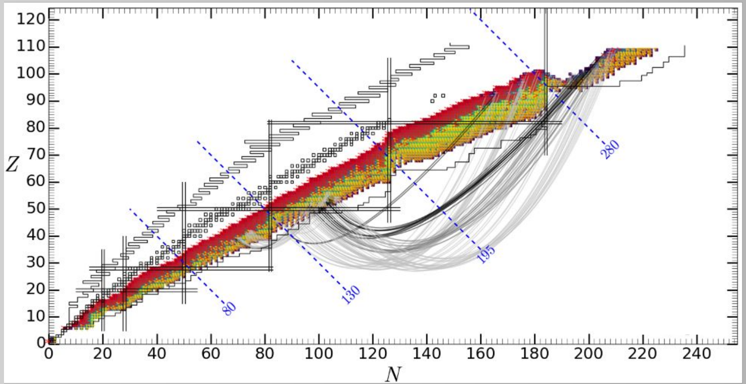
From nuclear structure to neutron star mergers

A unified finite-temperature equation of state with BSkG4

Guilherme Grams
University of Potsdam



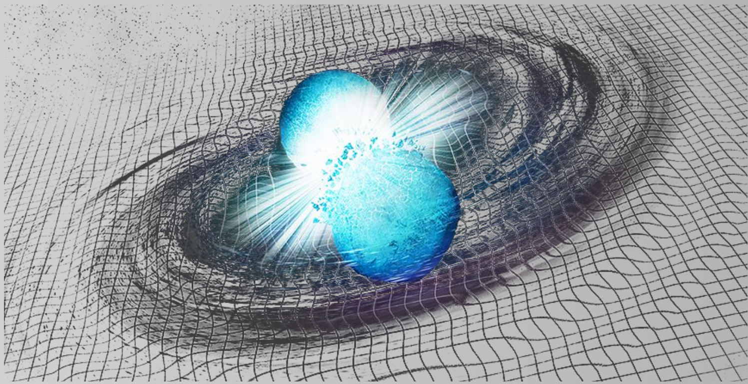


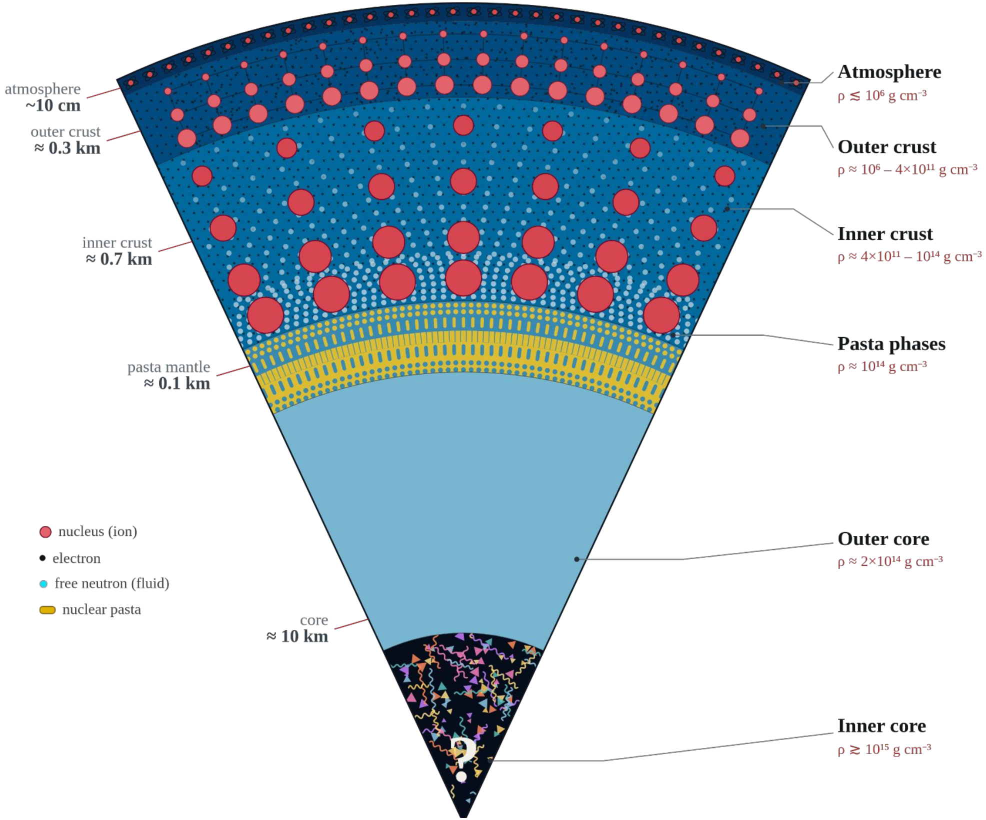
Image adapted from CXC/M. Weiss.

NPA-XII
10.09.2026 Cluj-Napoca

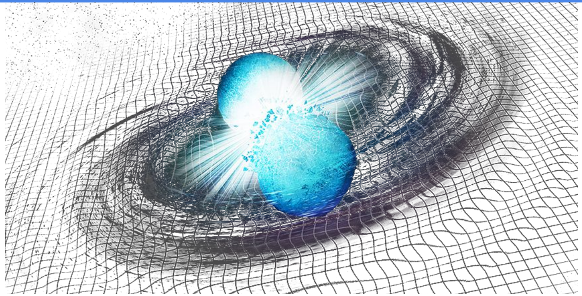
Universität Potsdam

Motivation

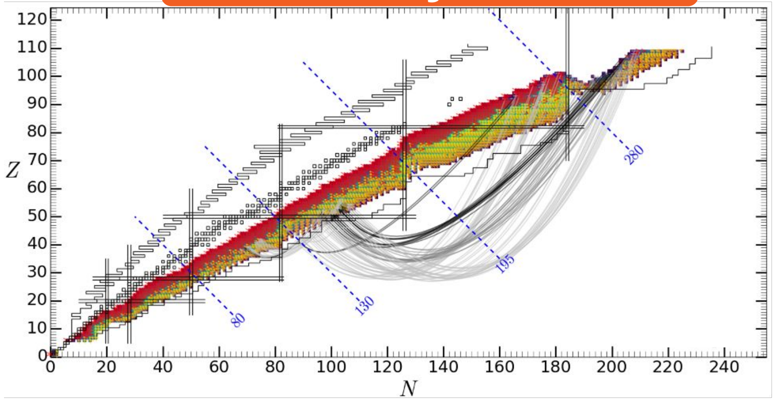
Neutron star EoS



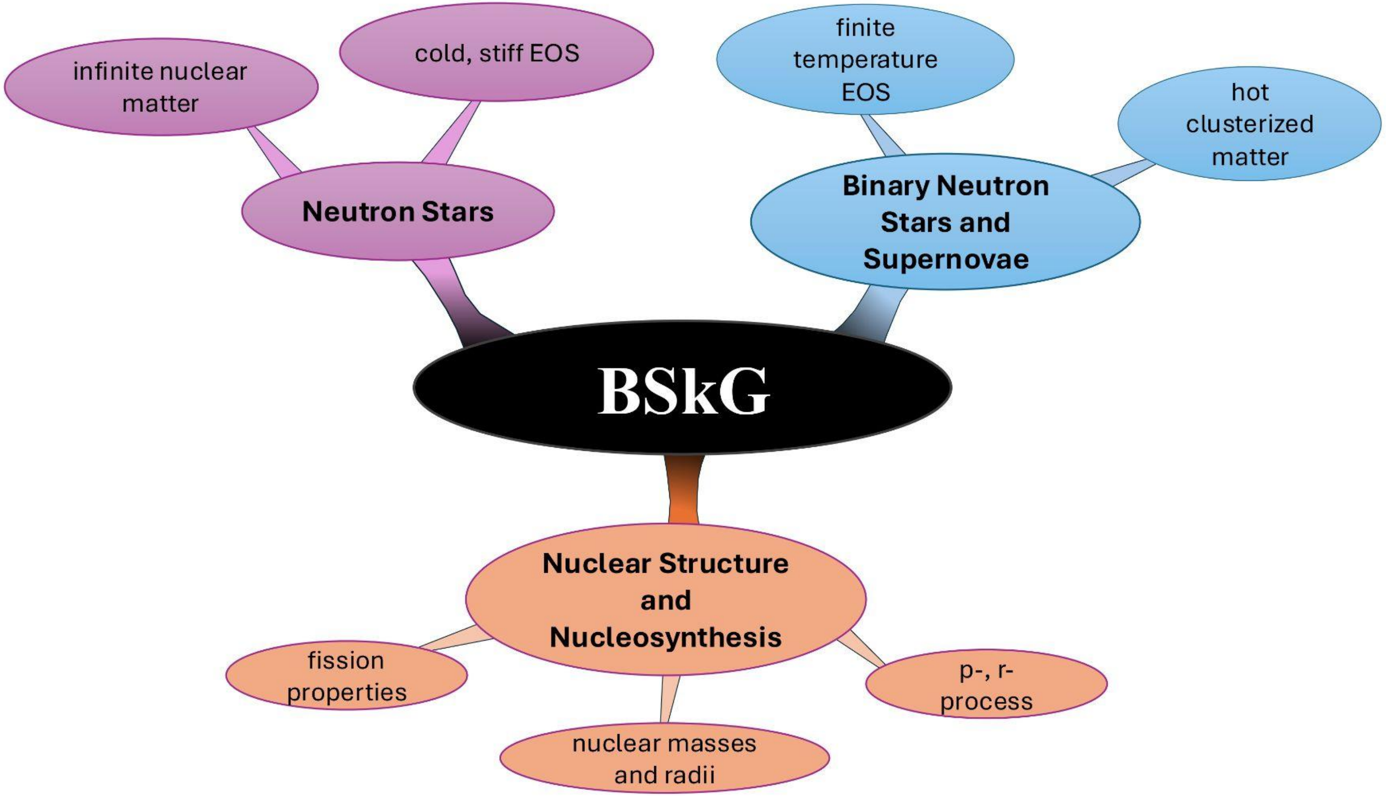
Binary neutron star mergers



Nucleosynthesis

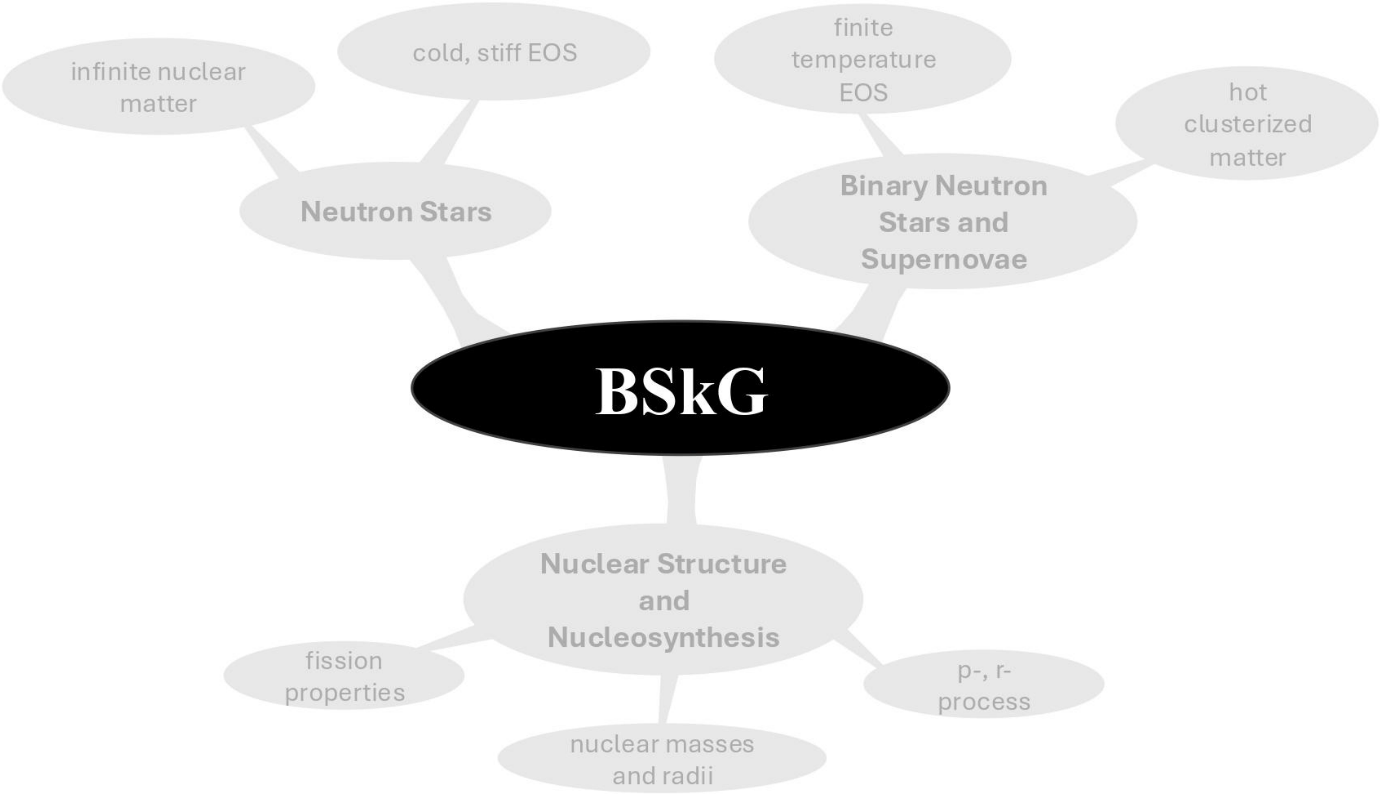


Motivation



BSkG
Neutron Stars
infinite nuclear matter
cold, stiff EOS
Binary Neutron Stars and Supernovae
finite temperature EOS
hot clusterized matter
Nuclear Structure and Nucleosynthesis
fission properties
nuclear masses and radii
p-, r- process

