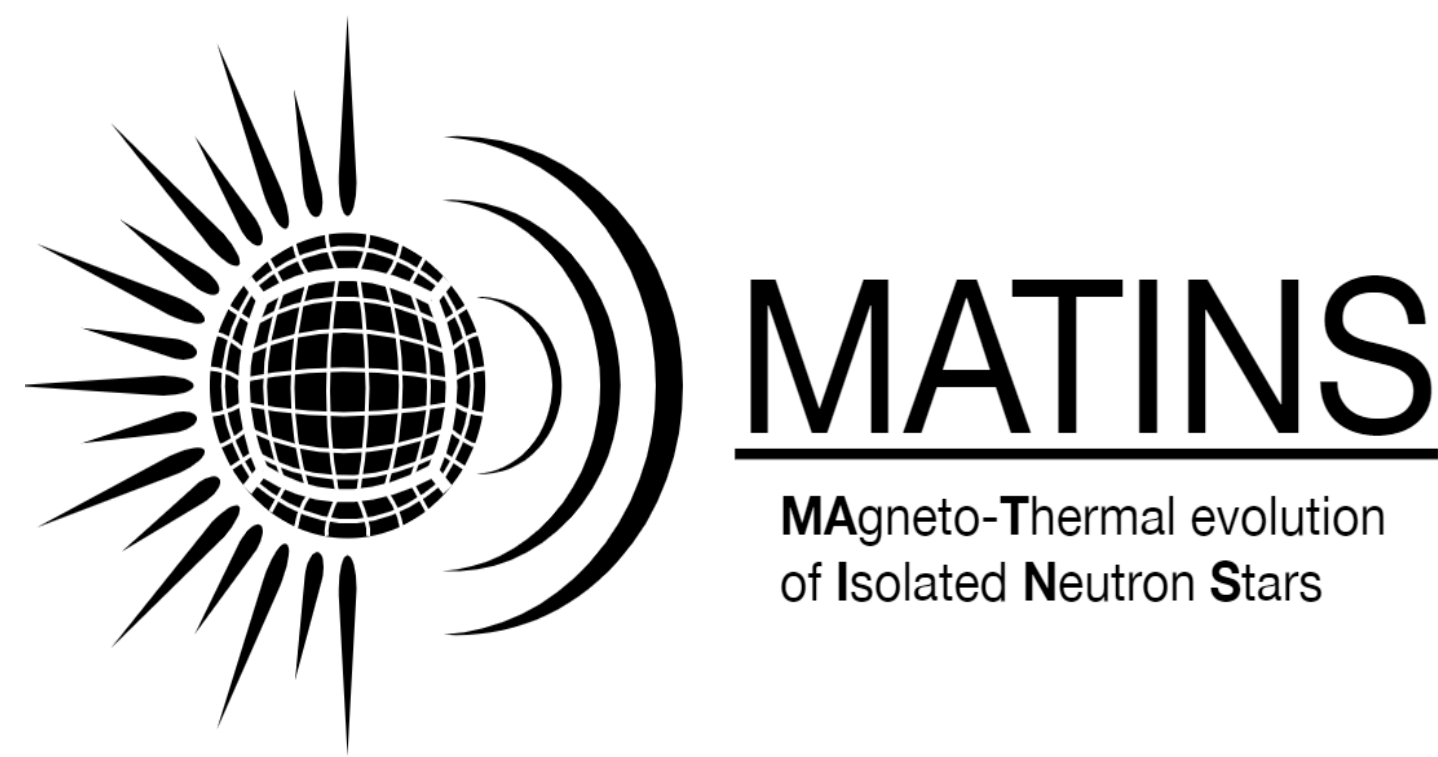


The Interior Physics of Neutron Stars: Magnetic and Thermal Evolution



September 10, 2026

