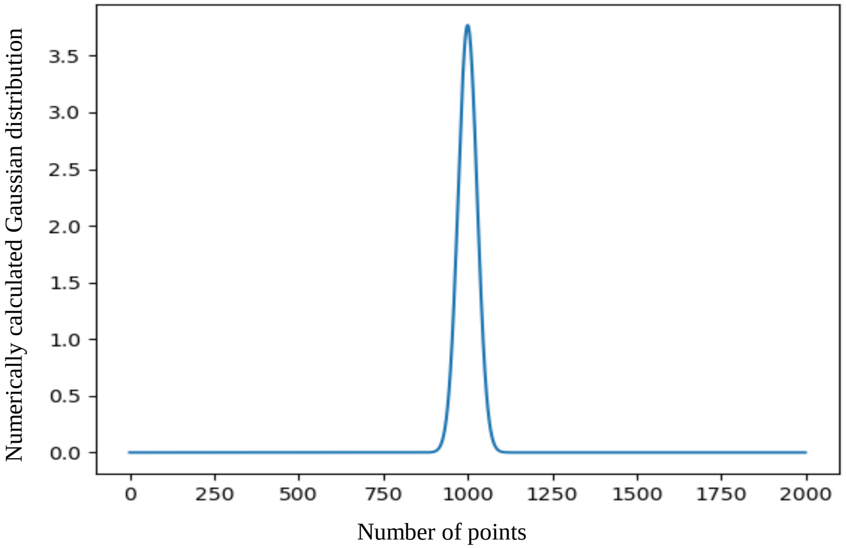


R=1.2, a=0.06

2. Characterization of Gaussian beam propagation

Comparison of Gauss profile after propagation for the analytical and numerical case respectively:



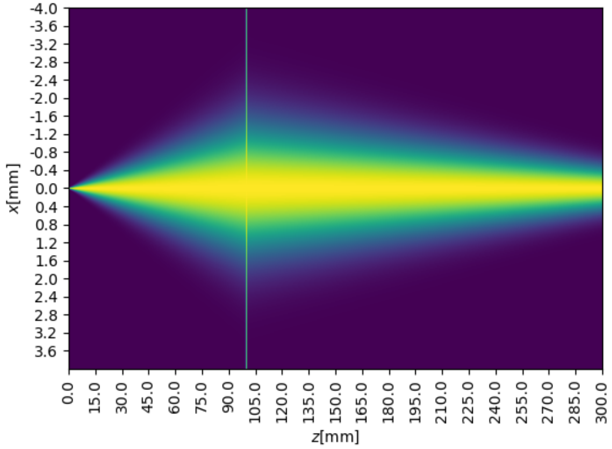


$$f_{analitic} = \frac{1}{\sqrt{2\pi}} \frac{\sqrt{k}}{\sqrt{iz + k\sigma^2}} \cdot e^{\left(\frac{-k(x_0 - x)^2}{2iz + 2k\sigma^2}\right)}$$

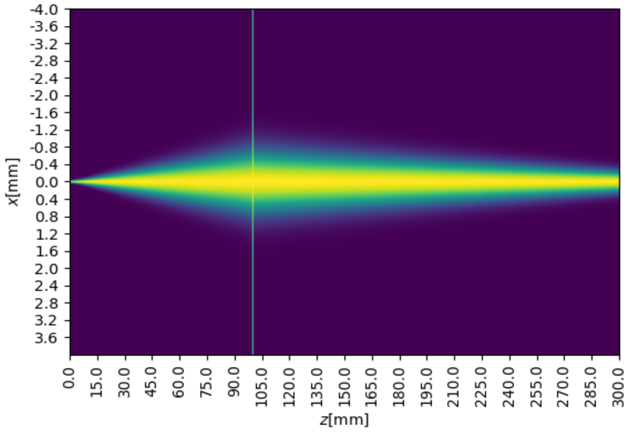
$$\max(|f_{analitic} - f_{numeric}|) = 1.24 \cdot 10^{-14}$$

Gaussian beam propagation for the following input beam diameters values:

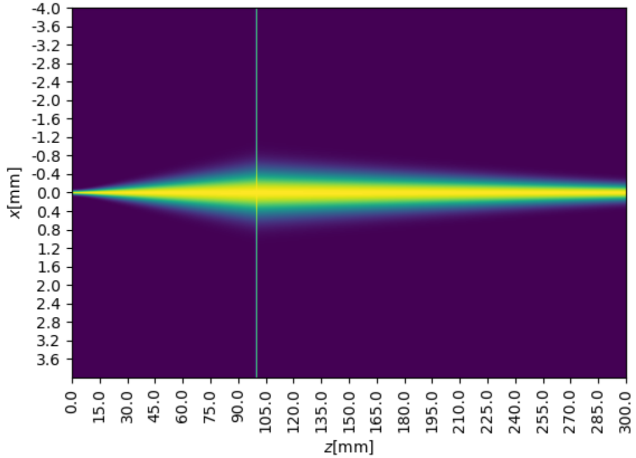
σ=0.01 mm;



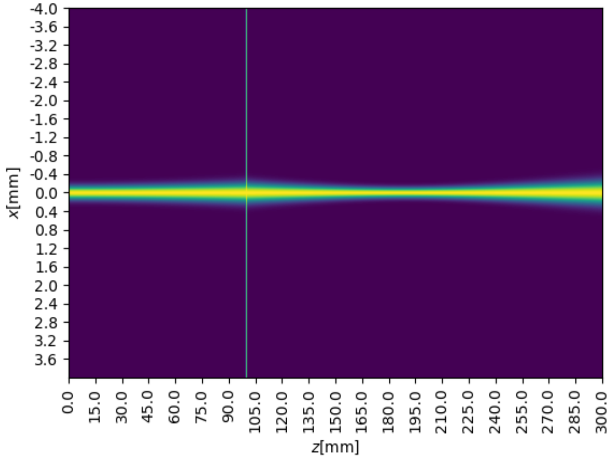
σ=0.02 mm;



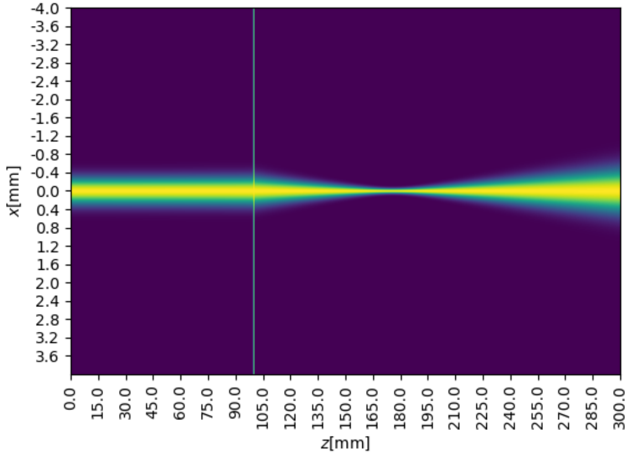
σ=0.03 mm;



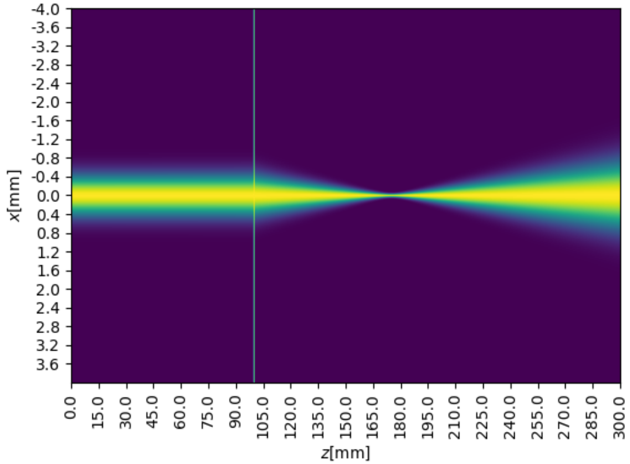
σ=0.1 mm;



σ=0.2 mm;



σ=0.3 mm.