Motivation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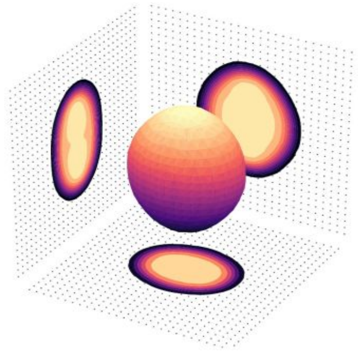
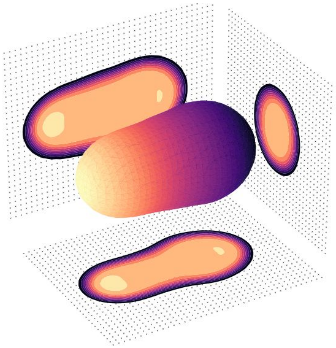
Brussels-Skyrme-on-a-grid (BSkG)

BSkG



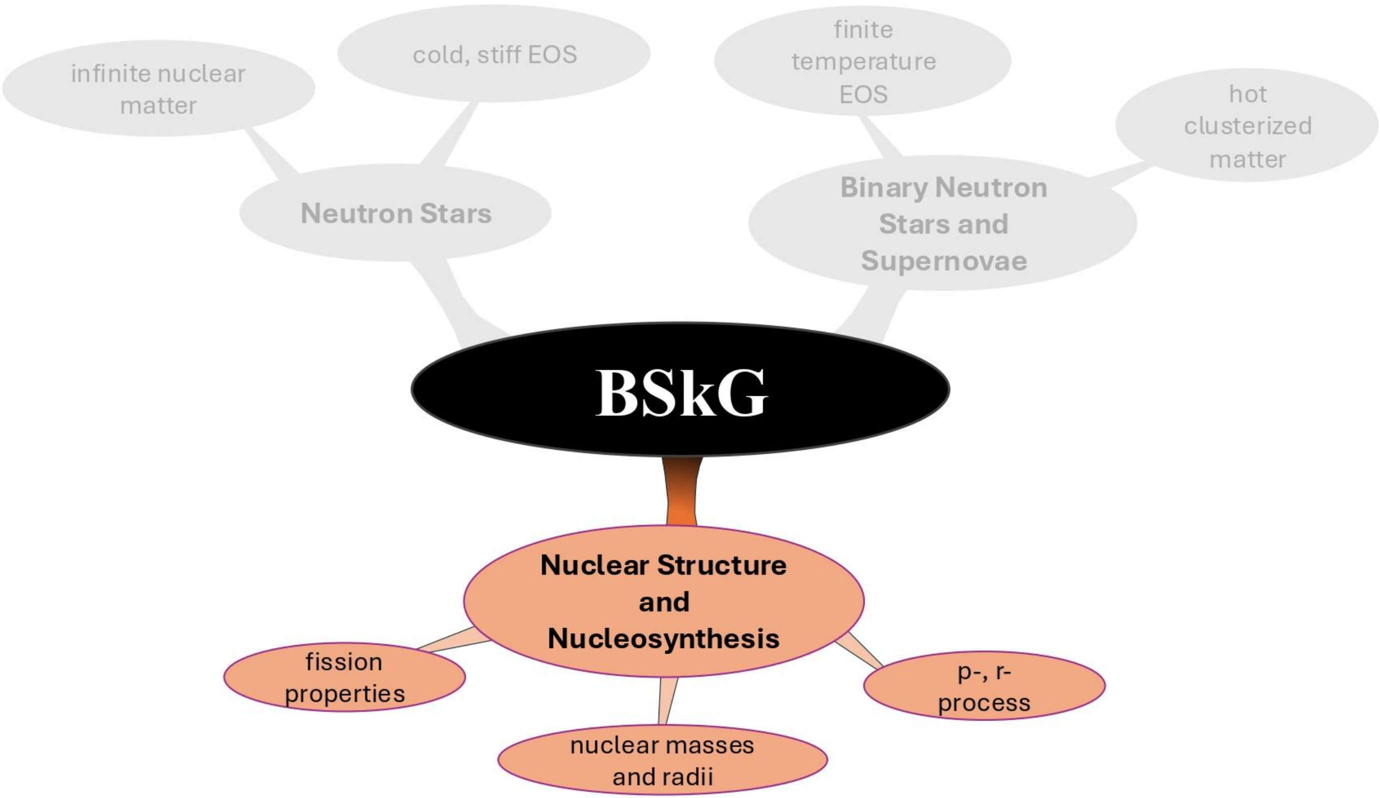
Large-scale model of nuclear structure:

- Interaction of **Skyrme-type;**
- Solve **HFB equations** in a **3D mesh;**
- Fitted to essentially **all experimental masses;**
- Constraints on **infinite nuclear matter** properties**.**



MOCCa: W. Ryssens and M. Bender, (2026) 10.5281/zenodo.20931651

Motivation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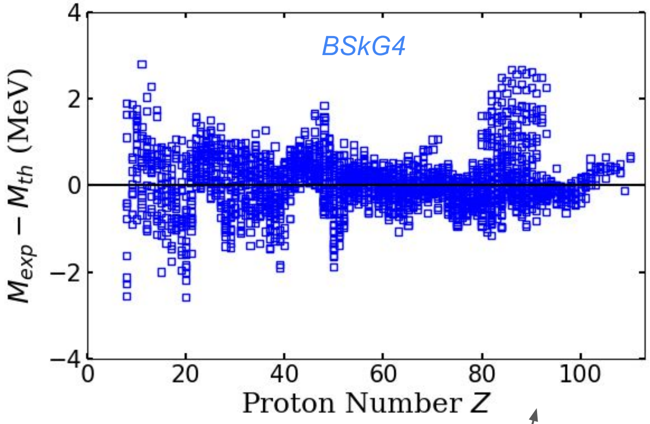
Nuclear structure with BSkGs

- ➢ BSkG1/2: accurate **nuclear structure** data: masses, radii, fission barriers;
- ➢ BSkG3: + stiff EoS to **neutron stars;**
- ➢ BSkG4: + improve **pairing gaps** at asymmetric matter.

Nuclear Structure and Nucleosynthesis



All 2457 nuclei in the AME2020.

σ (M) < 700 KeV
σ (R_C) < 0.025 fm
σ (E_I) < 400 KeV

Scamps+ EPJA (2021)
Ryssens+ EPJA (2022)

Grams+ EPJA (2023)

Grams+ EPJA (2025)

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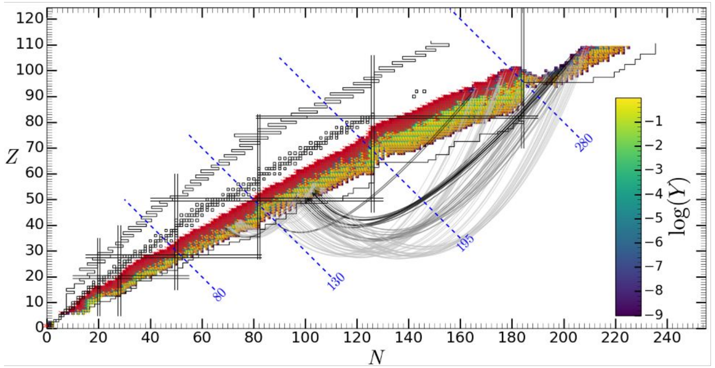
Nuclear Structure and Nucleosynthesis



σ (M) < 700 KeV
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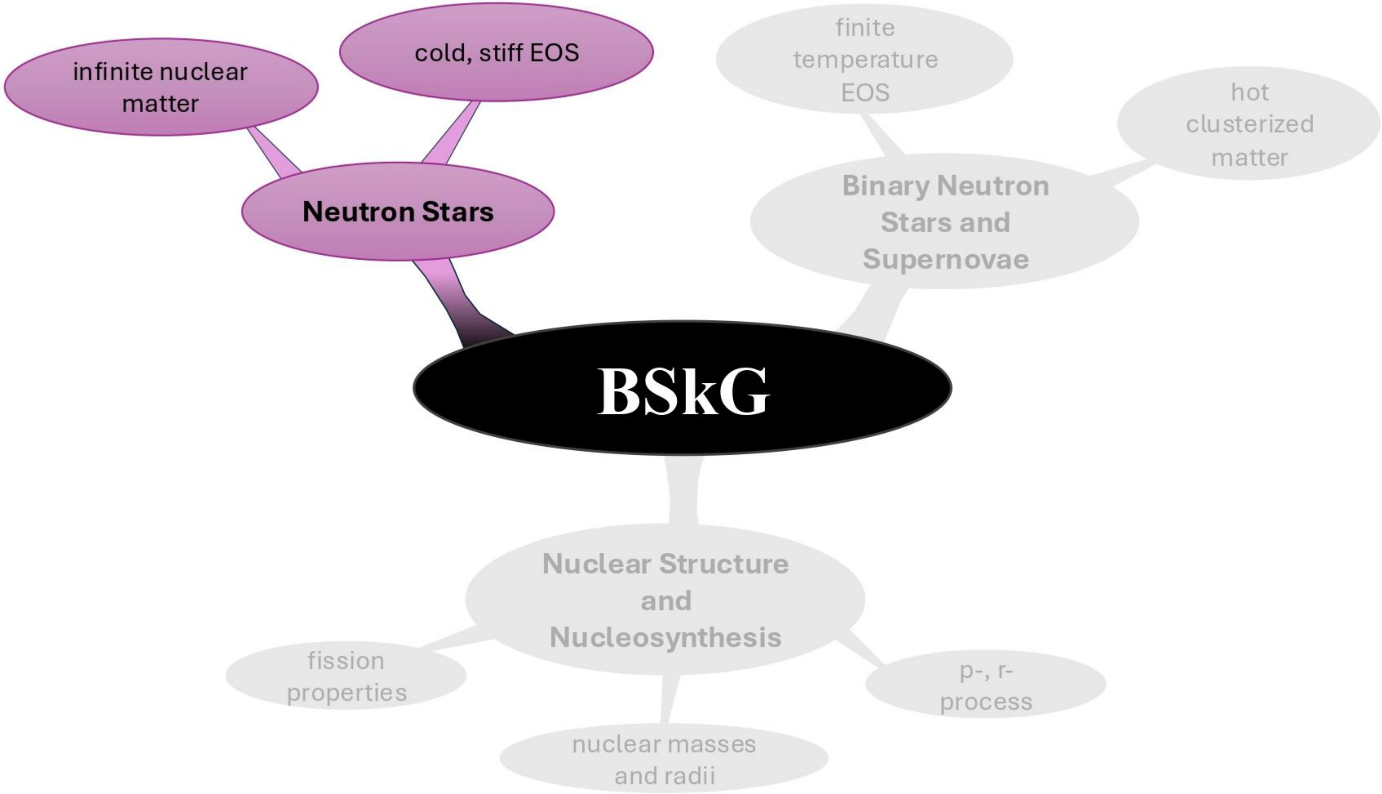
σ (E_I) < 400 KeV

Nuclear inputs for astrophysical simulations public available:

- **Mass Tables:** Global ground-state properties (masses, deformation, radii, pairing gaps, …) of ~9000 nuclei with 8 ≤ Z ≤ 118 lying between neutron and proton drip lines.
- **Fission Tables:** Fission barriers and isomer excitation energy for all 45 nuclei with Z ≥ 90 in the RIPL-3 database.

Grams+ EPJA (2023) | Grams+ EPJA (2025)

RIPL-3: R. Capote et al., Nucl. Data Sheets 110, 3107 (2009).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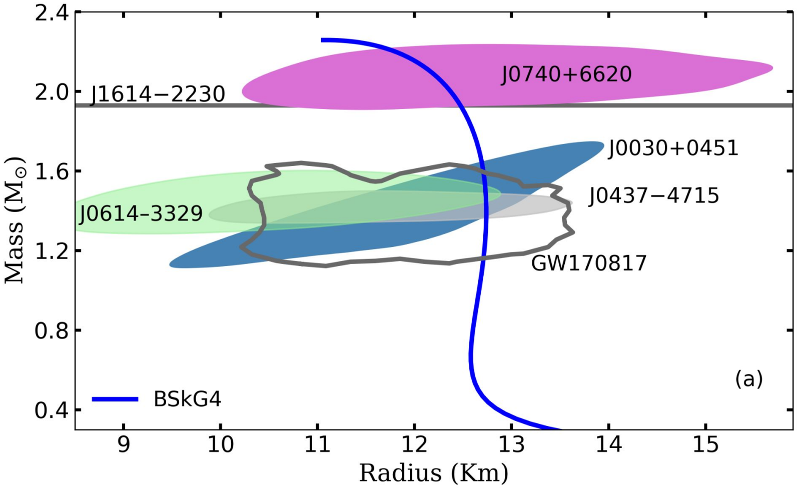
Neutron Stars