Nuclear Physics in Astrophysics XII

Cluj-Napoca, Babeş – Bolyai University

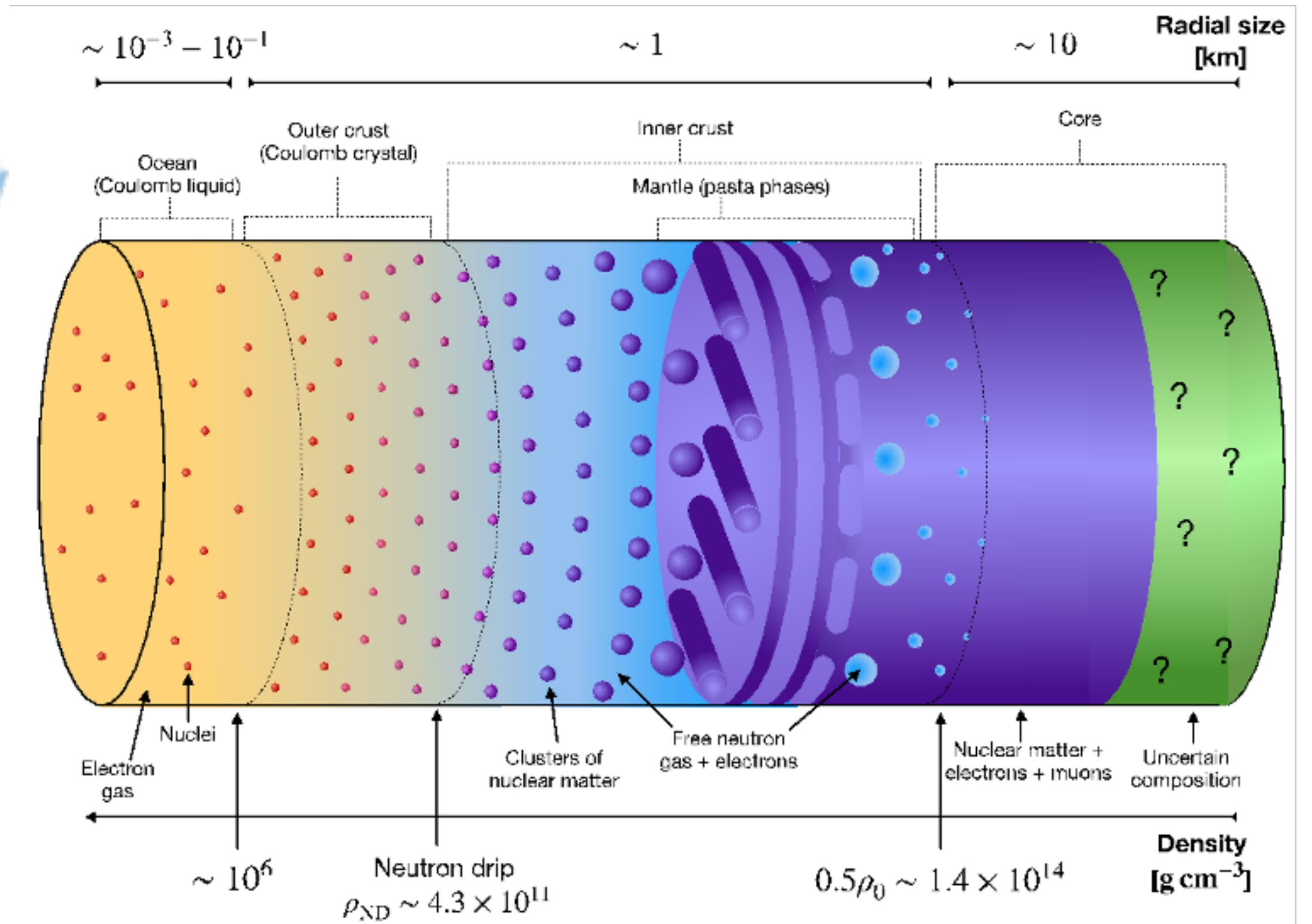
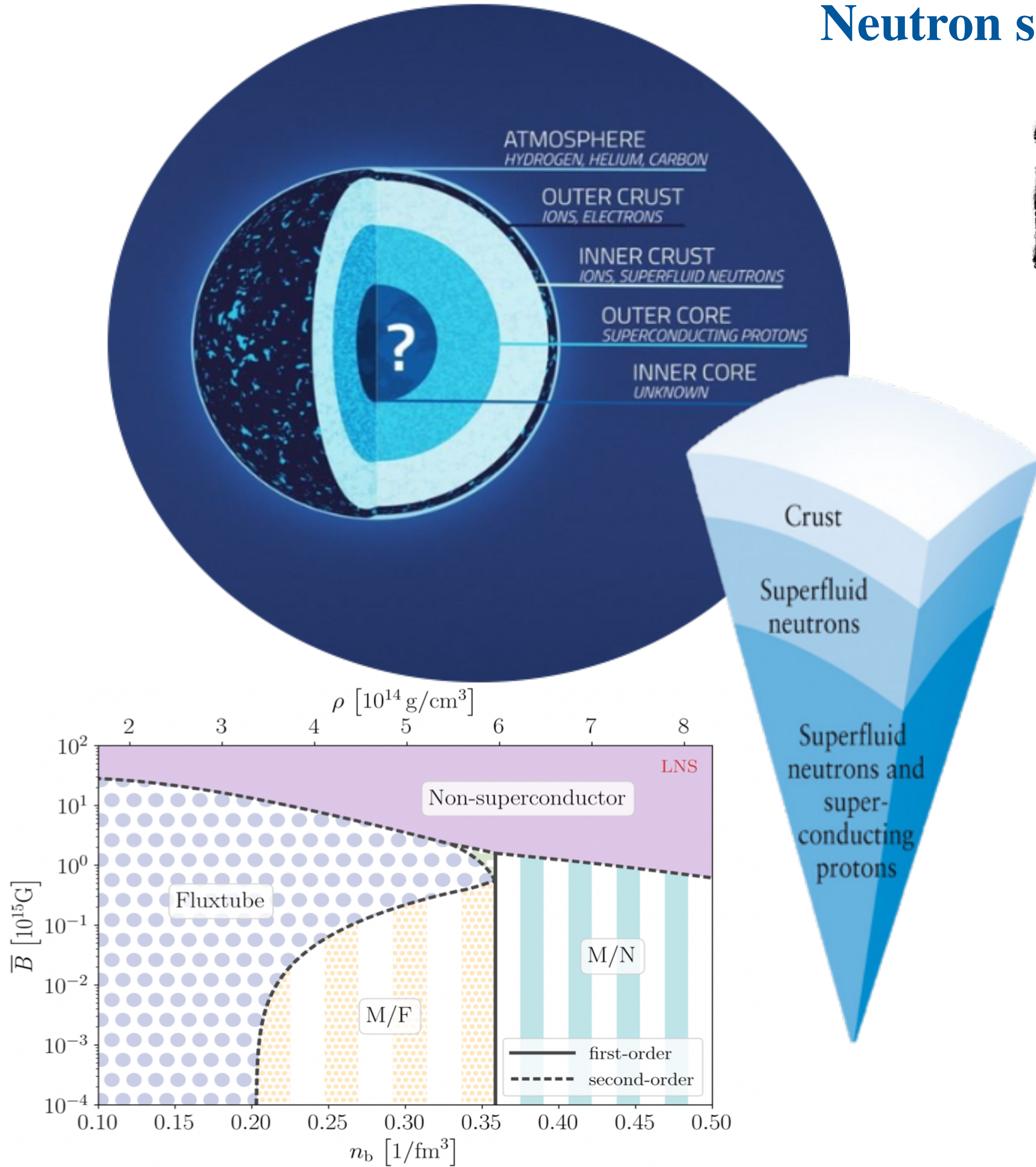
Clara Dehman