Establishing the method of obtaining the same value of the beam width at the input

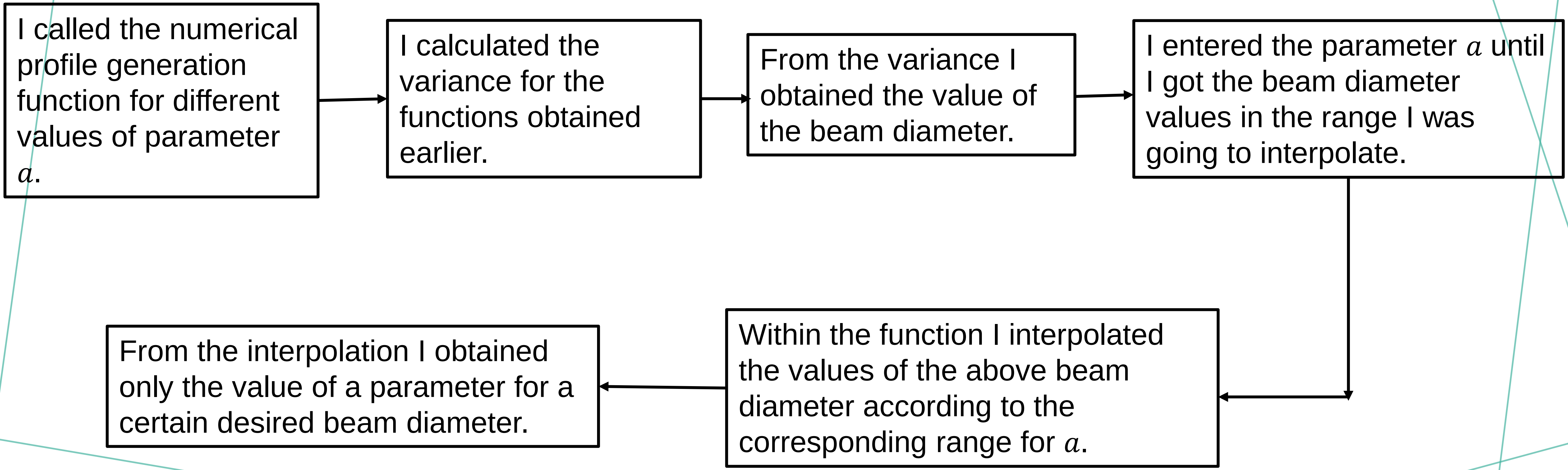
a) For the Gaussian intensity distribution this only needs to be fixed.

$$f_{Gauss}(x) = \frac{1}{\sigma \cdot \sqrt{2\pi}} e^{\frac{-1}{2} \cdot \left(\frac{x-\mu}{\sigma}\right)^2}$$

b) In the case of top-hat intensity distribution it can be determined analytically.

$$f_{top-hat} = \begin{cases} \frac{1}{\sqrt{2a}}, & -a < x < a \\ 0, & otherwise \end{cases}$$

c) For the Lorenz and Woods-Saxon functions I have implemented a function that takes as arguments the beam diameter I want to have as input and returns the values of the corresponding function parameters.

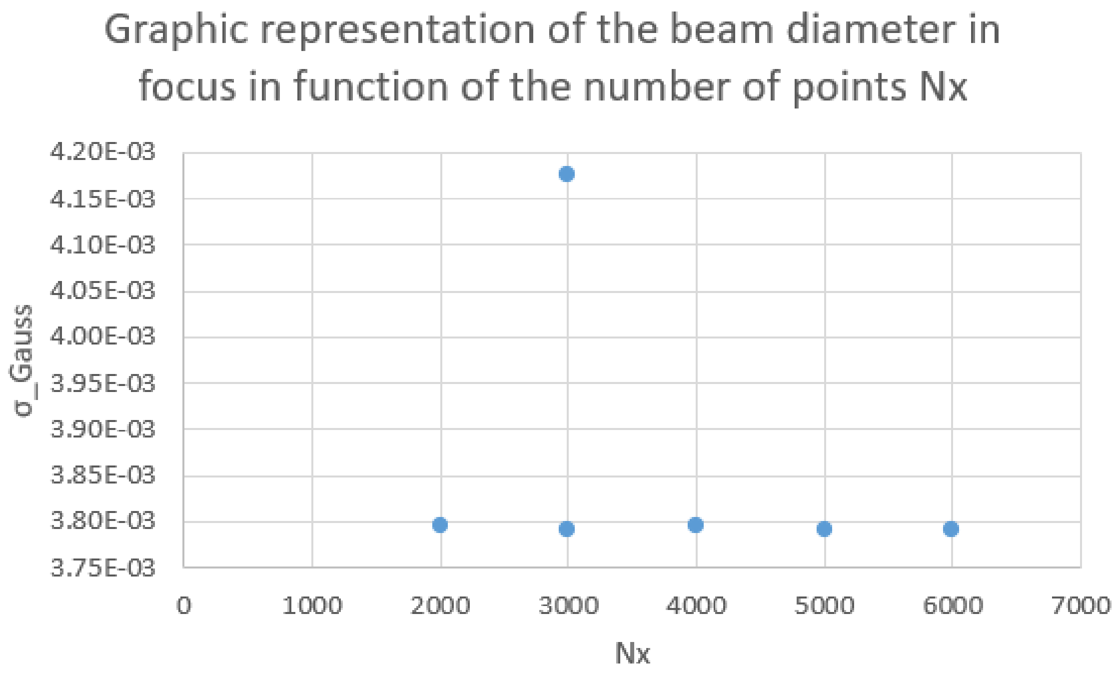


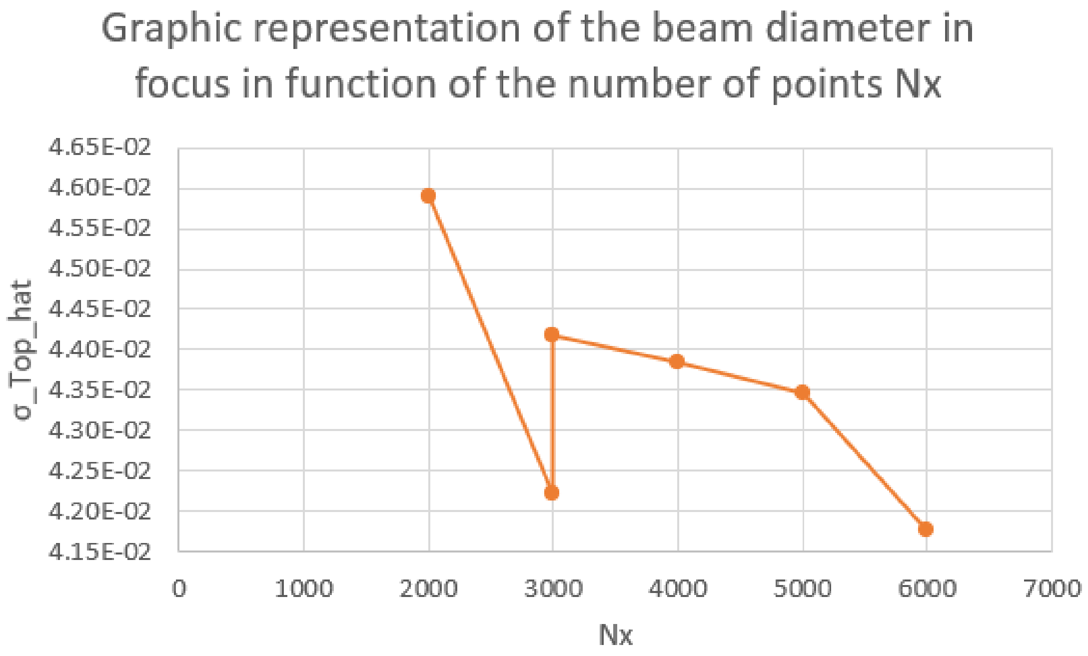
3. Comparing the focus of several types of profiles

- Numerically, all functions must be centered and decrease towards 0.
- It follows that after varying the number of points in the domain of definition, Nx and the step dx, the beam diameter in the focus should be the same.