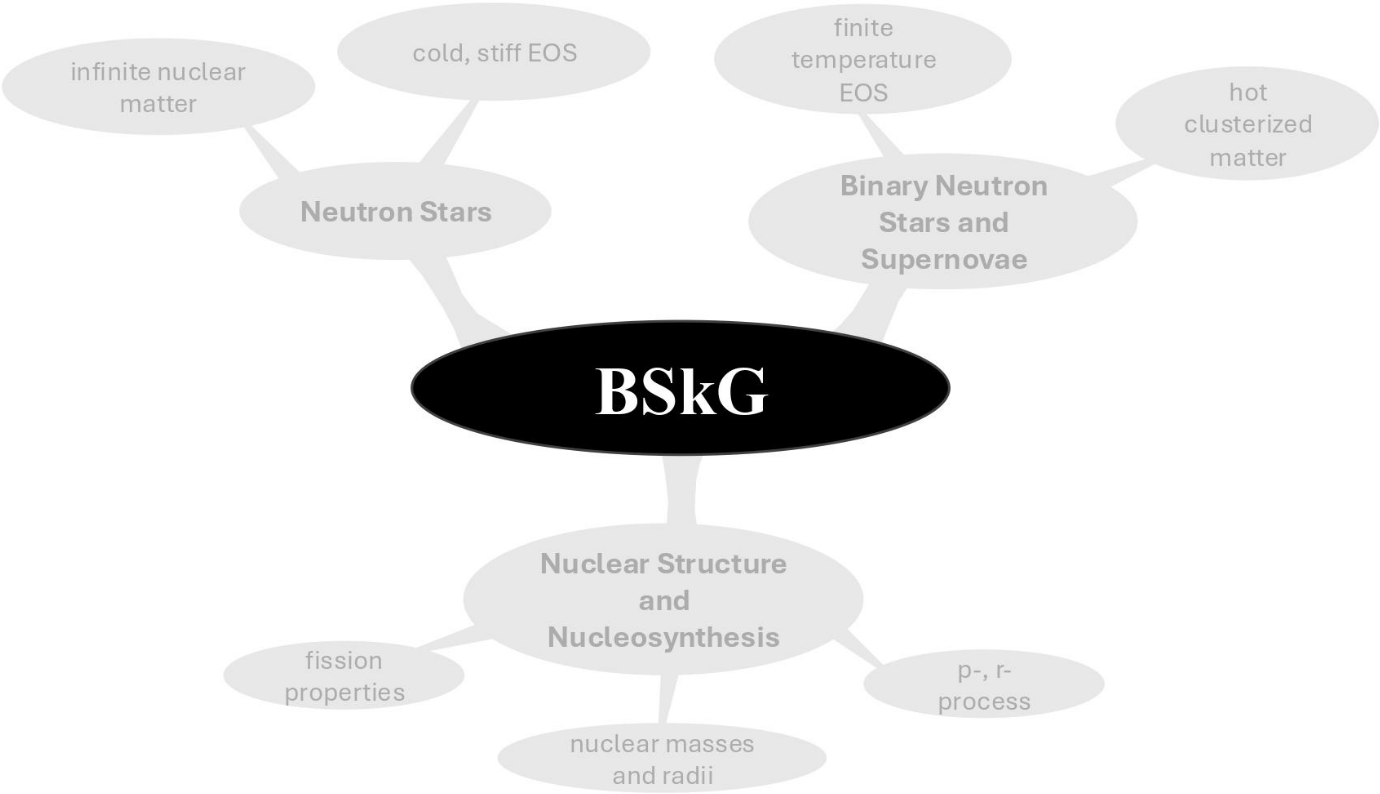
Beyond nuclear structure, BSkG3 and **BSkG4** provide:

- **New parameterizations** of extended-Skyrme force;
  - Compatible with nuclear matter properties, and neutron star observations.



Grams+ EPJA (2023) Grams+ EPJA (2025)

Motivation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BSkG5 -> Next-to-next-to-leading-order (N2LO) EDF

BSkG

❖ Systematic expansion on gradient terms on the EDF.

- Maintain high accuracy in **nuclear structure** similar to BSkG4.
- Stiff neutron matter energy, compatible with **neutron stars.**
- With 2 less parameters and less density dependency.

Grams+ PLB (2026)

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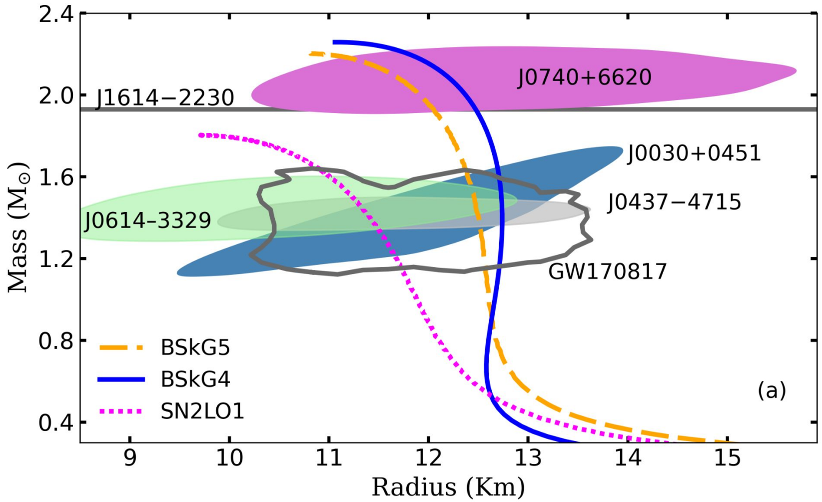
Nuclear Structure and Nucleosynthesis

Neutron Stars



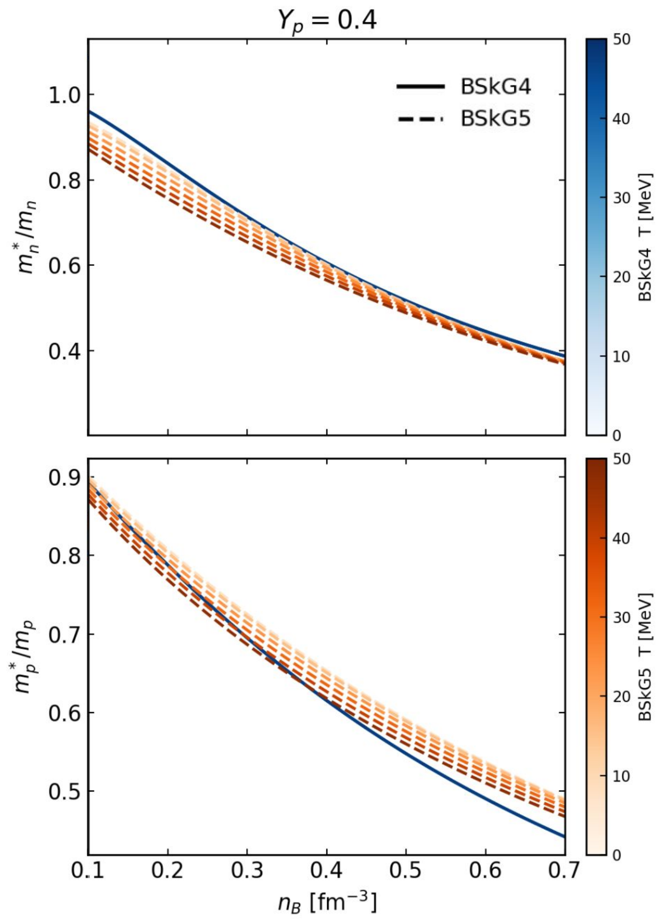
Grams+ PLB (2026)

SN2LO1: Becker+ PRC (2017)

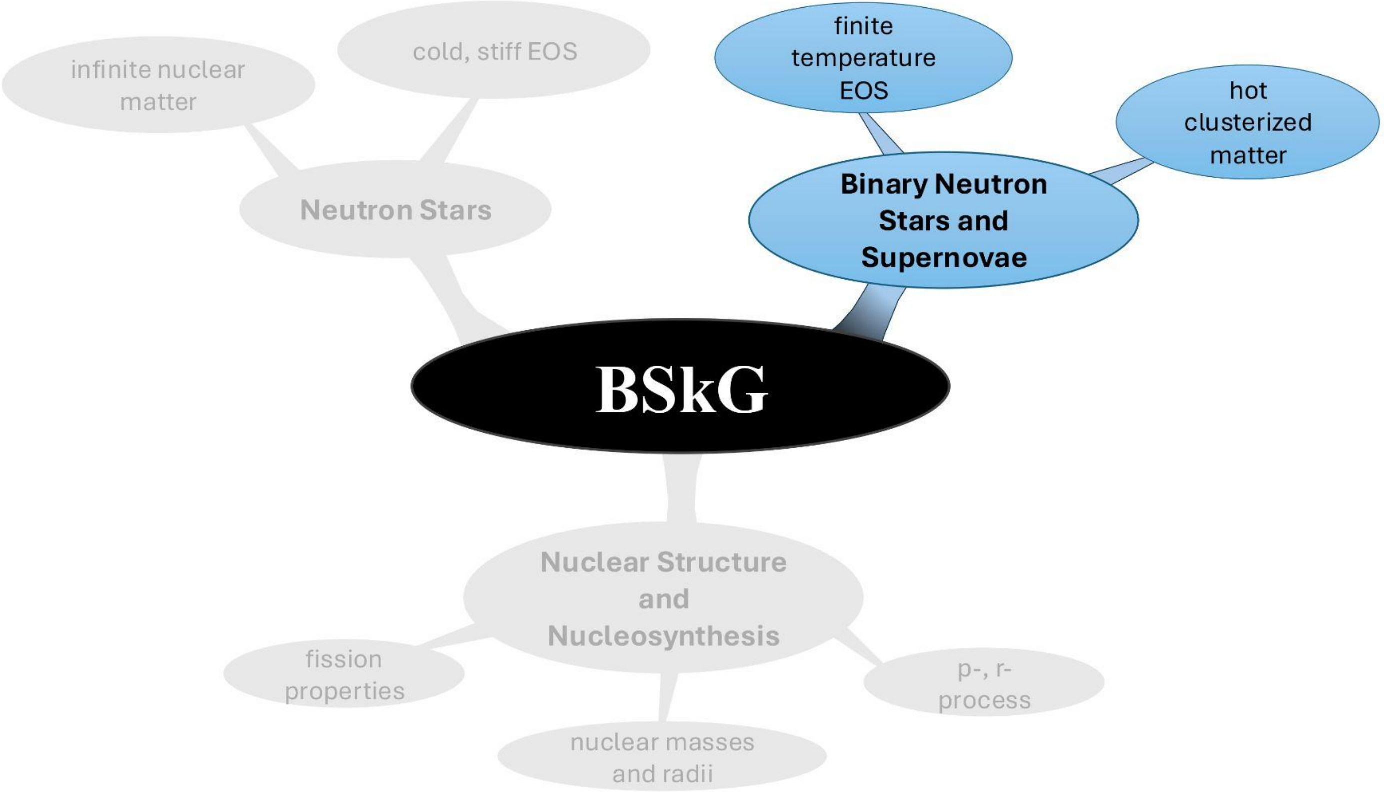
BSkG5

❖ The N2LO terms lead to a momentum and ***temperature dependent effective mass.***

New for Skyrme forces.



Grams+ PLB (2026)



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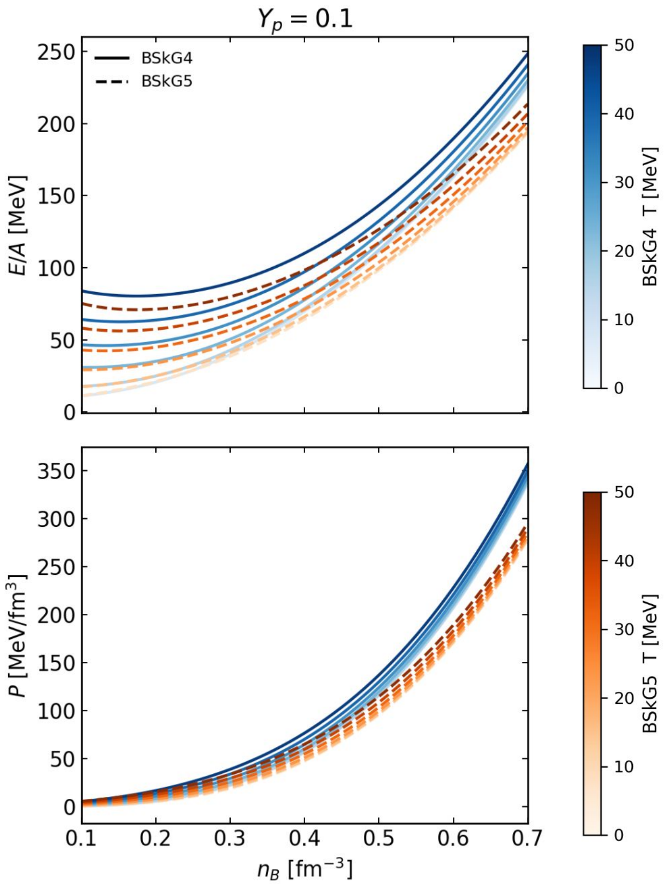
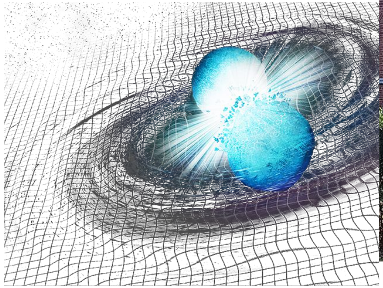
Homogeneous matter at finite temperature



Binary Neutron Stars and Supernovae

We simulate a 1.2-1.2 M_{sun} binary neutron star merger using the numerical relativity code BAM.