clara.dehman@ua.es

Juan de la Cierva Fellow

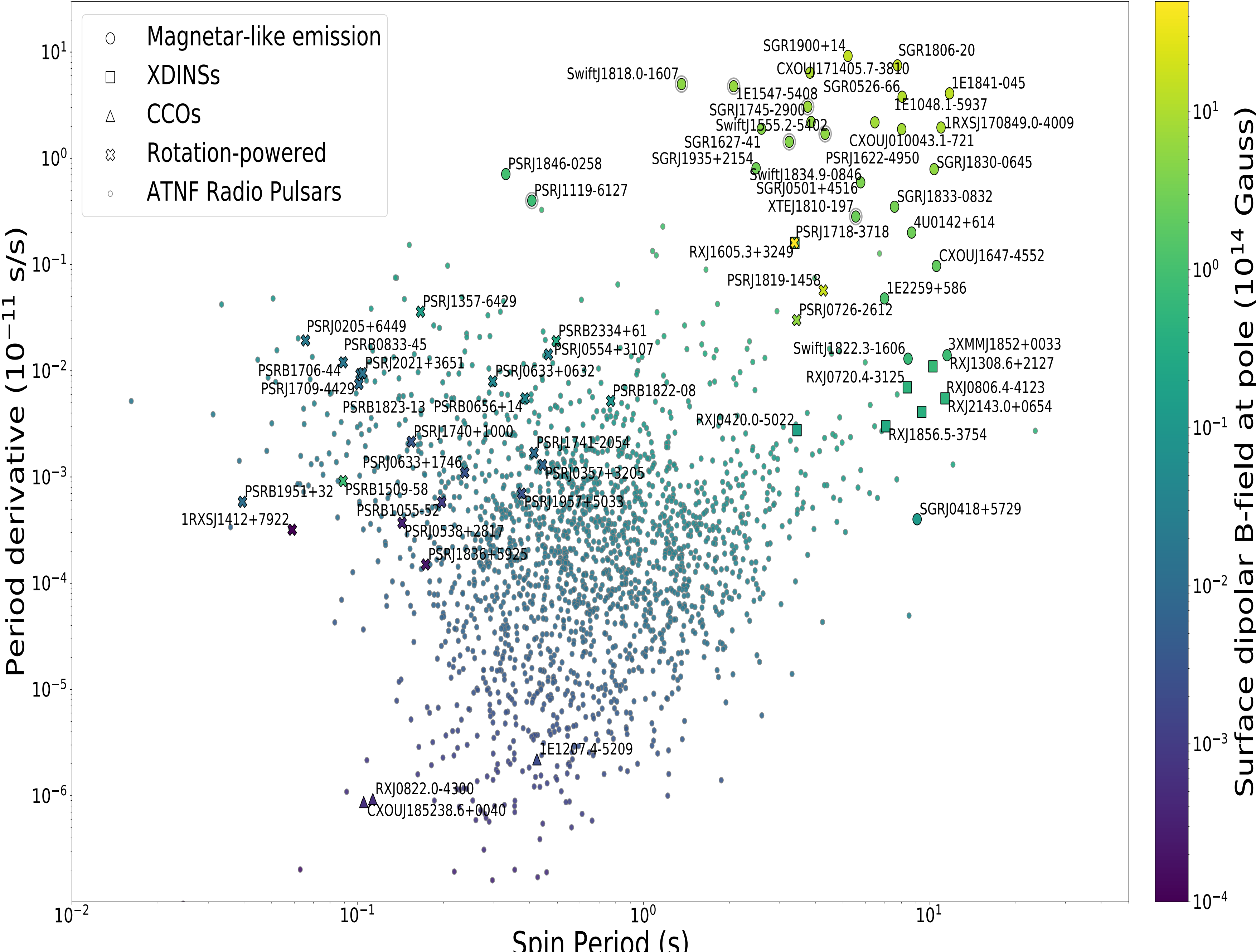
Neutron stars

Neutron stars form in core-collapse supernovae, pass through a proto-neutron star (PNS) phase, and settle into compact objects with typical masses $\approx 1.4 M_{\odot}$ and radii $\approx 10 - 12$ km.



[Ascenzi, Graber & Rea, 2024]

P-Pdot diagram for isolated neutron stars



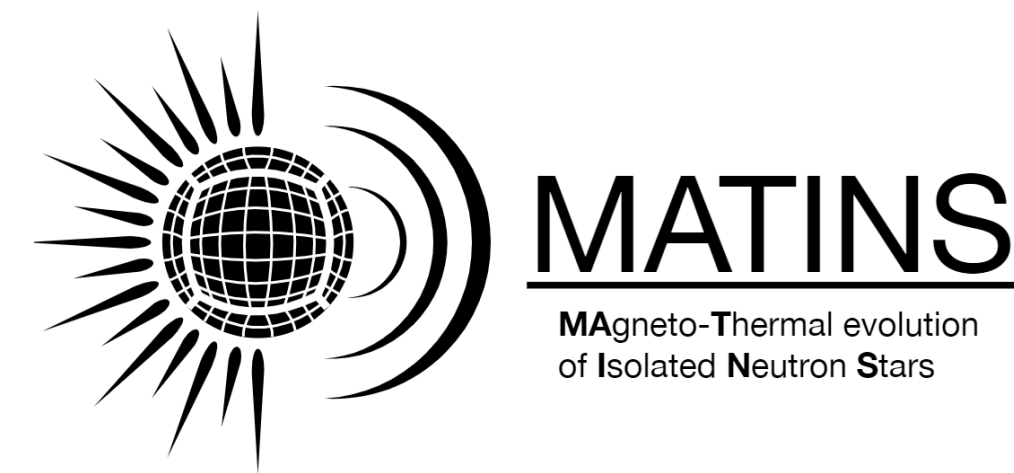
**Neutron stars form are
observed across the
electromagnetic spectrum**

Dipolar magnetic field at the pole of the star:

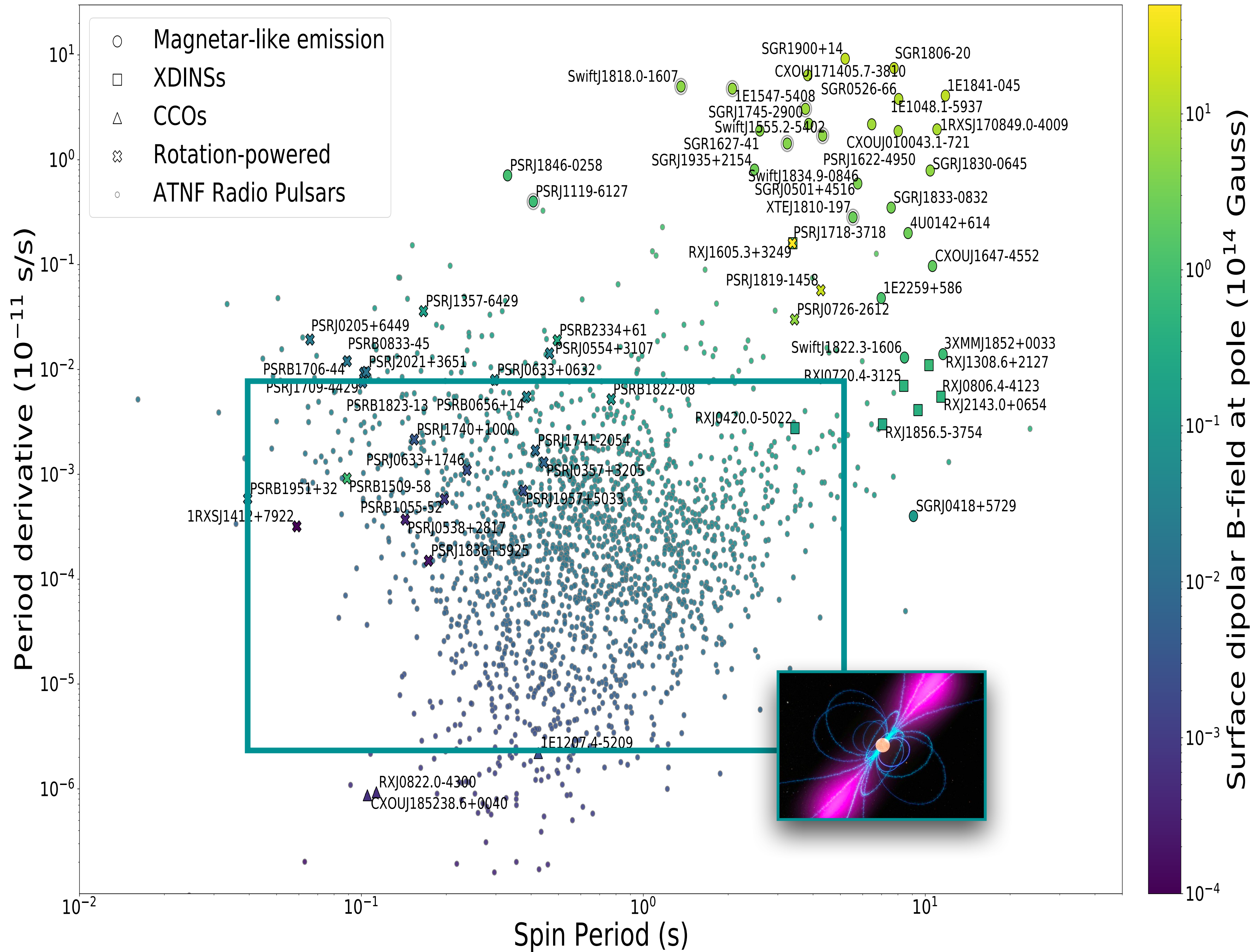
$$B_{dip} = 6.4 \times 10^{19} \sqrt{P\dot{P}}$$

Characteristic age:

$$\tau_c = \frac{P}{2\dot{P}}$$



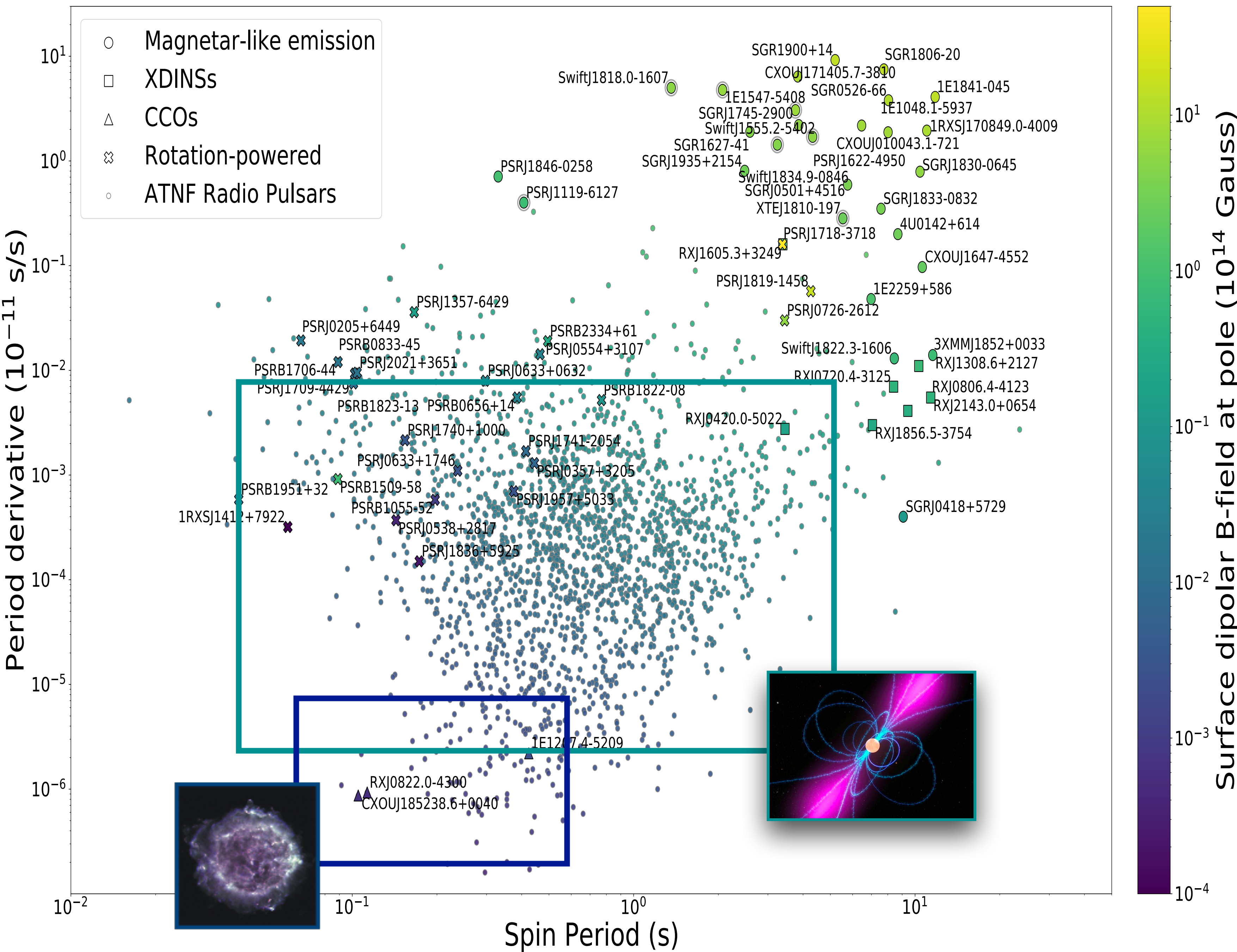
P-Pdot diagram for isolated neutron stars