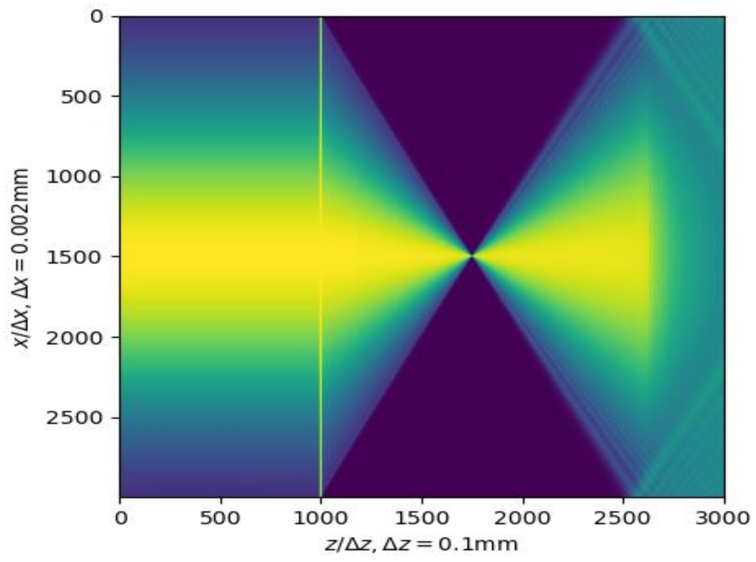
- From the value of 1 mm the beam width from the focal point show changes when varying the mentioned parameters.

- At low number of points and small step, it is higher than the rest of the situations for the same initial beam diameter.

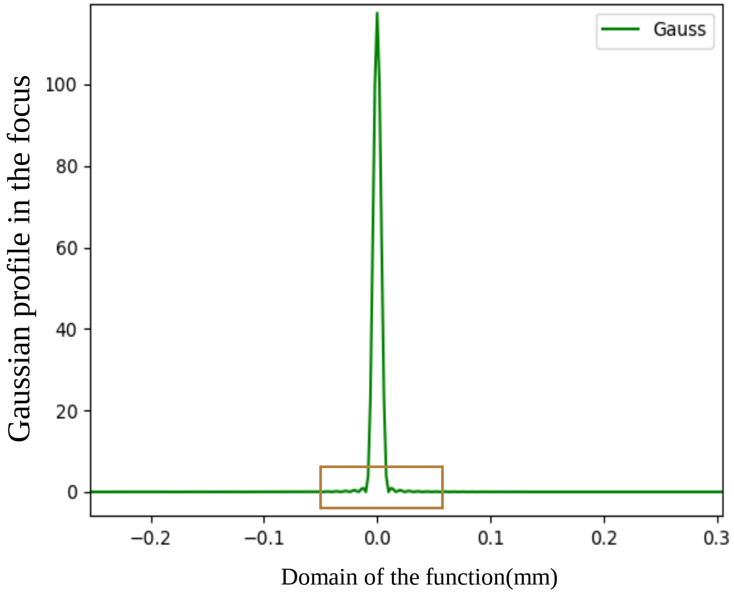


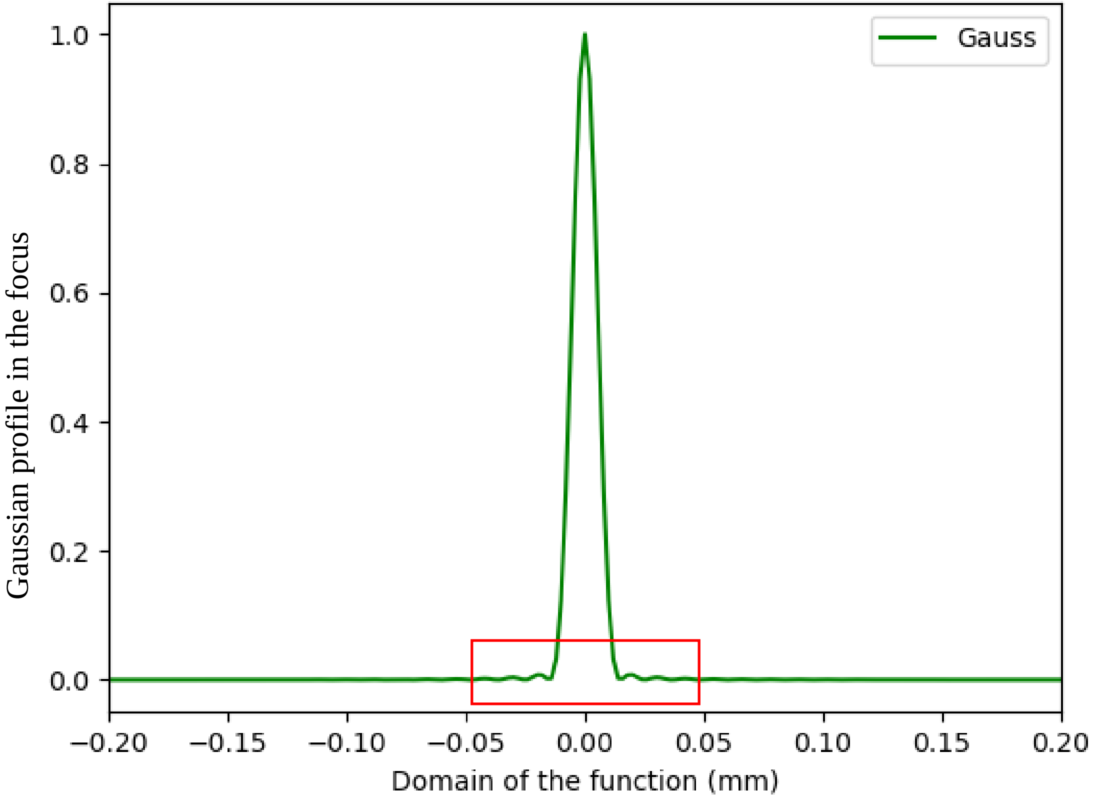


a) Gaussian beam propagation



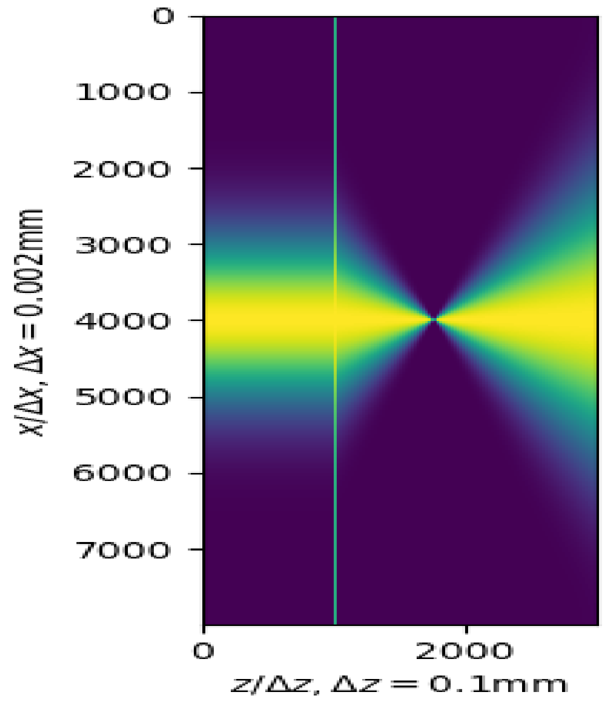
b) Gaussian profile in focus with a beam diameter of $w = 4.18 \cdot 10^{-3}\ mm$