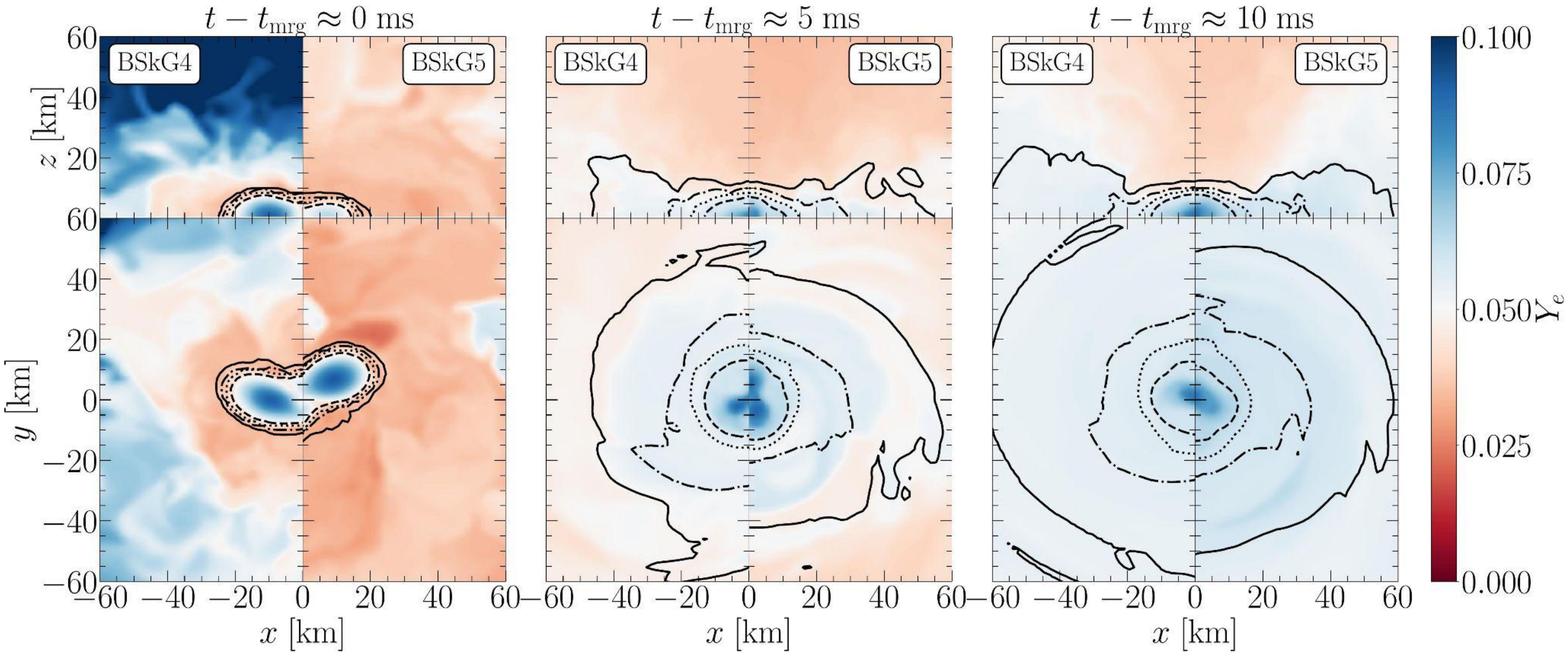


H. Gieg

BAM: Dietrich+ PRD(2015); Gieg+ PRD(2025).

Binary neutron star mergers: first simulations

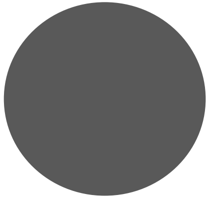
Binary neutron star mergers



Building **unified EoS** with semiclassical calculations

Finite-size with temperature-dependent **extended Thomas-Fermi** approach
Nuclear force: **BSkG4**

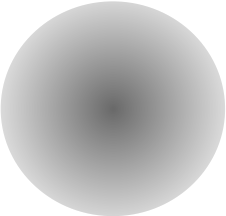
CLDM



$$n_{cl} = constant$$
$$r_p = r_n = r_{cl}$$

$$E_{\mathrm{CLDM}} = E_{\mathrm{bulk}} + E_{\mathrm{surf}}(params) + \dots$$

ETF



$$n_{cl} = n_{cl}(r)$$
$$r_p \neq r_n$$

$$E_{\mathrm{ETF}} = E(n_q(r), \nabla n_q(r), \dots)$$

N.N. Shchechilin

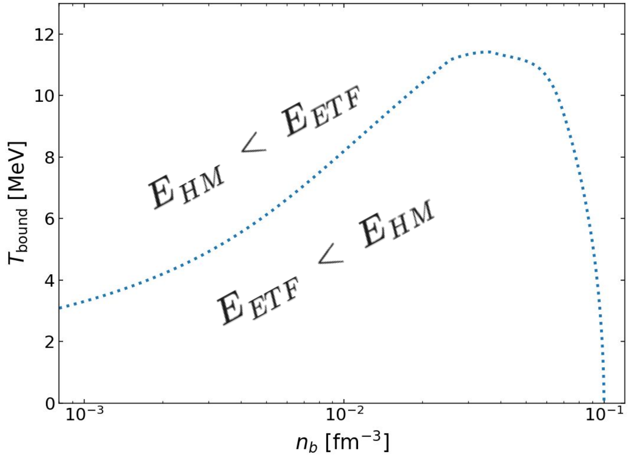
Grams+ Universe (2025)
Grams+ *in prep* (2026)

Unified EoS with semiclassical approach

Phase space
TETF-BSkG4

Binary Neutron Stars and Supernovae

Y_p=0.3



Phase space
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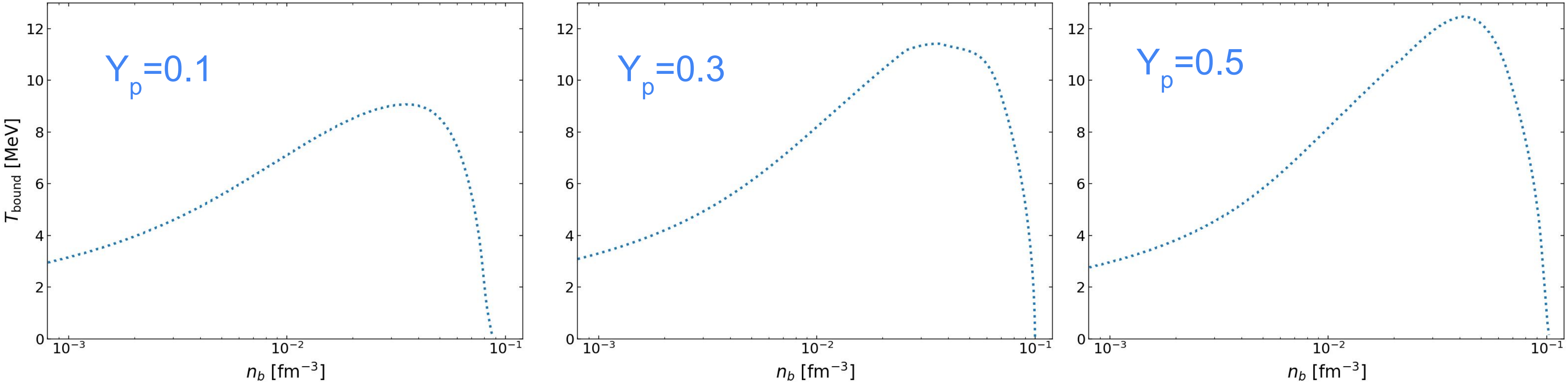
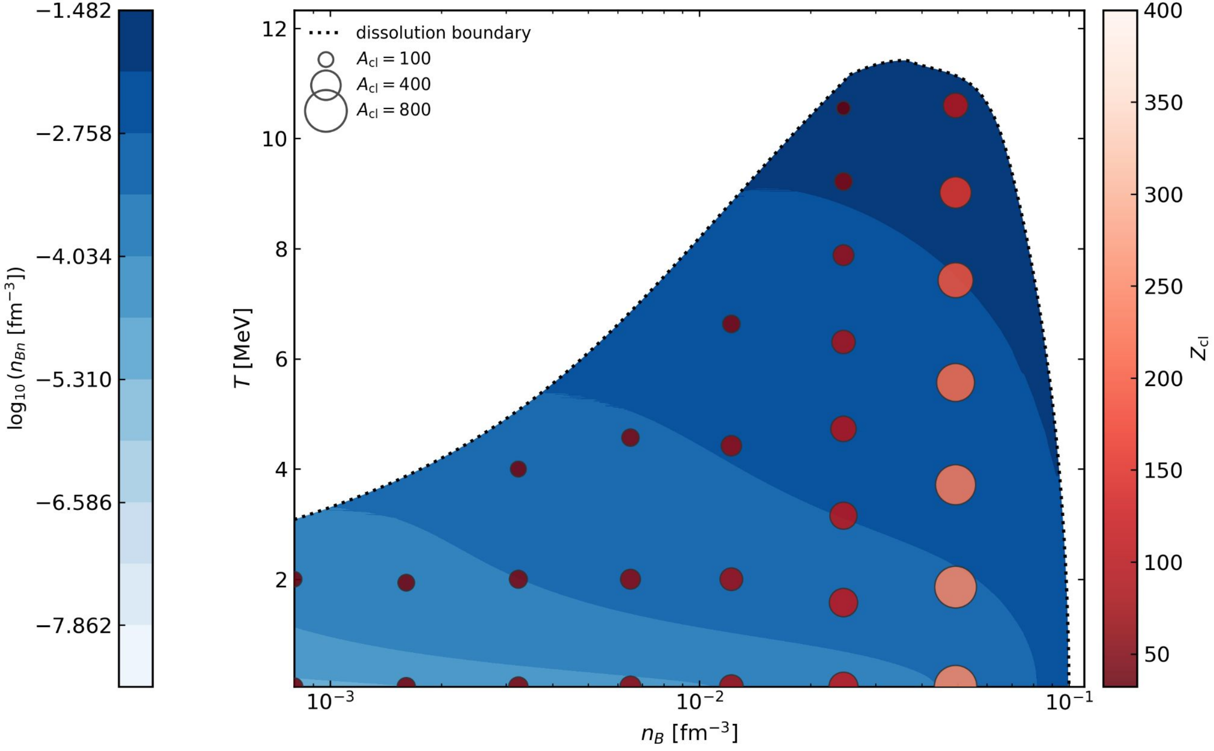


Hot clustered matter with semiclassical approach

Binary Neutron Stars and Supernovae

Y_p=0.3



Summary

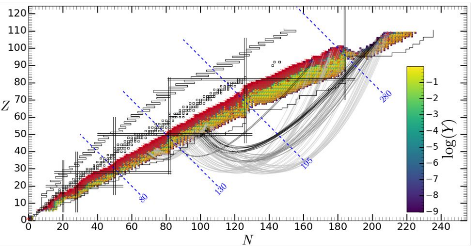
- **Same nuclear force to nuclear structure, neutron stars, binary neutron star mergers, supernovae and nucleosynthesis simulations.**

Extended Skyrme and N2LO-Skyrme parametrizations compatible with **neutron stars**.

First general purpose EoS based on extended Thomas-Fermi approach. Applications in **neutron star mergers**.

BSkG

High accuracy in **nuclear structure** properties: masses, radii, fission,… Input for **nucleosynthesis** simulations.



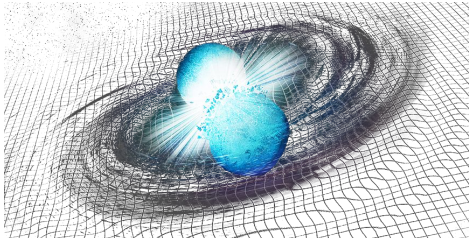


Image adapted from CXC/M. Weiss.

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GANIL, France

- Anthea F. Fantina
- Théau Diverrès

Laboratoire de Physique Corpusculaire (LPC) - France

- Francesca Gulminelli

extra slides

EoS

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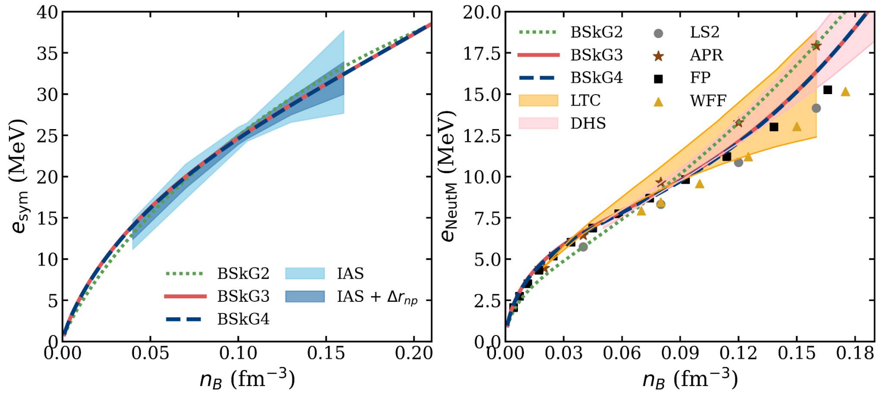
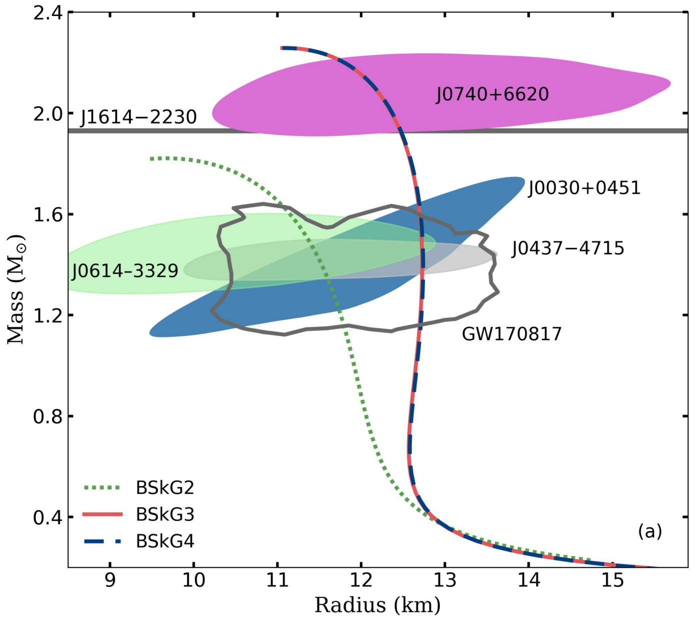
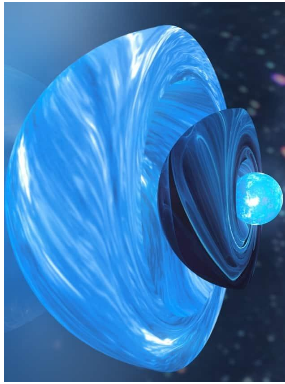




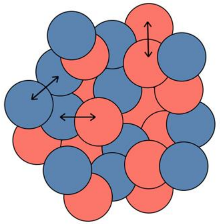
Grams+ EPJA (2023) Grams+ EPJA (2025)

BSkG4:

Improved description of pairing gaps at arbitrary asymmetry.