Rotational-Powered Pulsars

Powered by rotational energy. Typically emitting in radio.

P-Pdot diagram for isolated neutron stars

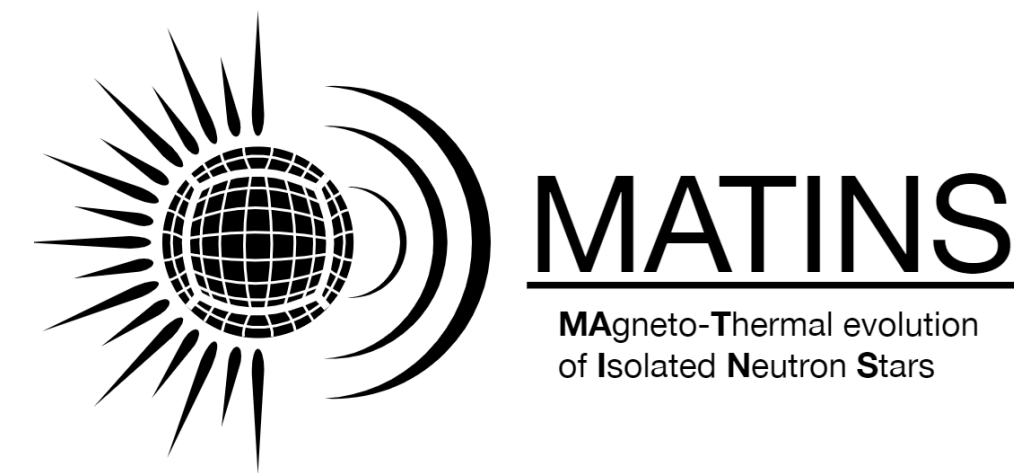


Central Compact Objects

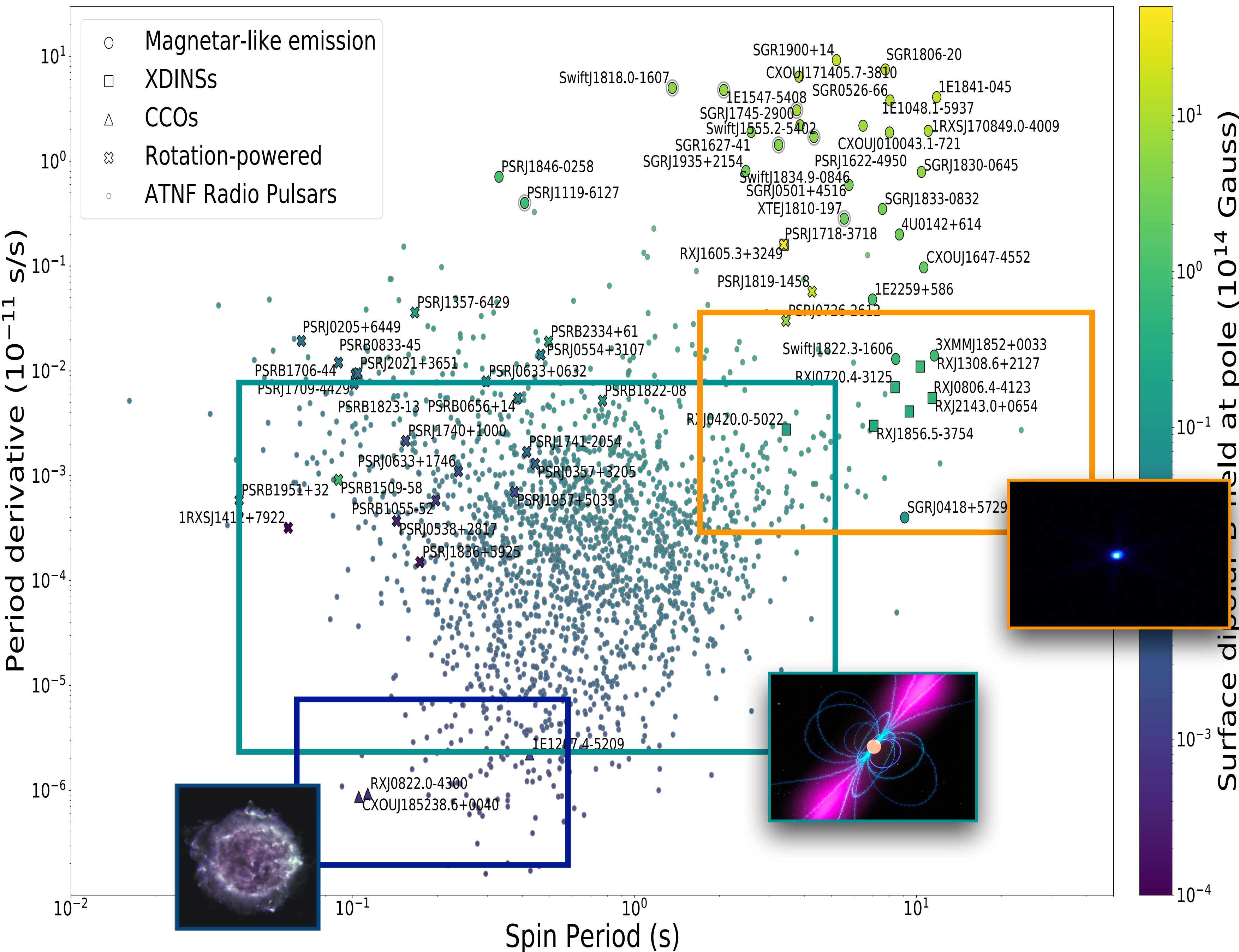
Powered by magnetic energy. Young with bright SNRs. Typically emitting in X-rays