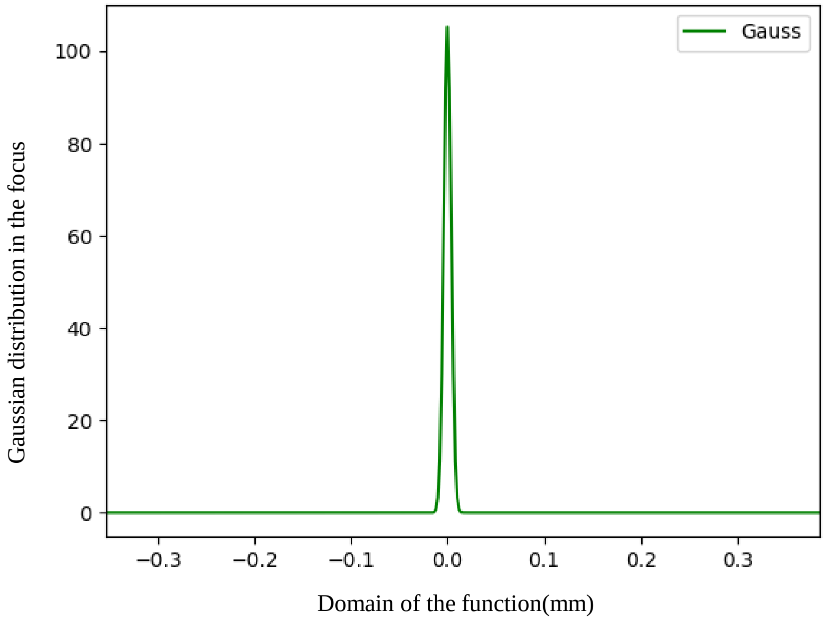


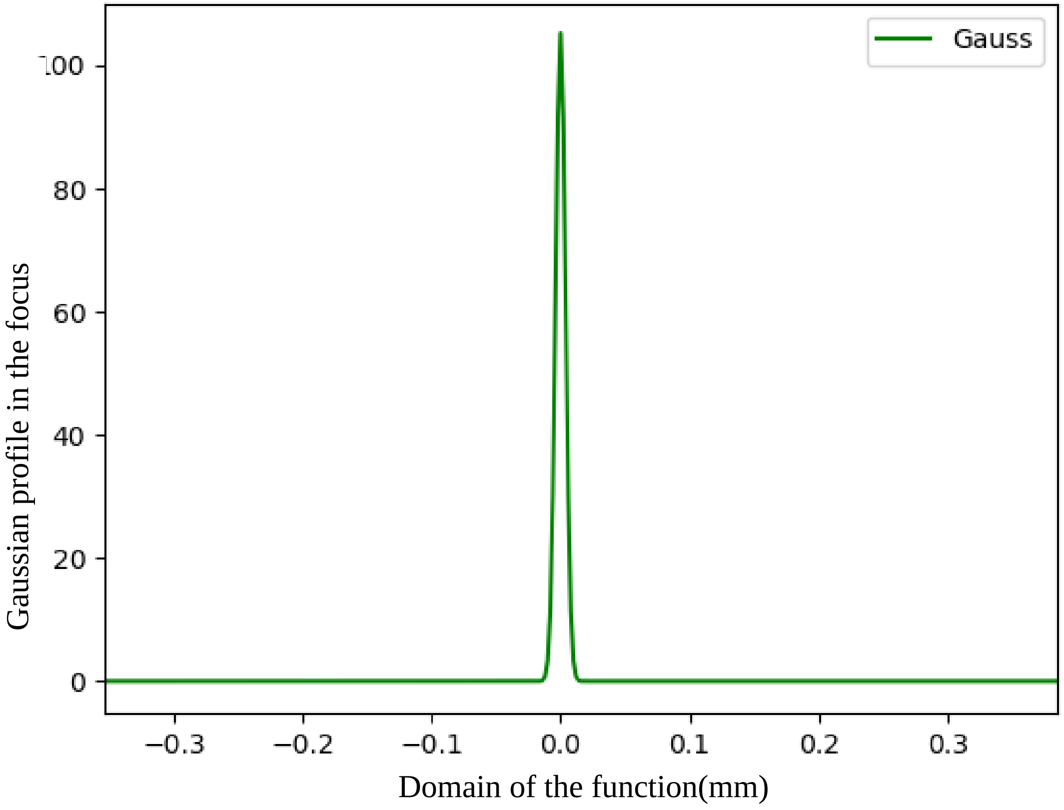


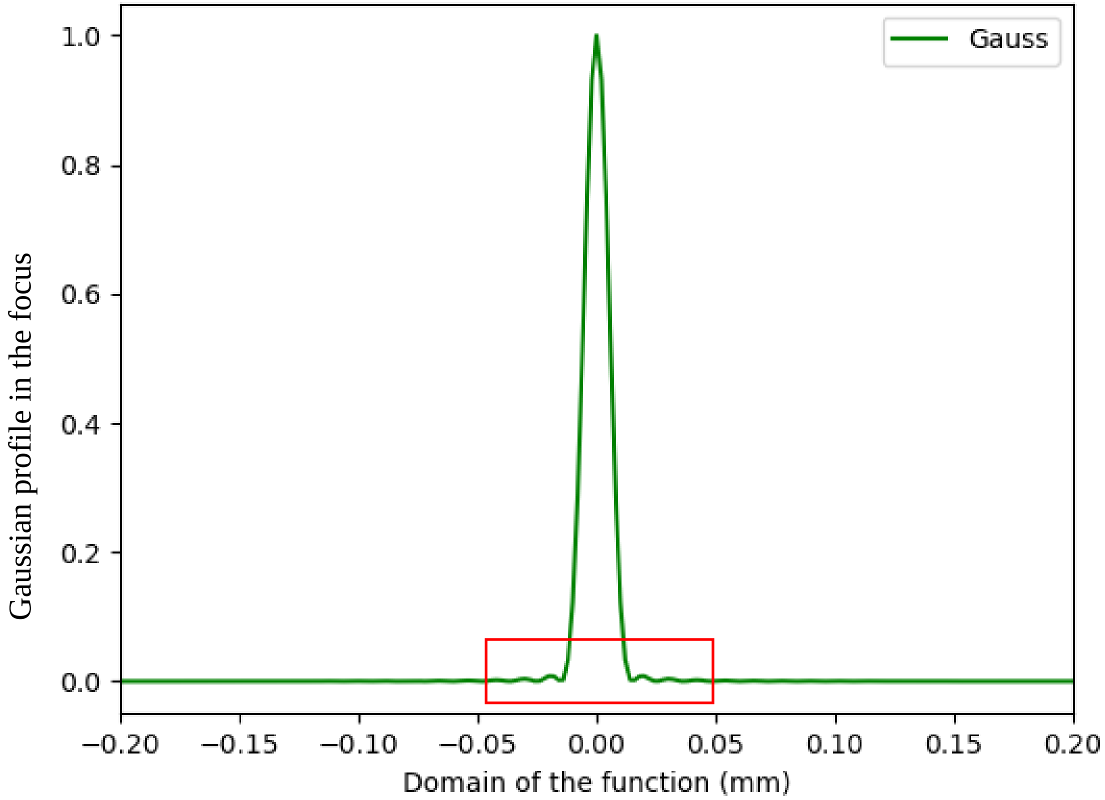
Gauss
1.0
0.8
0.6
0.4
0.2
0.0
Gaussian profile in the focus
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20
Domain of the function (mm)