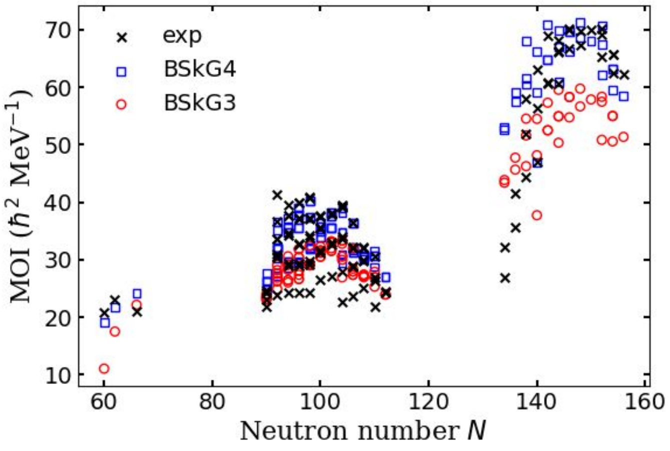


Pairing gaps **impacts both finite nuclei and neutron stars (superfluidity).**

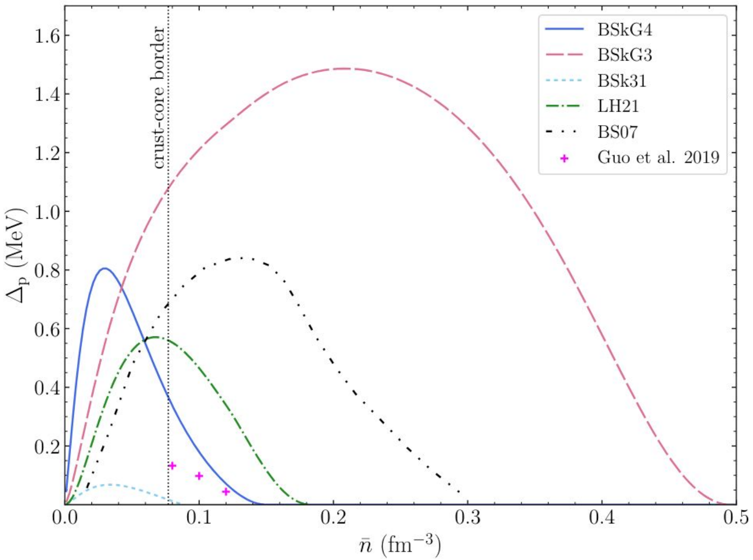


BSkG4 -> Improved description of pairing gaps at arbitrary asymmetry.

Moment of Inertia of ***finite nuclei***



Proton pairing gap in ***neutron star*** matter

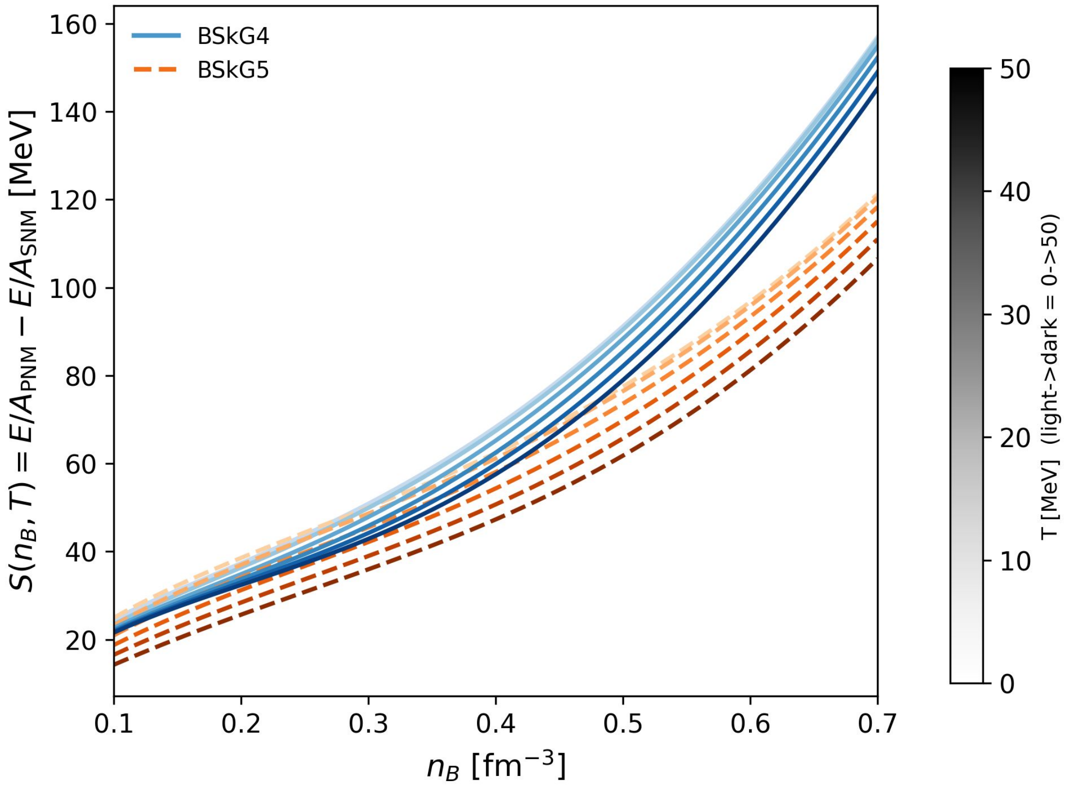


LH21: Y. Lim & J. W. Holt, PRC 103, 025807 (2021).
BS07: M. Baldo and H. J. Schulze, PRC, 75, 025802 (2007).
Guo19: W. Guo, et al., NPA, 986, 18 (2019).

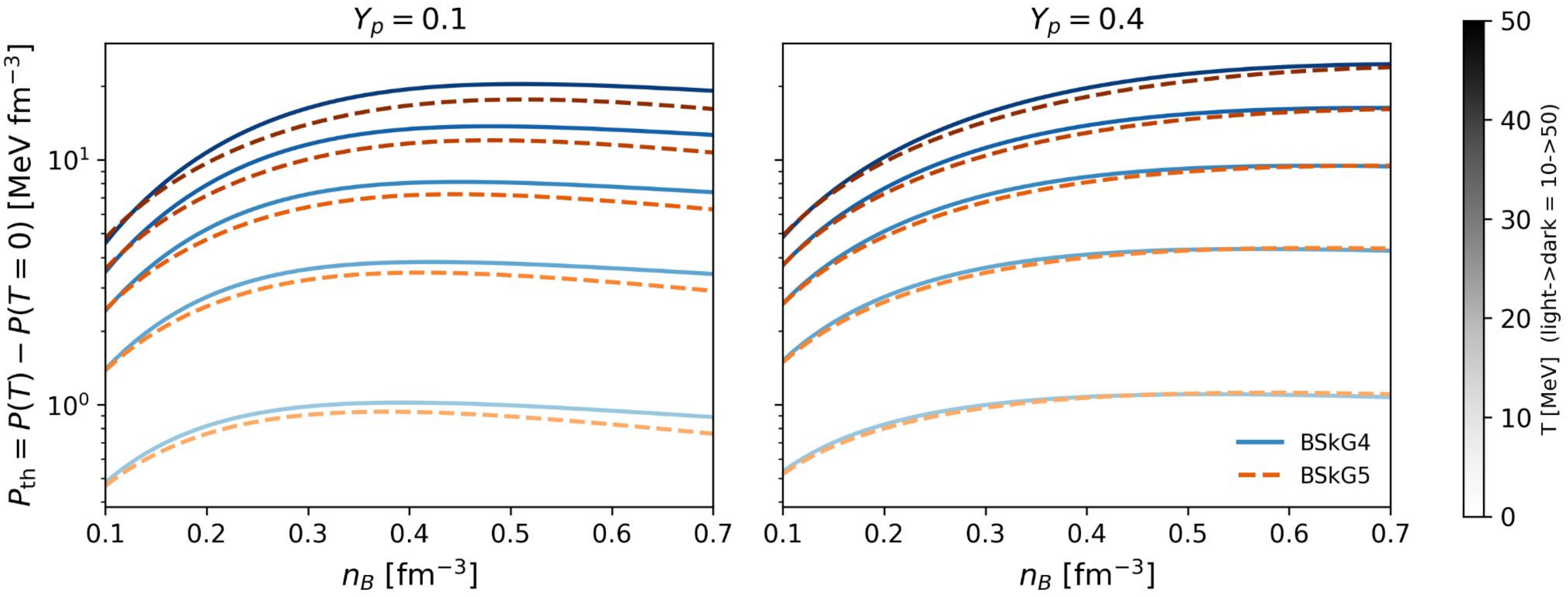
Grams+ EPJA (2025)

Energy, pressure and symmetry energy

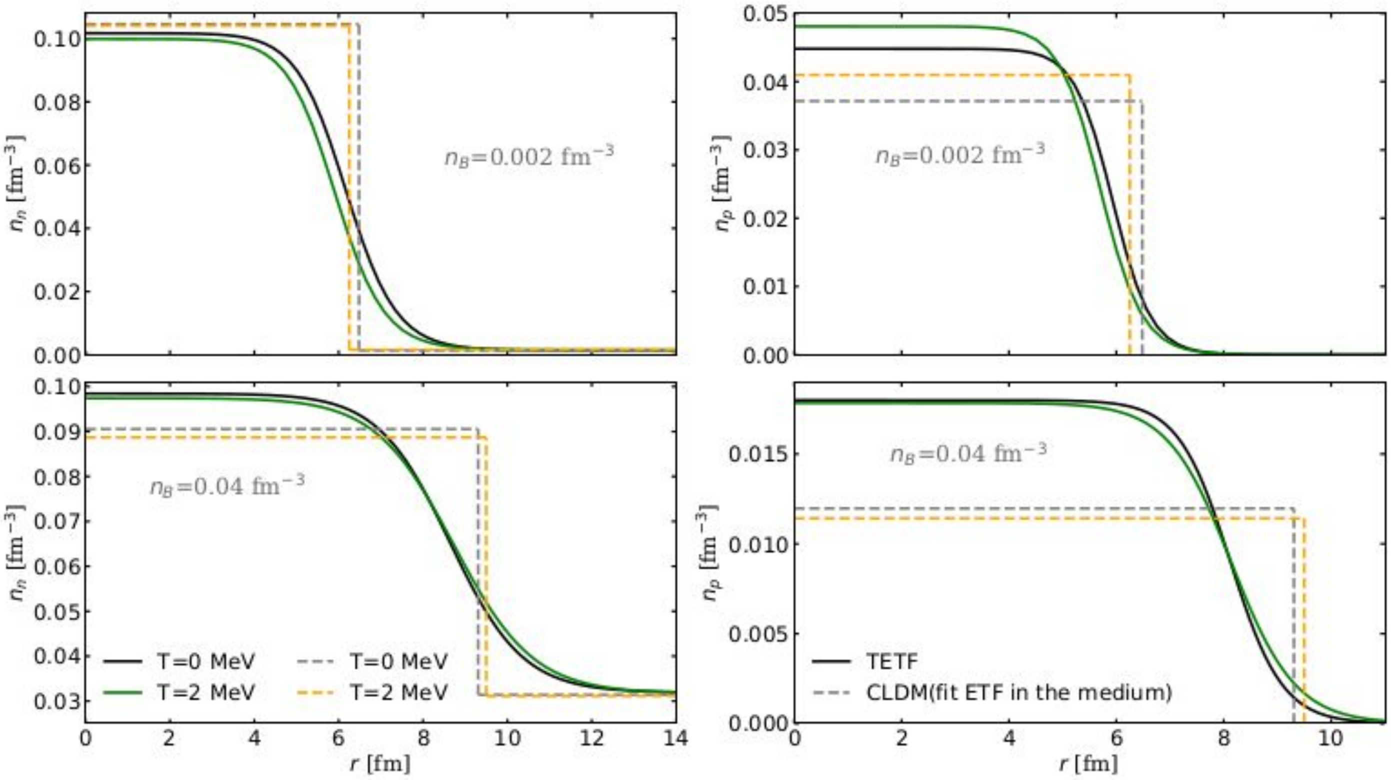
Binary neutron star mergers



Thermal pressure

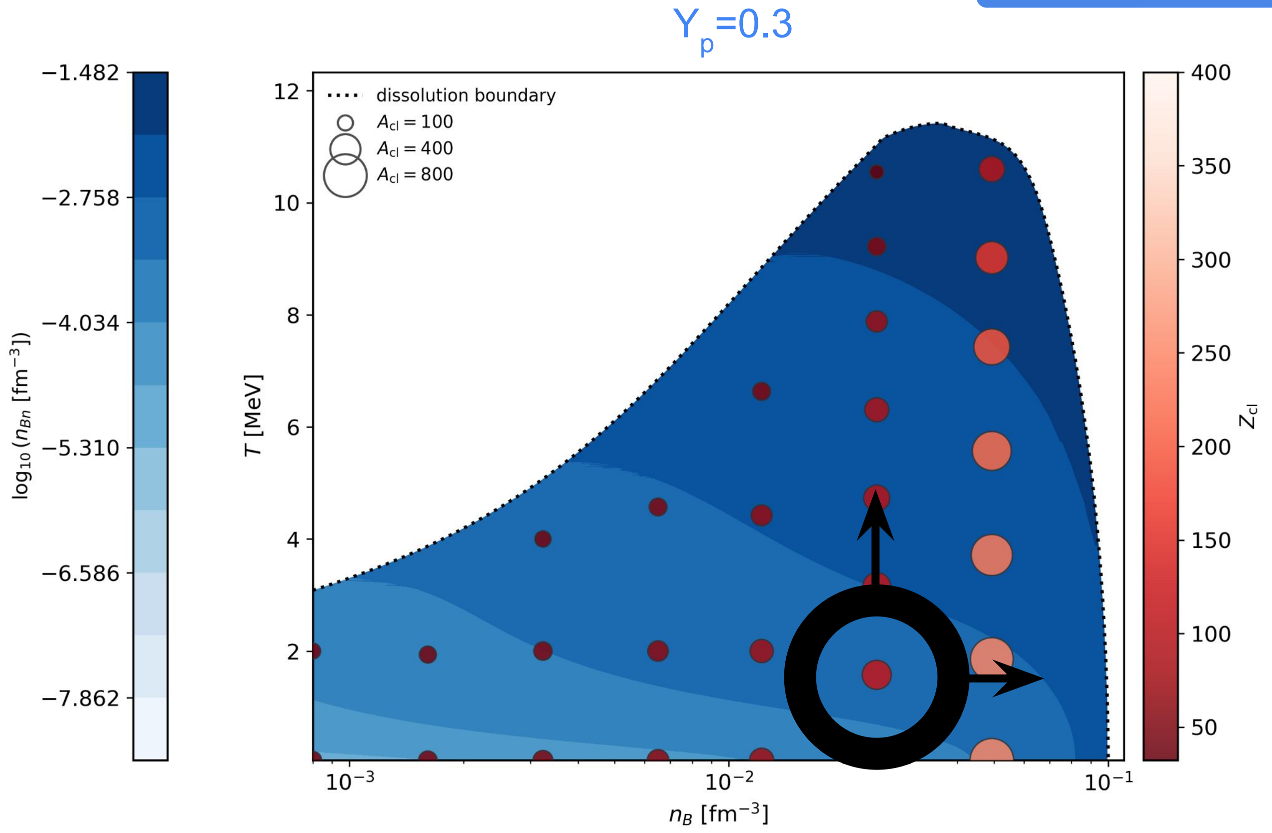


Building **unified EoS** with semiclassical calculations



Grams+ Universe (2025)