Rotational-Powered Pulsars

Powered by rotational energy. Typically emitting in radio.



P-Pdot diagram for isolated neutron stars



X-ray Dim Isolated Neutron Stars

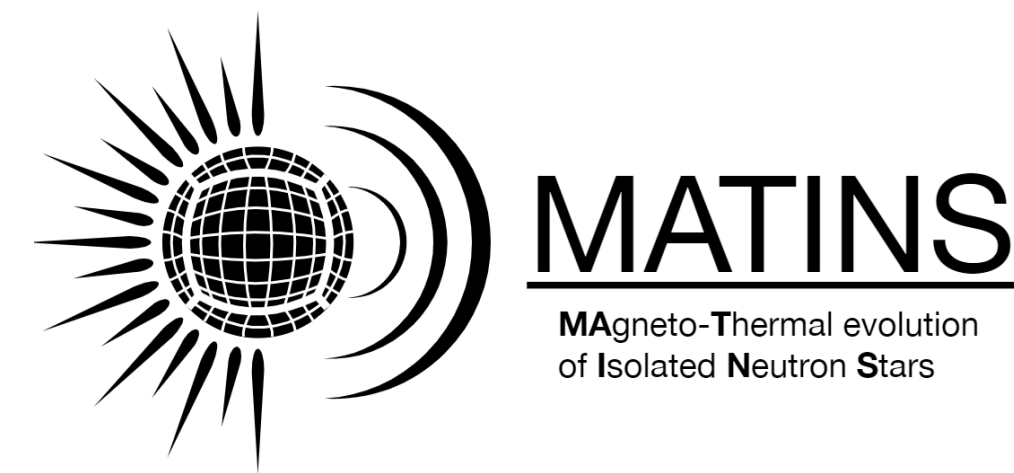
Powered by magnetic energy. Old, almost pure blackbodies. Typically emitting in X-rays

Central Compact Objects

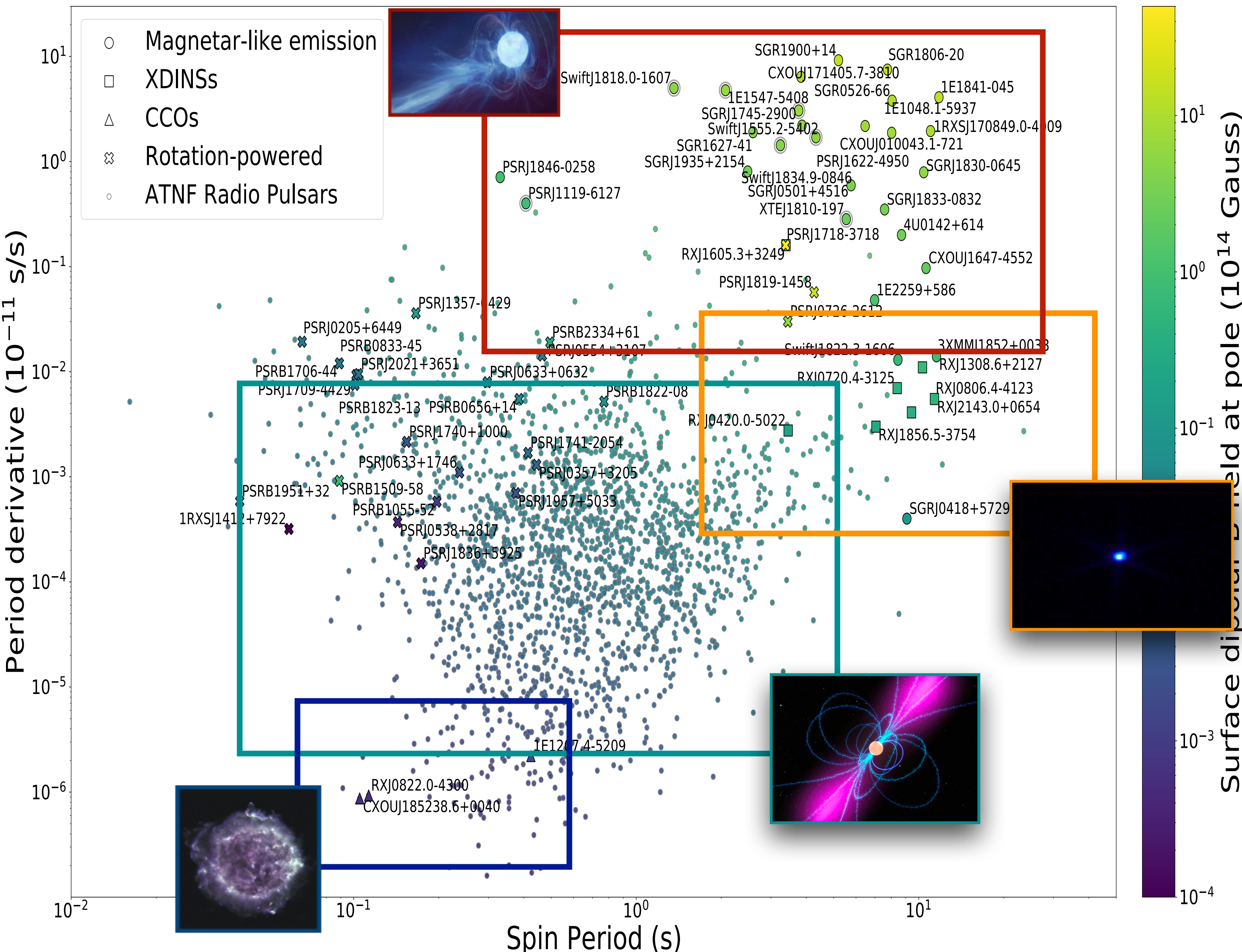
Powered by magnetic energy. Young with bright SNRs. Typically emitting in X-rays