For Nx=8000, dx=0.002:

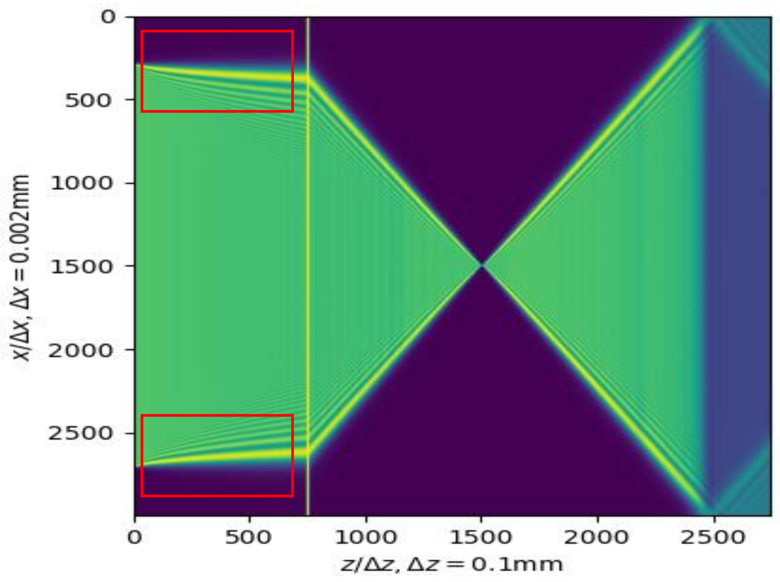


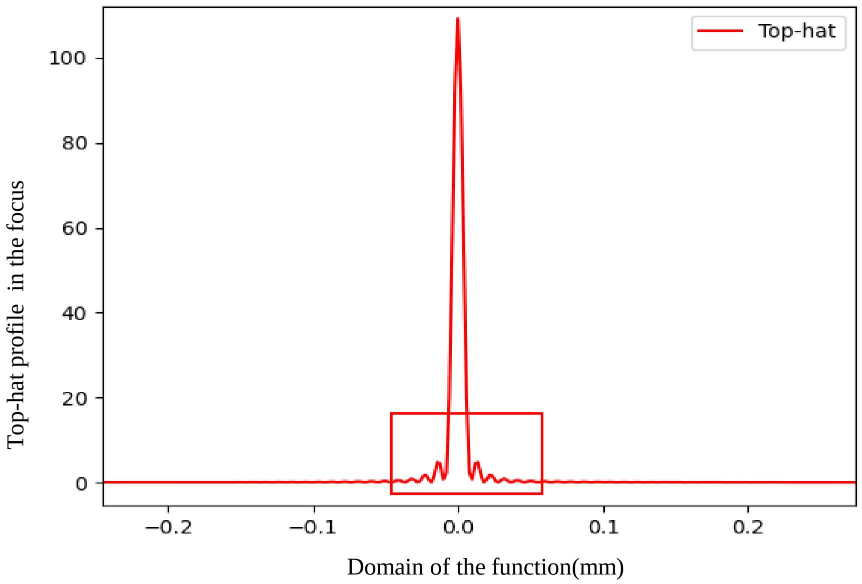



Gauss
100
80
60
40
20
0
−0.3
−0.2
−0.1
0.0
0.1
0.2
0.3
Gaussian profile in the focus
Domain of the function(mm)


Gauss
1.0
0.8
0.6
0.4
0.2
0.0
−0.20
−0.15
−0.10
−0.05
0.00
0.05
0.10
0.15
0.20
Gaussian profile in the focus
Domain of the function (mm)

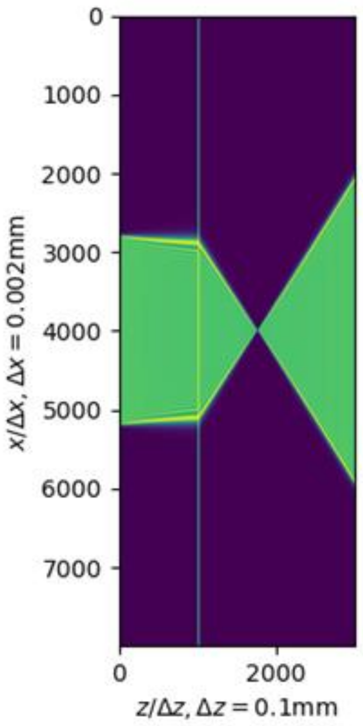
b) The beam diameter from the focus in the top-hat distribution varies to the second or third decimal place.

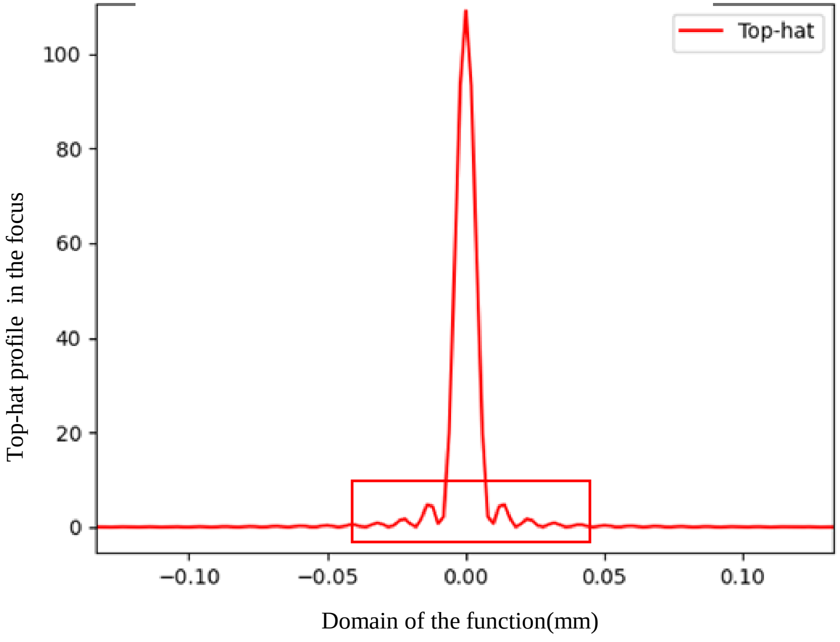




- Top-hat profile approaches the edges of the domain at low counts (3000 points).
- It shows the existence of side bites at the focal point that cause the diameter to change.