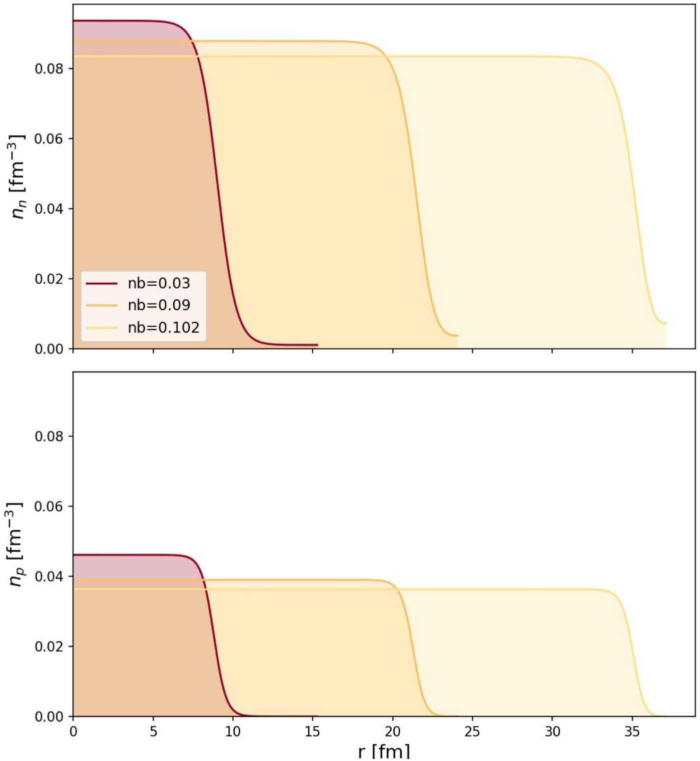
Binary neutron star mergers



Clustered matter with semiclassical approach

Profile densities at Y_p=0.3

T = 2 MeV, increase density



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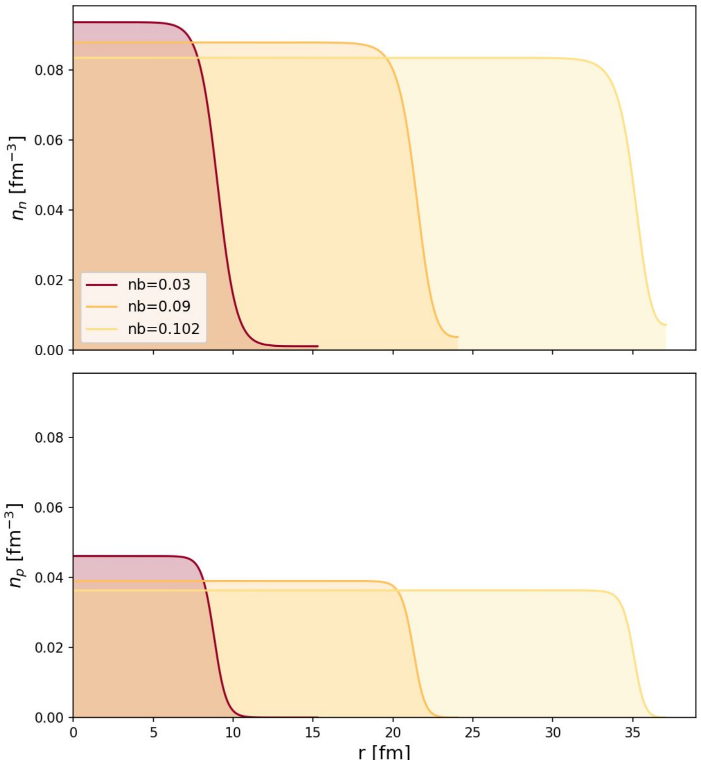
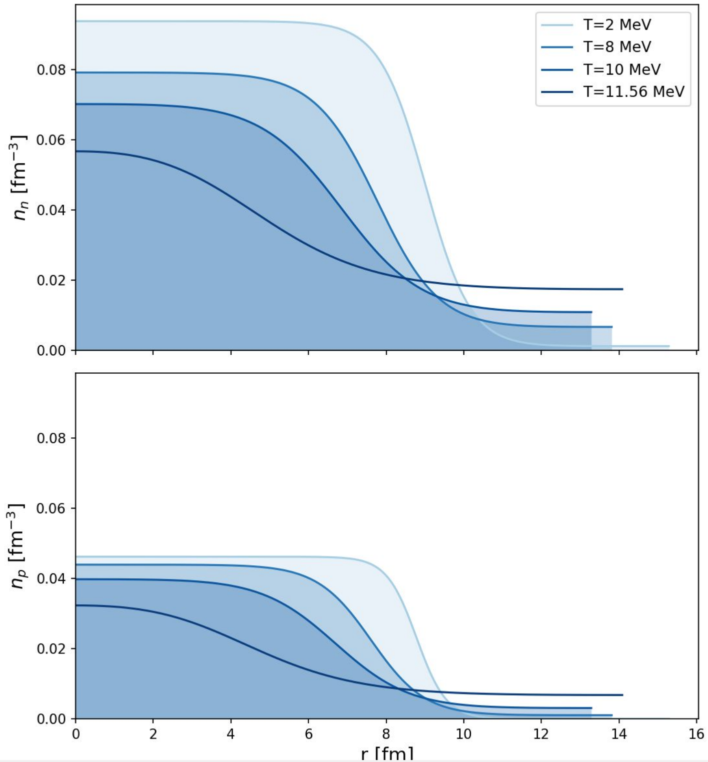
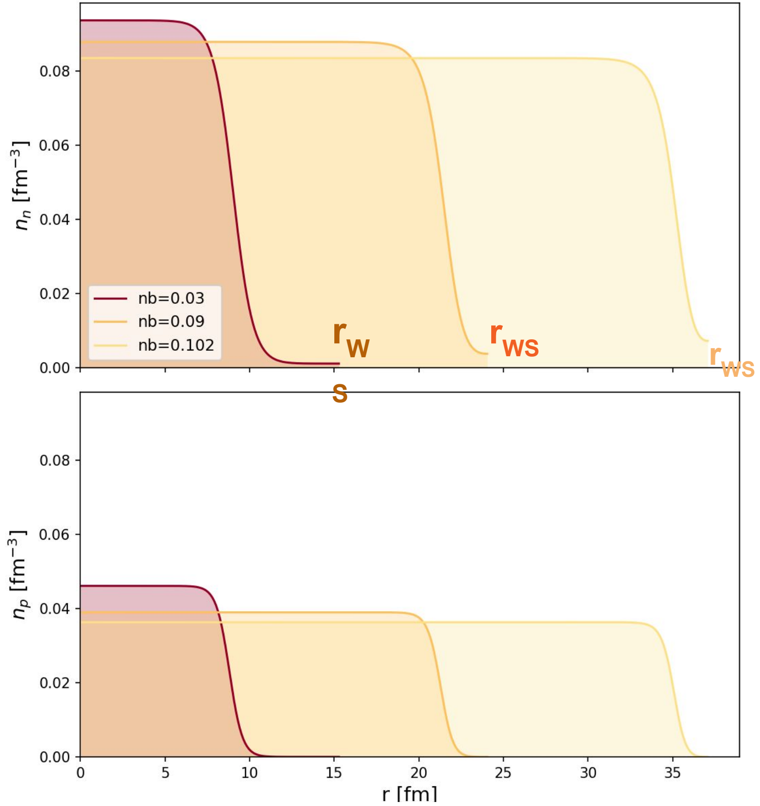


n_b = 0.03 fm^{-3}, increase temperature

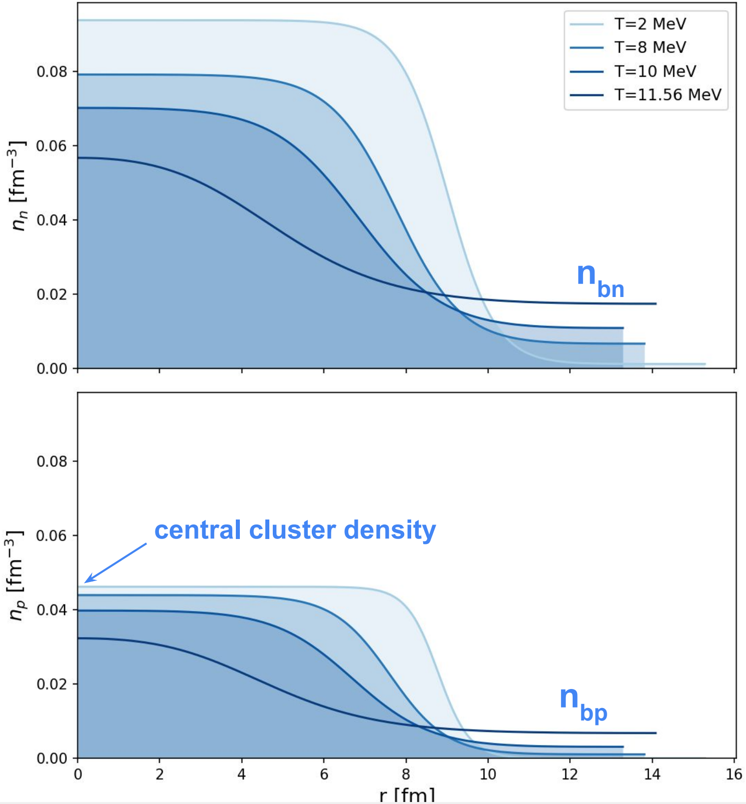


Clustered matter with semiclassical approach

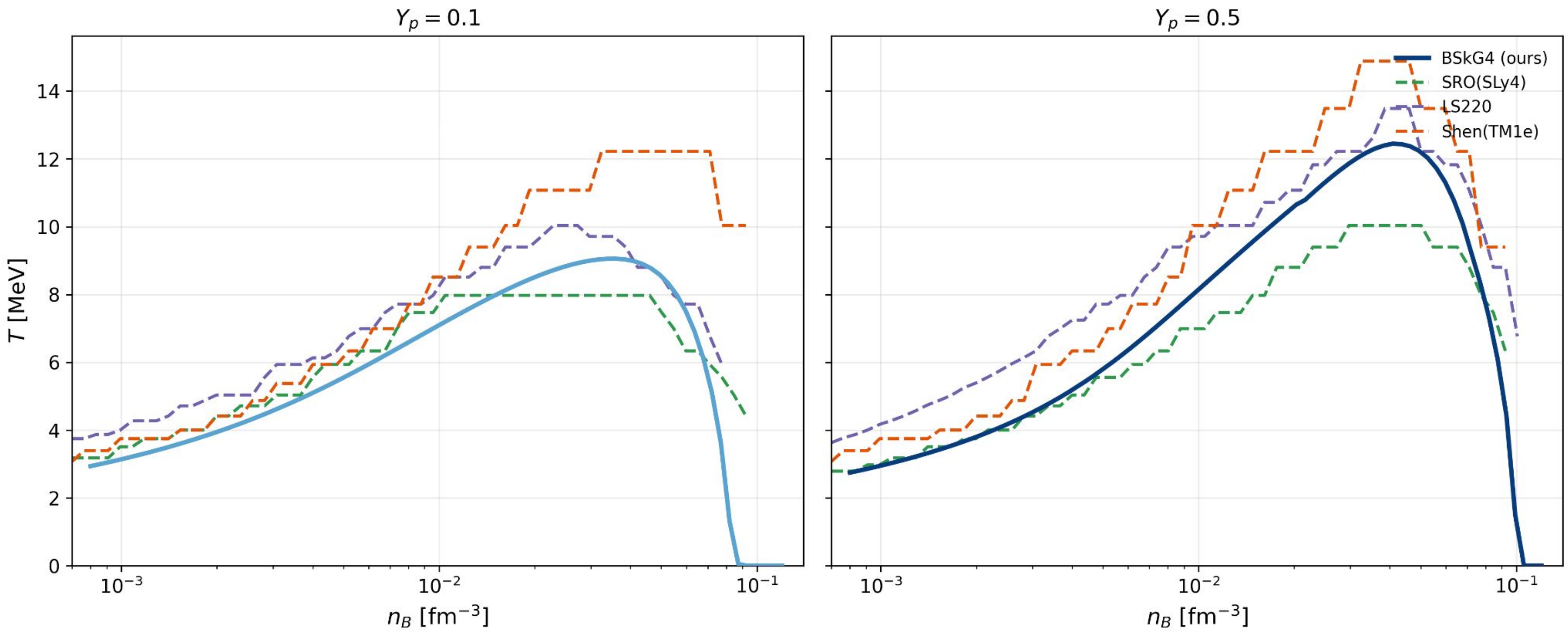
T = 2 MeV, increase density: cluster fill the whole cell