Rotational-Powered Pulsars

Powered by rotational energy. Typically emitting in radio.



P-Pdot diagram for isolated neutron stars



Magnetars

Powered by magnetic energy.
Characterised by outbursts and flares.
Typically emitting in X-rays

X-ray Dim Isolated Neutron Stars

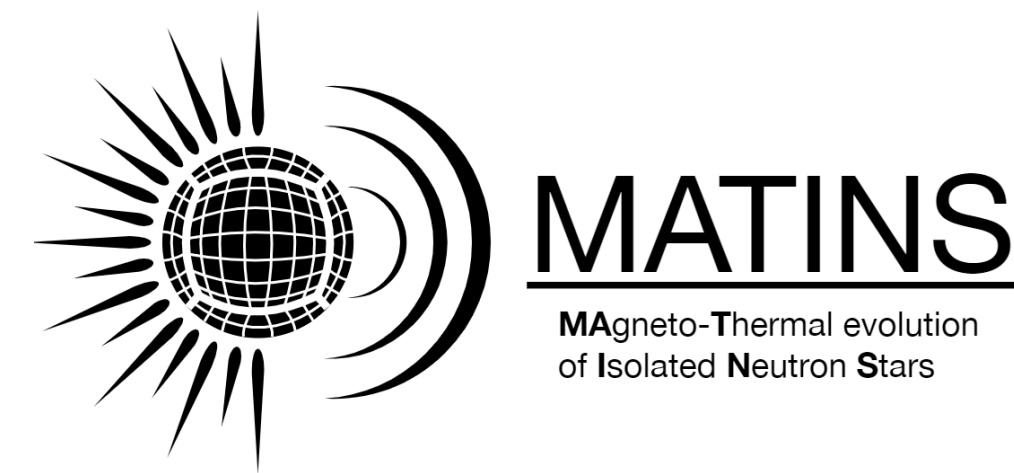
Powered by magnetic energy. Old, almost pure blackbodies. Typically emitting in X-rays

Central Compact Objects

Powered by magnetic energy. Young with bright SNRs. Typically emitting in X-rays

Rotational-Powered Pulsars

Powered by rotational energy. Typically emitting in radio.



Neutron star model & Cooling

The structure is provided by the TOV equations which solve the hydrostatic equilibrium assuming a static interior Schwarzschild metric

Cold matter EOS + central pressure: star structure & composition (fixed)

Thermal evolution:

$$c_V(T) \frac{\partial(Te^\nu)}{\partial t} = \vec{\nabla} \cdot (e^\nu \hat{k} \cdot \vec{\nabla}(e^\nu T)) + e^{2\nu}(Q_J - Q_\nu)$$

Ingredients:

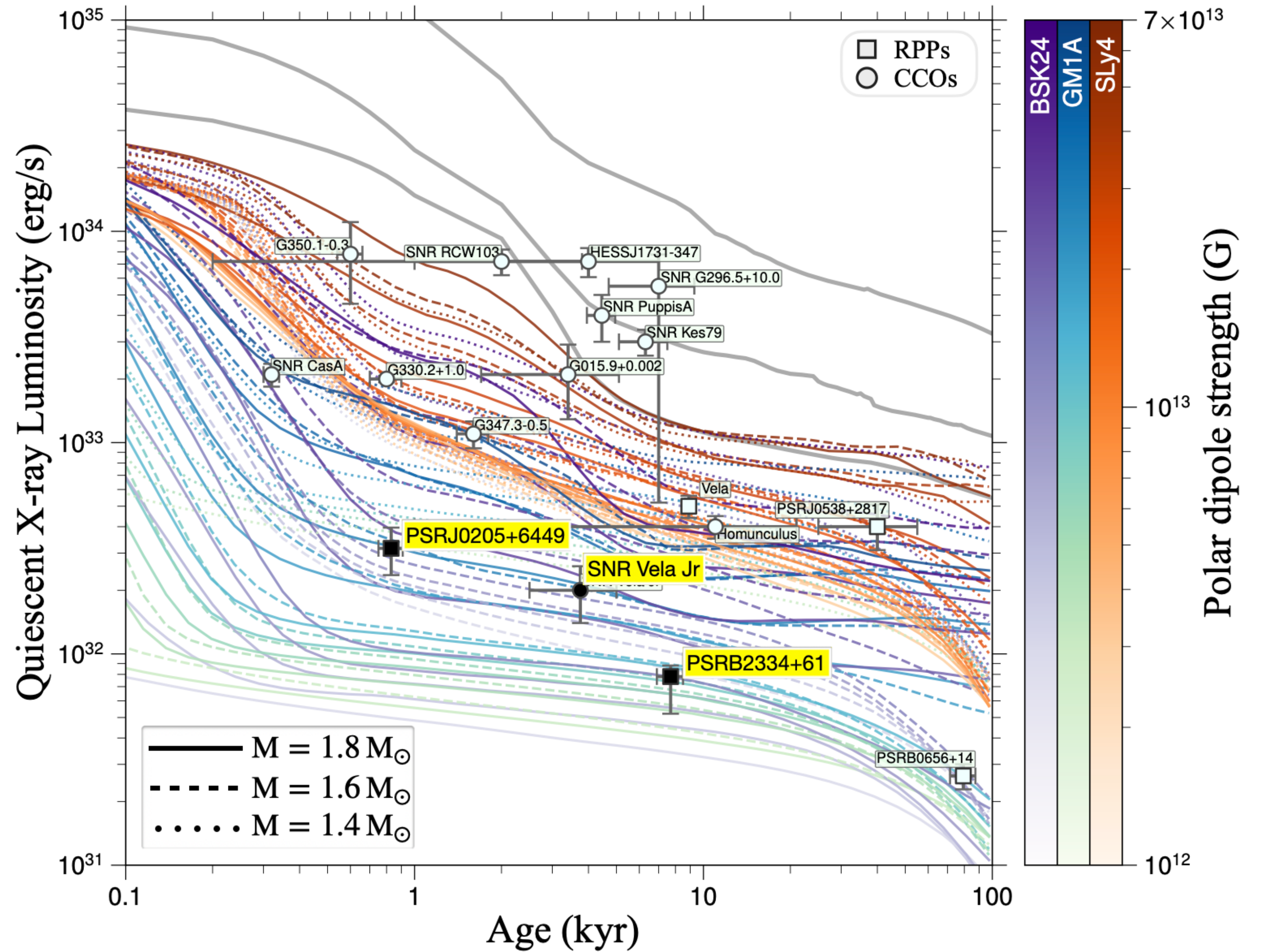
- Heat capacity $C_V(\rho, T)$: main contribution by neutrons in the core
- **Thermal conductivity** $\kappa(\rho, T, \mathbf{B})$ very large (star core rapidly isothermal), dominated by electrons, becomes **anisotropic** in presence of magnetic field
- Neutrino emissivity $Q_\nu(\rho, T, \mathbf{B})$
- Sources of internal heat Q_j : nuclear reactions, **Ohmic dissipation**, accretion...
- Hydrostatic equilibrium models of envelope (i.e., liquid outermost 100 m), that due to its stronger gradients of density and temperature has much faster timescales than the interior
- Emission model (atmosphere, **blackbody**, condensed surface...)