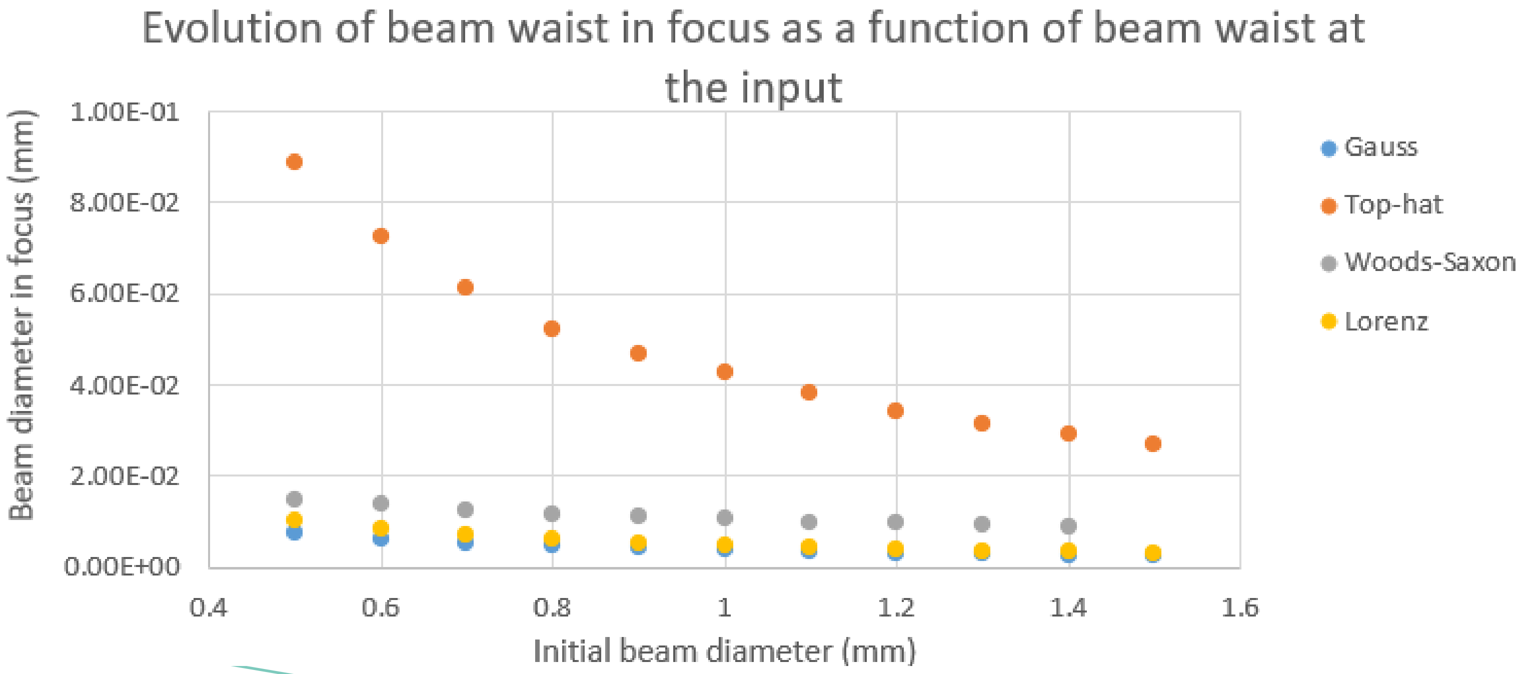
For Nx=8000, dx=0.002:

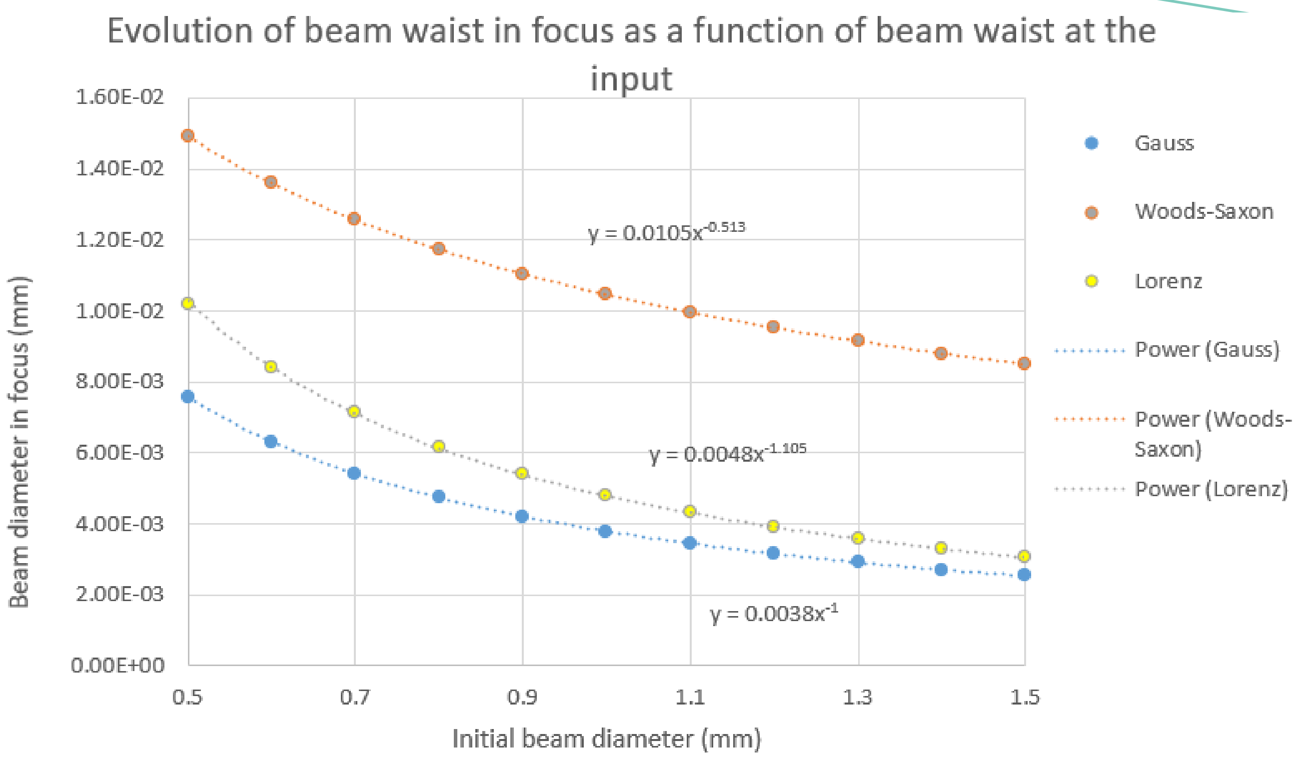




4. Comparison of the beam diameter of the focus for the four profiles

Plot of the evolution of the beam diameter in focus versus the beam diameter from the input:





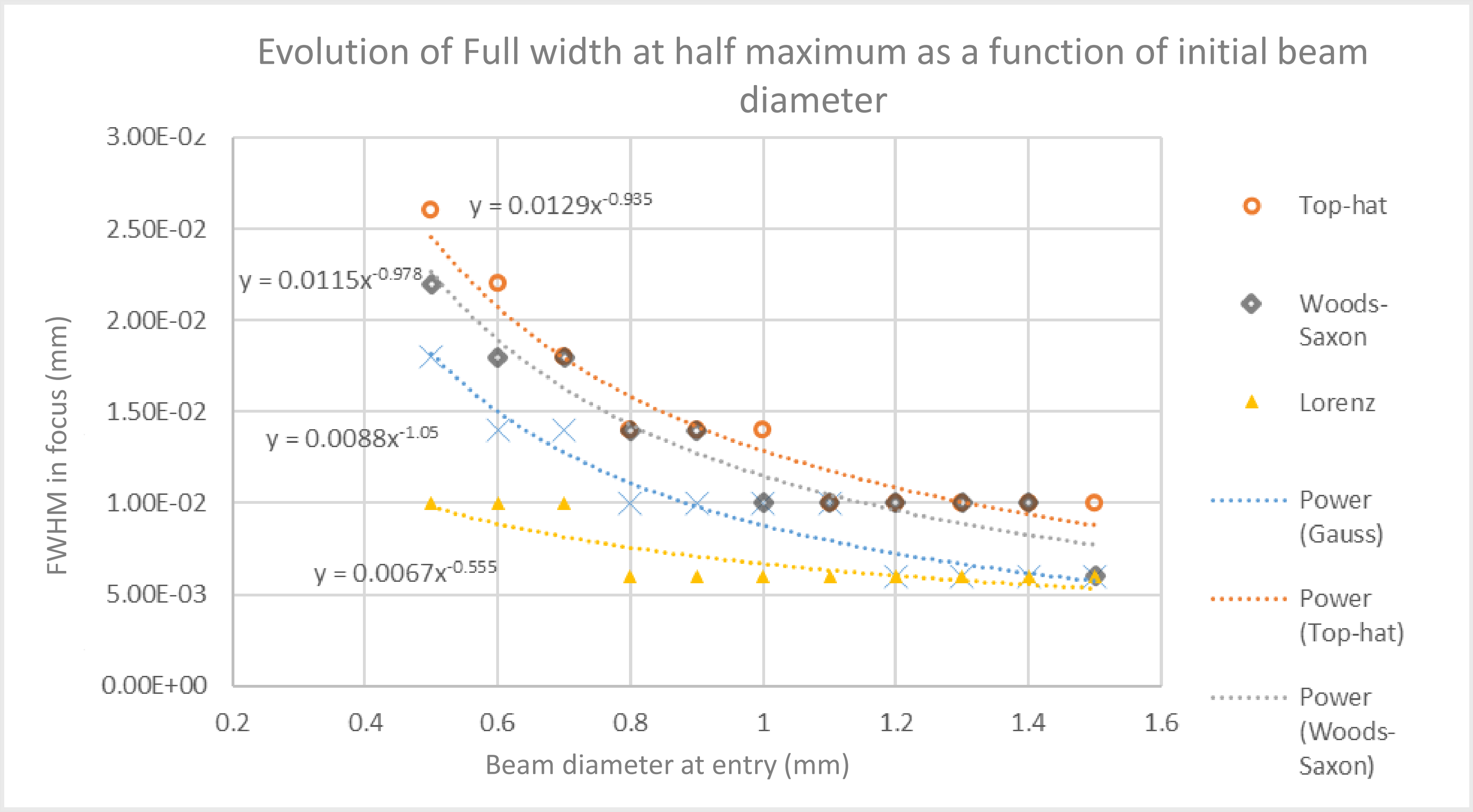
- The Woods-Saxon profile has the largest coefficient and the smallest exponent, which is why its beam diameter in focus is the largest.
- According to the equation, the Gauss profile has the smallest coefficient and is the one with the smallest beam width in focus.
- The beam width in focus of the Lorenz profile is between Gauss and Woods-Saxon profiles.

5. Comparison of the FWHM of the focus for the four profiles

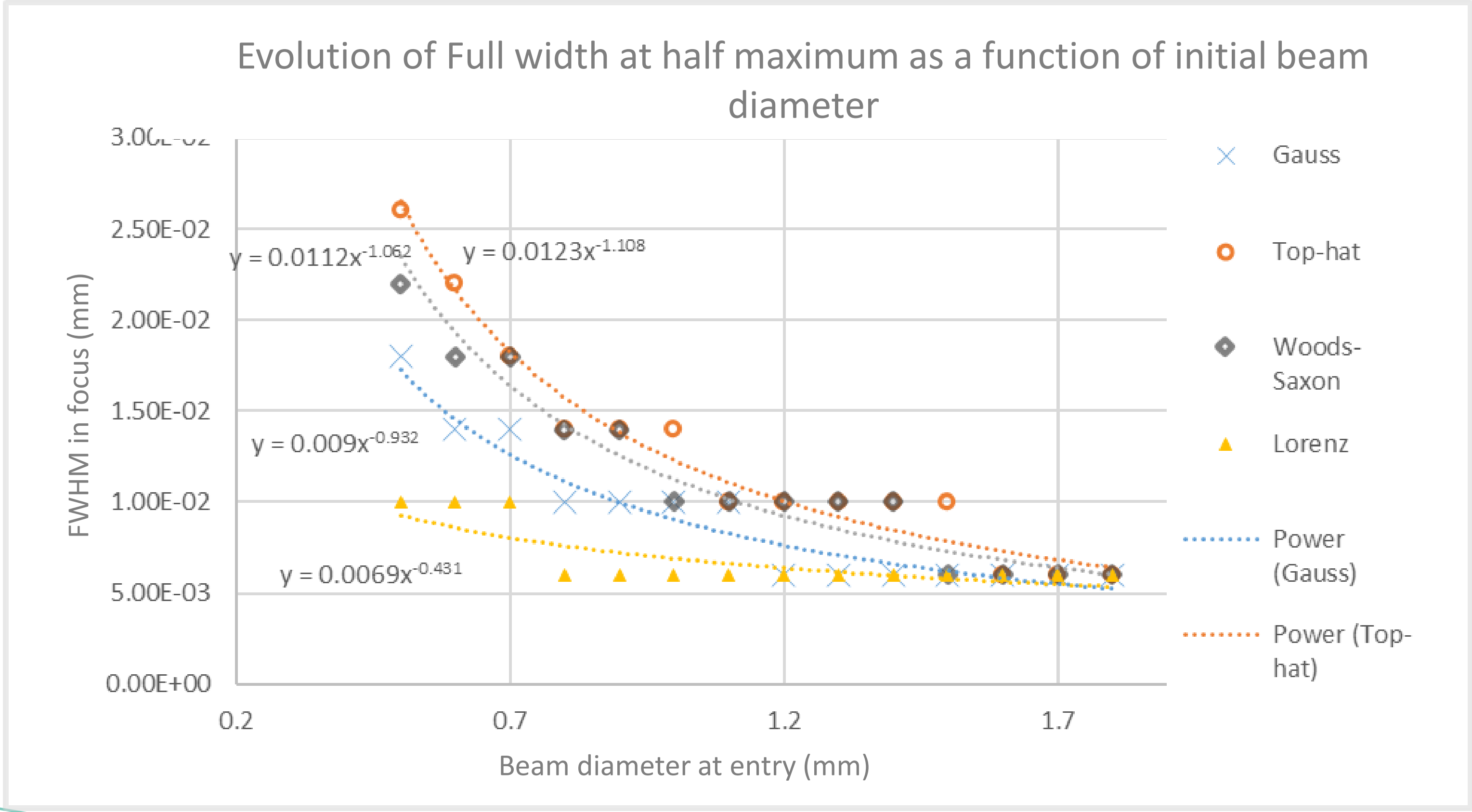
I compared how the four profiles focus according to full width at half maximum(FWHM) for the following reasons:

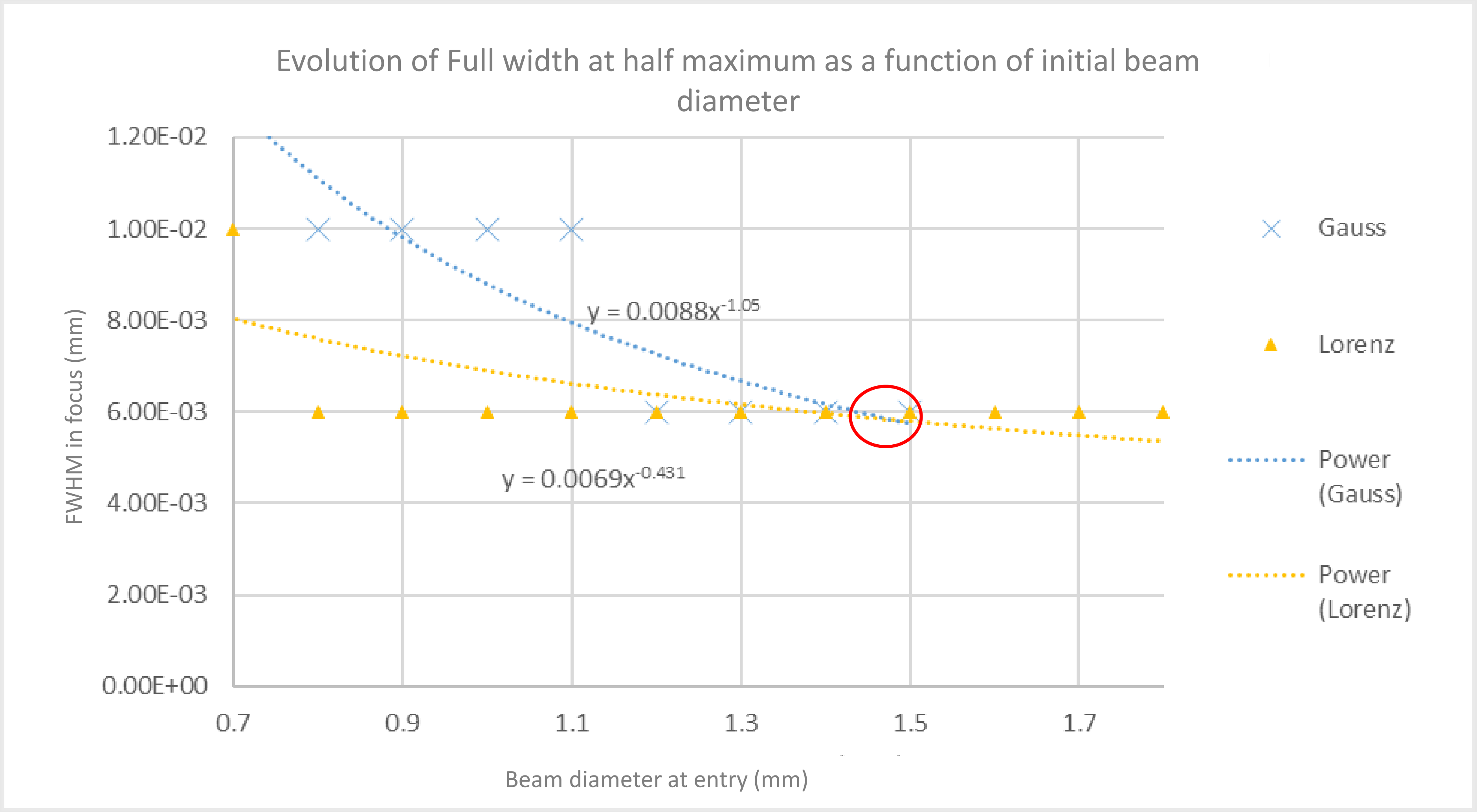
- Mathematically the beam diameter is not defined for the Lorenz distribution.
- It is necessary to check the simulation conditions because at a low point count domain the mentioned profiles scatter at the lens entrance.
- The top-hat profile has a larger beam waist than the rest due to the side bites.

Graphical representation of FWHM evolution:



- Following the calculations I obtained that the two graphs intersect at an initial beam diameter of about 1.73 mm.





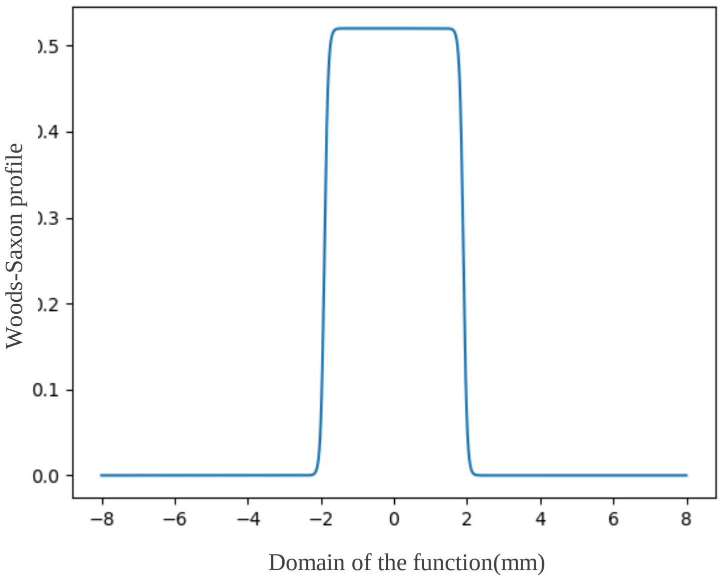
Evolution of Full width at half maximum as a function of initial beam diameter
FWHM in focus (mm)
1.20E-02
1.00E-02
8.00E-03
6.00E-03
4.00E-03
2.00E-03
0.00E+00
0.7
0.9
1.1
1.3
1.5
1.7
Beam diameter at entry (mm)
y = 0.0088x^-1.05
y = 0.0069x^-0.431
Gauss
Lorenz
Power (Gauss)
Power (Lorenz)

CONCLUSIONS

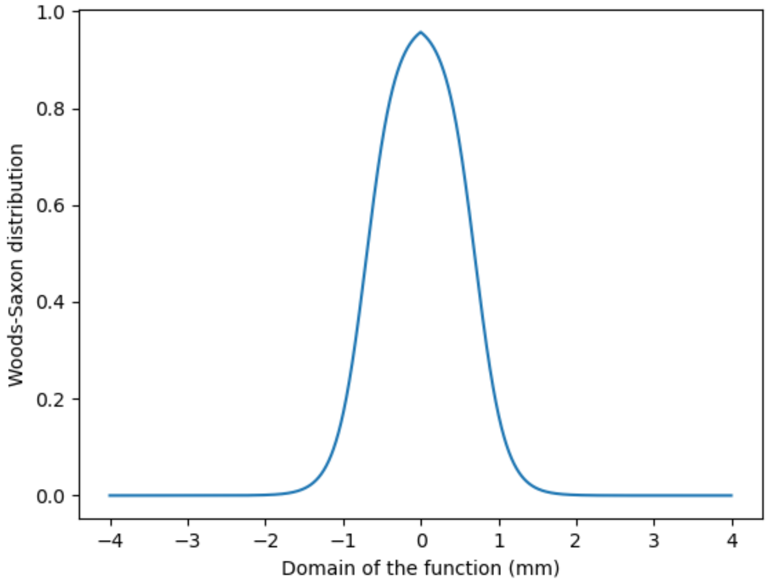
- After a comparison by beam diameter and FWHM, the Gauss and Lorenz profiles focus better than the Woods-Saxon and Top-hat.
- The Top-hat has smaller beam waist values when comparing by FWHM because the side bites do not enter into its calculation, but it is still below Lorenz and Gauss.
- The Woods-Saxon profile is between Top-hat and the other two in terms of focus.
- When comparing by beam waist the Gaussian profile focuses better.
- When comparing by full width al half maximum the Lorenz profile focuses better.

OUTLOOK

The Woods-Saxon profile has a focusing advantage because it involves a compromise between Gauss and Top-hat. Since for some parameters it is close to top-hat, and for others to Lorenz, which is similar to Gauss, just narrower, it may represent a perspective in beam focusing.



R=1.2, a=0.06



R=0.7, a=0.19

ACKNOWLEDGEMENT

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Thank you for your attention!