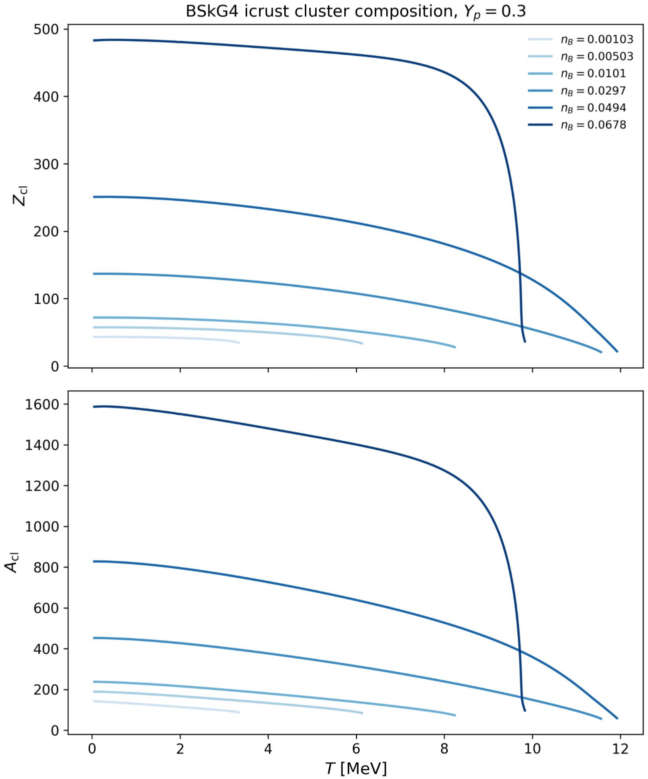


n_b = 0.03 fm^{-3}, increase temperature: cluster dissolve by profiles becoming flat

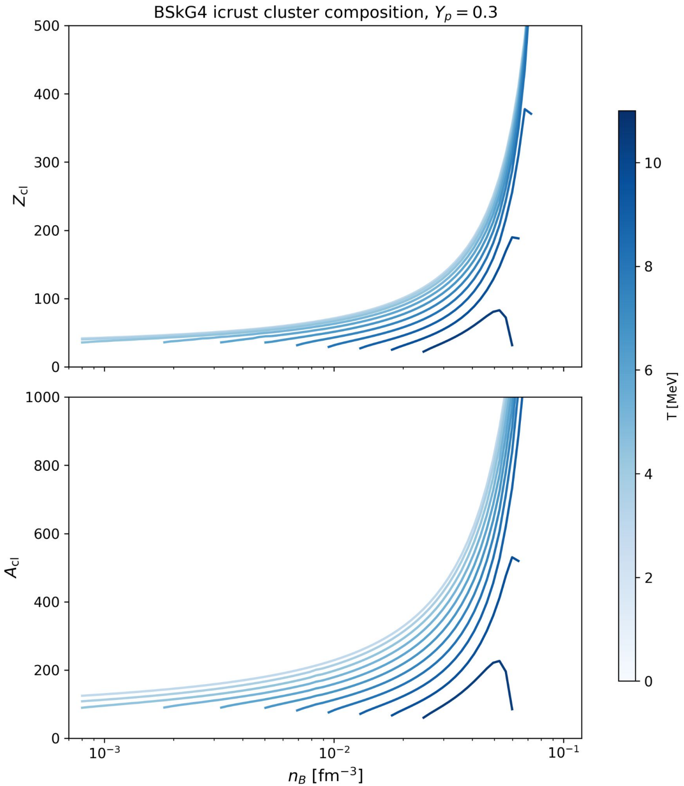


Heavy-cluster dissolution boundary: BSkG4 vs CompOSE SNA tables



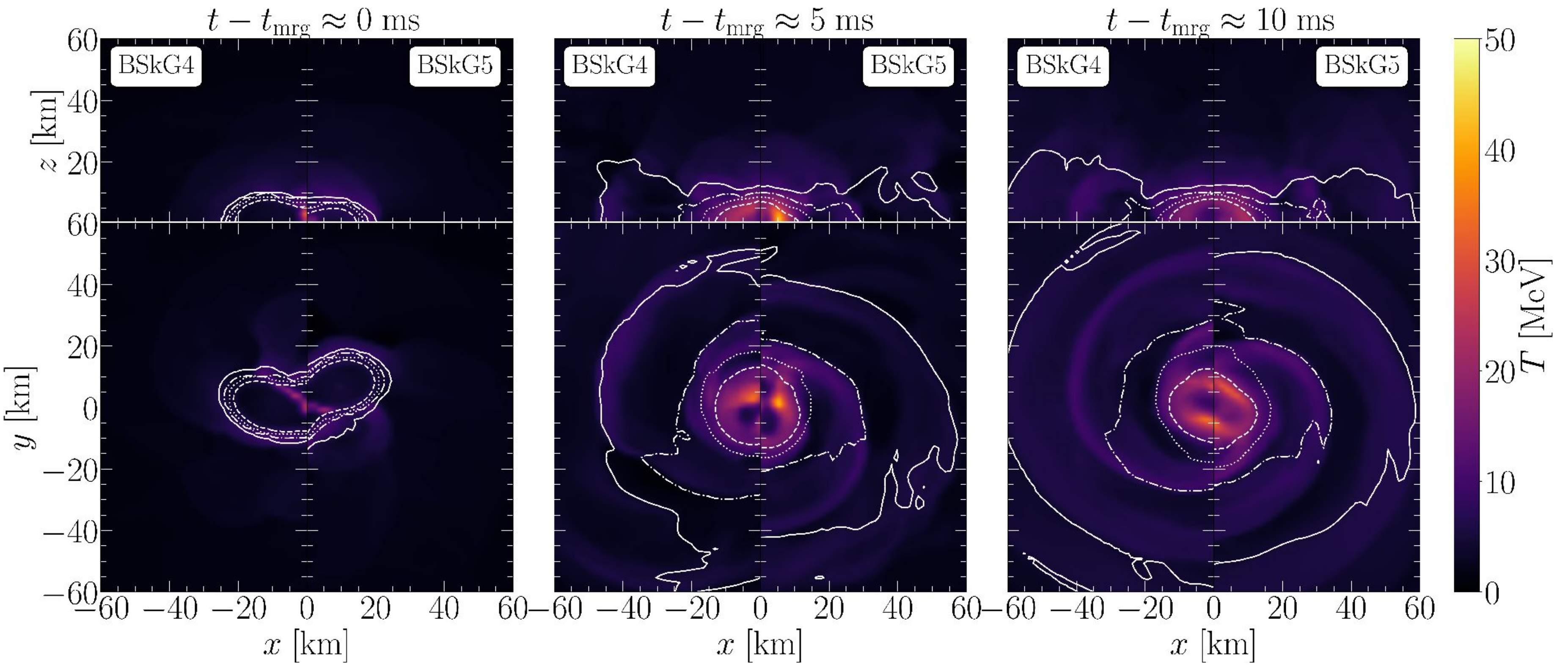


BSkG4 icrust cluster composition, $Y_p = 0.3$
$n_B = 0.00103$
$n_B = 0.00503$
$n_B = 0.0101$
$n_B = 0.0297$
$n_B = 0.0494$
$n_B = 0.0678$
Z_{cl}
A_{cl}
T [MeV]



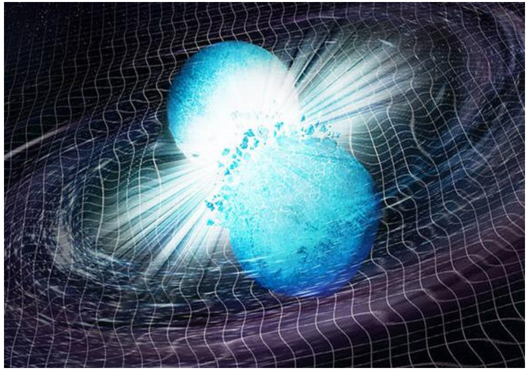
BSkG4 icrust cluster composition, $Y_p = 0.3$
Z_{cl}
A_{cl}
n_B [fm^{-3}]
T [MeV]

Binary neutron star merger: first simulation tests



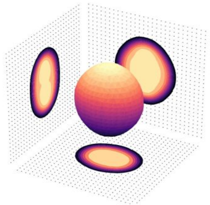
We build **finite temperature EoSs** with BSkG4 and BSKG5.

- First only homogeneous matter, matched to RG(SLy4) crust.



RG(SLy4): Raduta & Gulminelli NPA (2019) extra slides

nuclear structure



Energy

Coupling constants

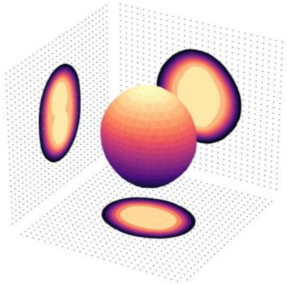
Local densities

Gradients of densities

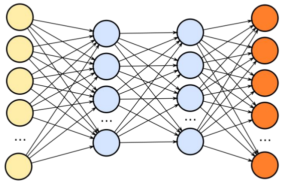
$$E = \boxed{C_1}\rho^2 + C_2\boxed{\rho\tau} + C_3\boxed{\nabla\rho\nabla\rho} + \dots$$

Brussels-Skyrme-on-a-grid (BSkG)

- Solve HFB equations with MOCCa 3D code;
- Fitted to essentially all experimental masses;
- Constraints on infinite nuclear matter properties;
- Machine learning to accelerate the fit.



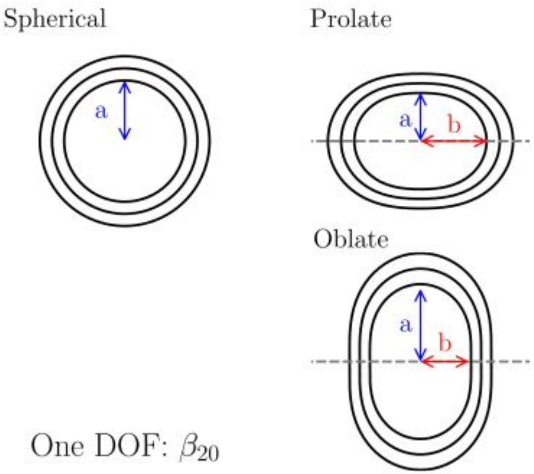
+

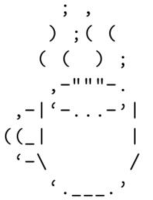


MOCCa: W. Ryssens and M. Bender, (2026) 10.5281/zenodo.20931651

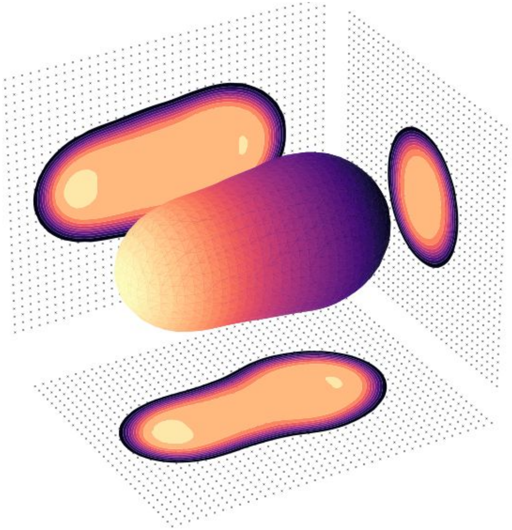
HFB solver : MOCCa

Nuclear shapes



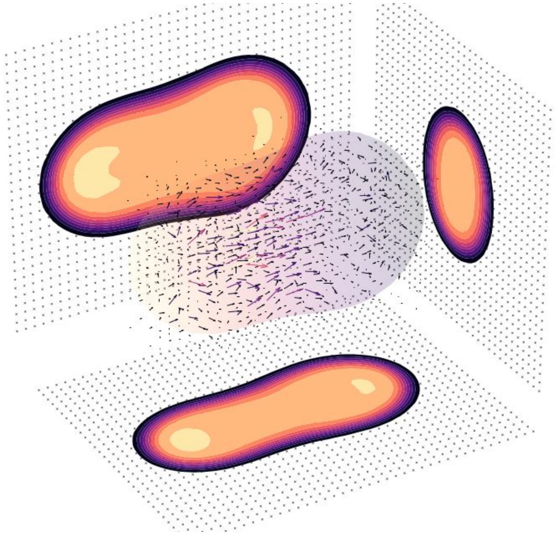


Triaxial + octupole



β_{20}, β_{22} and β_{30}

Spin densities and currents



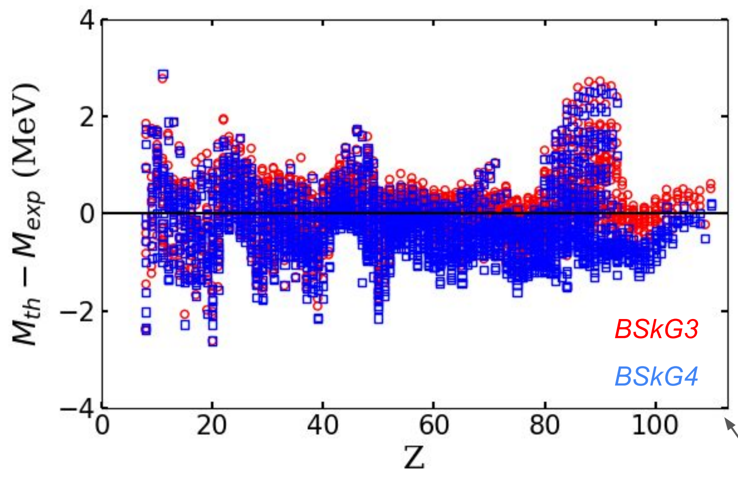
Time-reversal breaking

HFB solver at **3D coordinate-space.**
W. Ryssens PhD Thesis, ULB (2016).

Nuclear structure with BSkGs

Reproduction of known nuclear **masses**, **radii**, **fission** barriers heights.

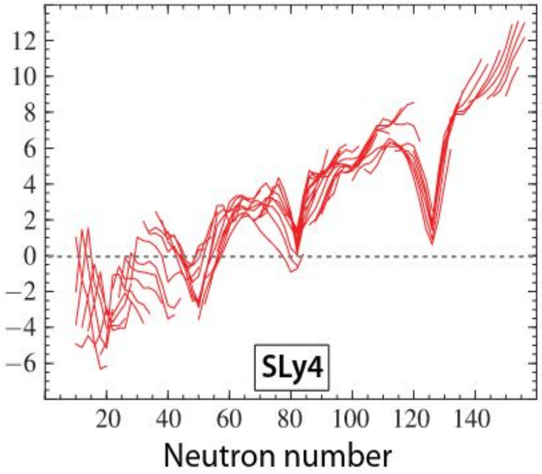
σ (M) < 700 KeV
σ (R_C) < 0.025 fm
σ (E_I) < 400 KeV



All 2457 nuclei in the AME2020.

Comparison:

SLy4
σ (M) = 4.80 MeV
UNEDF0
σ (M) = 1.45 MeV



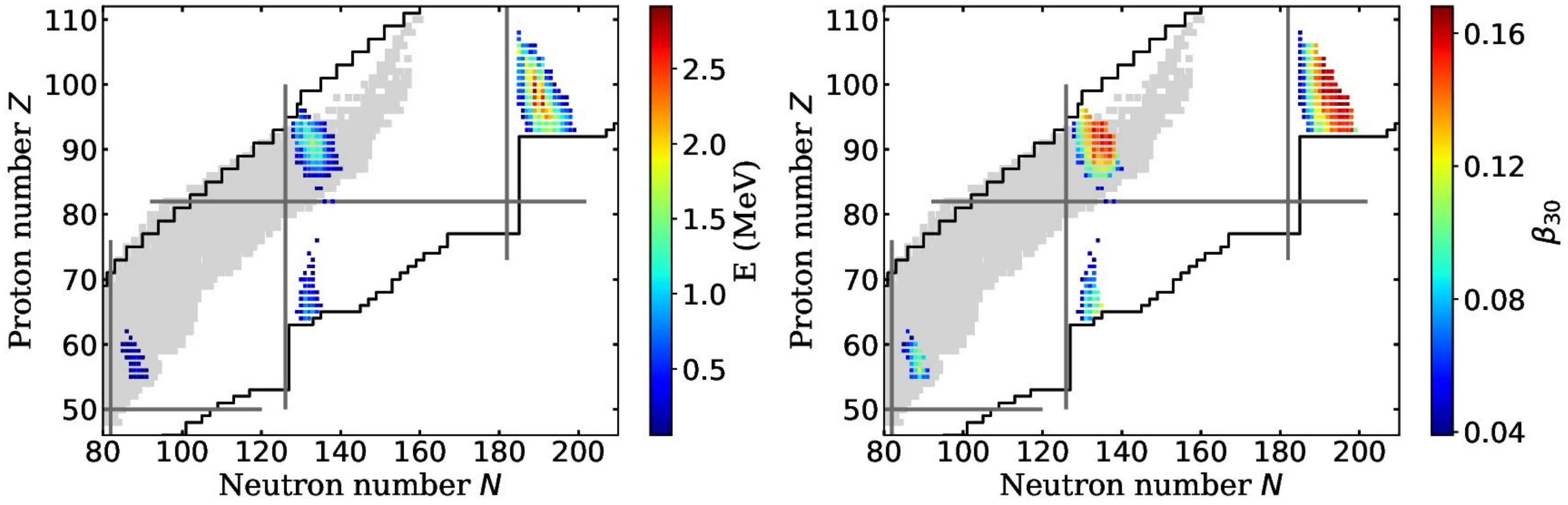
SLy4: E. Chabanat, et al., Phys. Scr. T 56, 231 (1995).
UNEDF0: M. Kortelainen, et al., Phys. Rev. C 82, 024313 (2010).

Grams+ EPJA (2023) Grams+ EPJA (2025)

The BSkG3 mass model

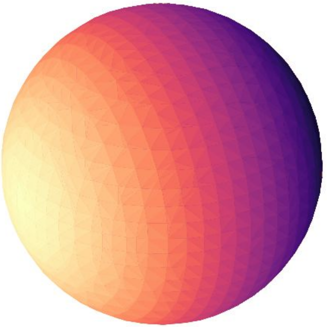
Break reflection symmetry: octupole shapes in ground state

Most general global study of **octupole** deformation and the **first** to systematically include odd–mass and odd–odd nuclei.



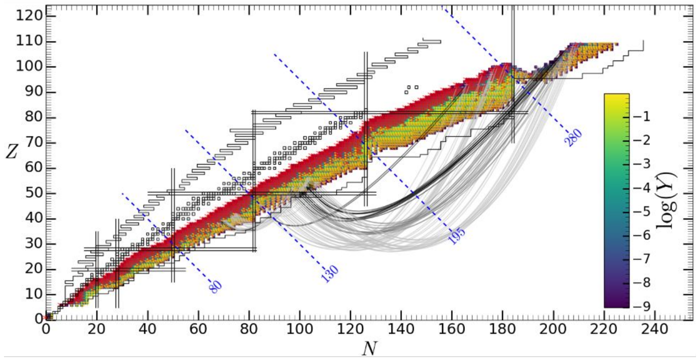
GG, W. Ryssens, G. Scamps, S. Goriely and N. Chamel, EPJA **59**, 270 (2023).

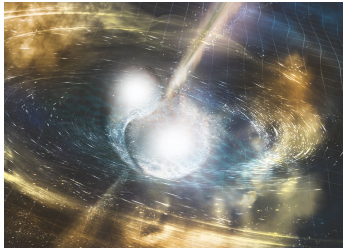
BSkG3 fission

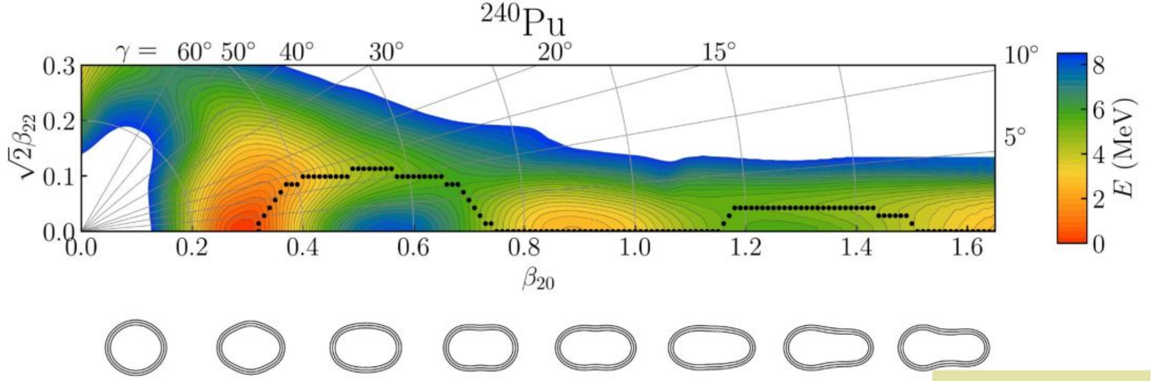


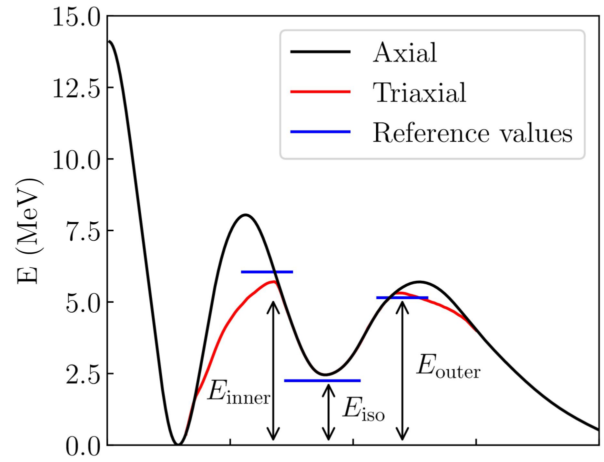
Impact r-process:

- "**fission recycling**";
- the **r-process abundances** in the 110 ≤ A ≤ 170 region;
- the production of cosmic chronometers such as Th and U;
- the **heating rate of kilonovae**.









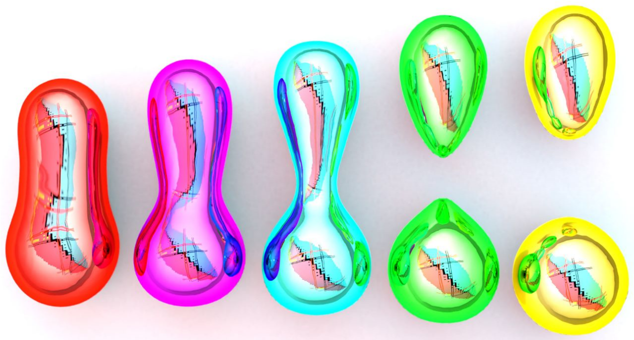


Fig from G. Scamps.

BSkG5 -> Next-to-next-to-leading-order (N2LO) EDF

❖ Systematic expansion on gradient terms on the EDF.

Order of the gradients in the EDF

$$E_{Sk} = \int d^3\mathbf{r} \sum_{t=0,1} \left[\mathcal{E}_t^{(0)}(\mathbf{r}) + \mathcal{E}_t^{(2)}(\mathbf{r}) + \mathcal{E}_t^{(4)}(\mathbf{r}) \right]$$

LO NLO N2LO

Grams+ PLB (2026)

BSkG5 -> Next-to-next-to-leading-order (N2LO) EDF

Order of the gradients in the EDF

Energy

$$E_{\text{Sk}}^{\text{N2LO}} = \int d^3\mathbf{r} \sum_{t=0,1} \left[\mathcal{E}_t^{(0)}(\mathbf{r}) + \mathcal{E}_t^{(2)}(\mathbf{r}) + \mathcal{E}_t^{(4)}(\mathbf{r}) \right]$$

Term with four gradients appear only at N2LO

Local densities

$$\mathcal{E}_t^{(4)}(\mathbf{r}) = C_t^{(4),M\rho} \left[\rho_t(\mathbf{r})\mathbf{Q}(\mathbf{r}) + \tau_t^2(\mathbf{r}) + 2\sum_{\mu\nu} \tau_{t,\mu\nu}\tau_{t,\mu\nu}(\mathbf{r}) \right] + \dots$$

Coupling constants

Fourth order kinetic density Q

BSkG5 -> Next-to-next-to-leading-order (N2LO) EDF

effective mass

$$\frac{m}{m_q^*(k)} = 1 + \frac{m}{k}\frac{\partial}{\partial k}U_{qs}(\varepsilon, k)\Big|_{k=k(\varepsilon)}$$

Since the only momentum dependence is in τ, at NLO m* is normally define as:

$$\left[\frac{m}{m_q^*(k)}\right]^{NLO} = \frac{2m_q}{\hbar^2}\frac{\partial \mathcal{E}^{NLO}}{\partial \tau_q}$$

At NLO: m* has no momentum dependence
== when T > 0, the m* has no temperature dependence.

BSkG5 -> Next-to-next-to-leading-order (N2LO) EDF

effective mass

$$\frac{m}{m_q^*(k)} = 1 + \frac{m}{k}\frac{\partial}{\partial k} U_{qs}(\varepsilon, k)\Big|_{k=k(\varepsilon)}$$

Local densities

$$\mathcal{E}_t^{(4)}(\mathbf{r}) = C_t^{(4),M\rho}\left[\rho_t(\mathbf{r})\mathbf{Q}(\mathbf{r}) + \tau_t^2(\mathbf{r}) + 2\sum_{\mu\nu}\tau_{t,\mu\nu}\tau_{t,\mu\nu}(\mathbf{r})\right] + \dots$$

Coupling constants

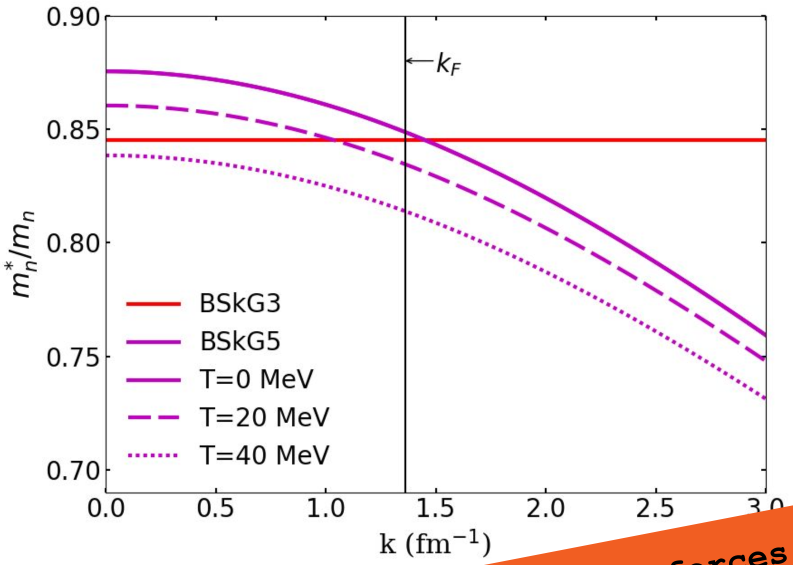
Fourth order kinetic density Q

Thanks to the high order kinetic densities, at NL2O, m* has a temperature dependence.

BSkG5 -> Next-to-next-to-leading-order (N2LO) EDF

- ❖ Systematic expansion on gradient terms on the EDF.
- ❖ Mass table with:
  - ➢ mass rms < 700 KeV.
  - ➢ primary fission barrier rms < 400 KeV.
- ❖ ***Momentum and temperature dependent effective mass.***

Neutron effective mass at symmetric matter and density = 0.17 fm^{-3}.



New for Skyrme forces.