Enhanced cooling and constraining nuclear EoS

This marks the first direct measurement of enhanced cooling. Magneto-thermal simulation and machine learning techniques were used to understand this finding.

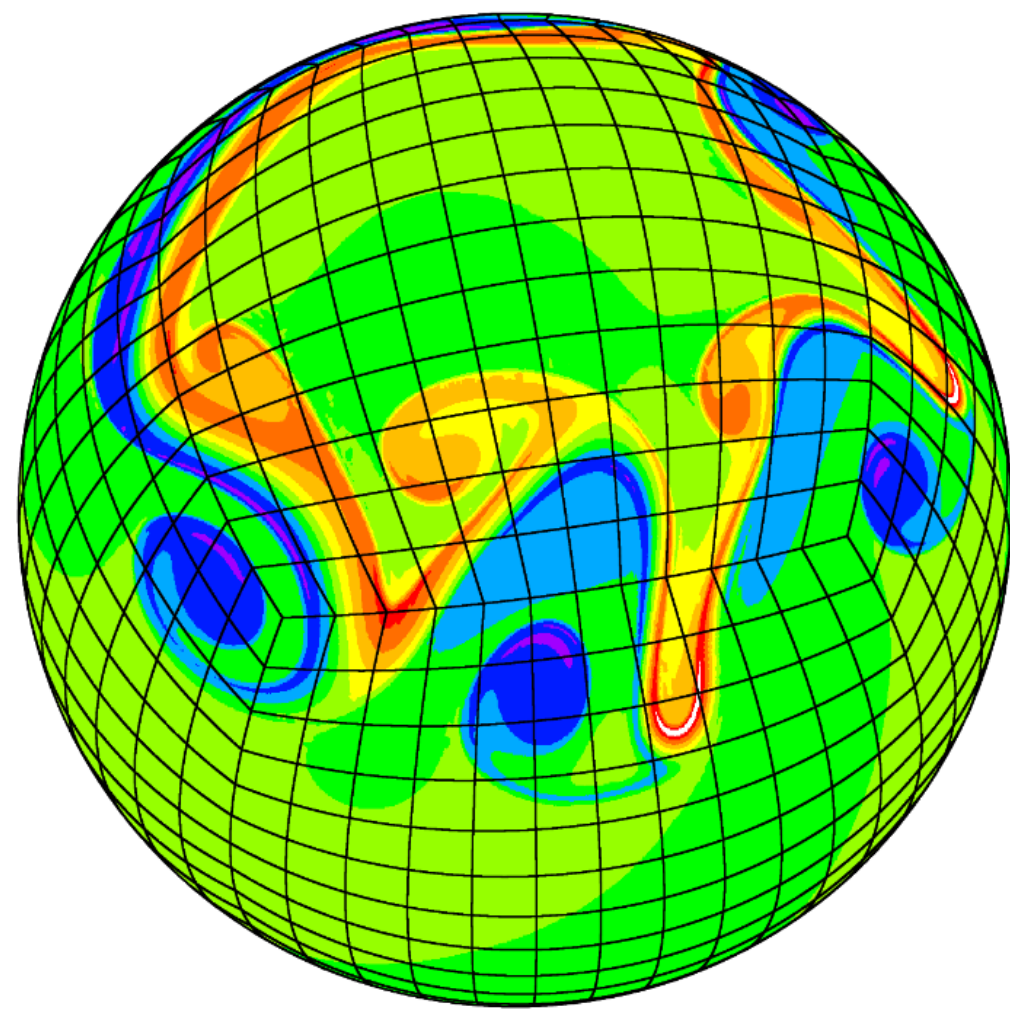
A suitable EoS should elucidate both exceptionally bright objects like magnetars (Joule heating) and extremely faint objects at young ages.

Taking into account a simplified meta-modeling approach (Margueron et al., 2018), the excluded range is estimated to encompass approximately 75% of the proposed nuclear EoSs.

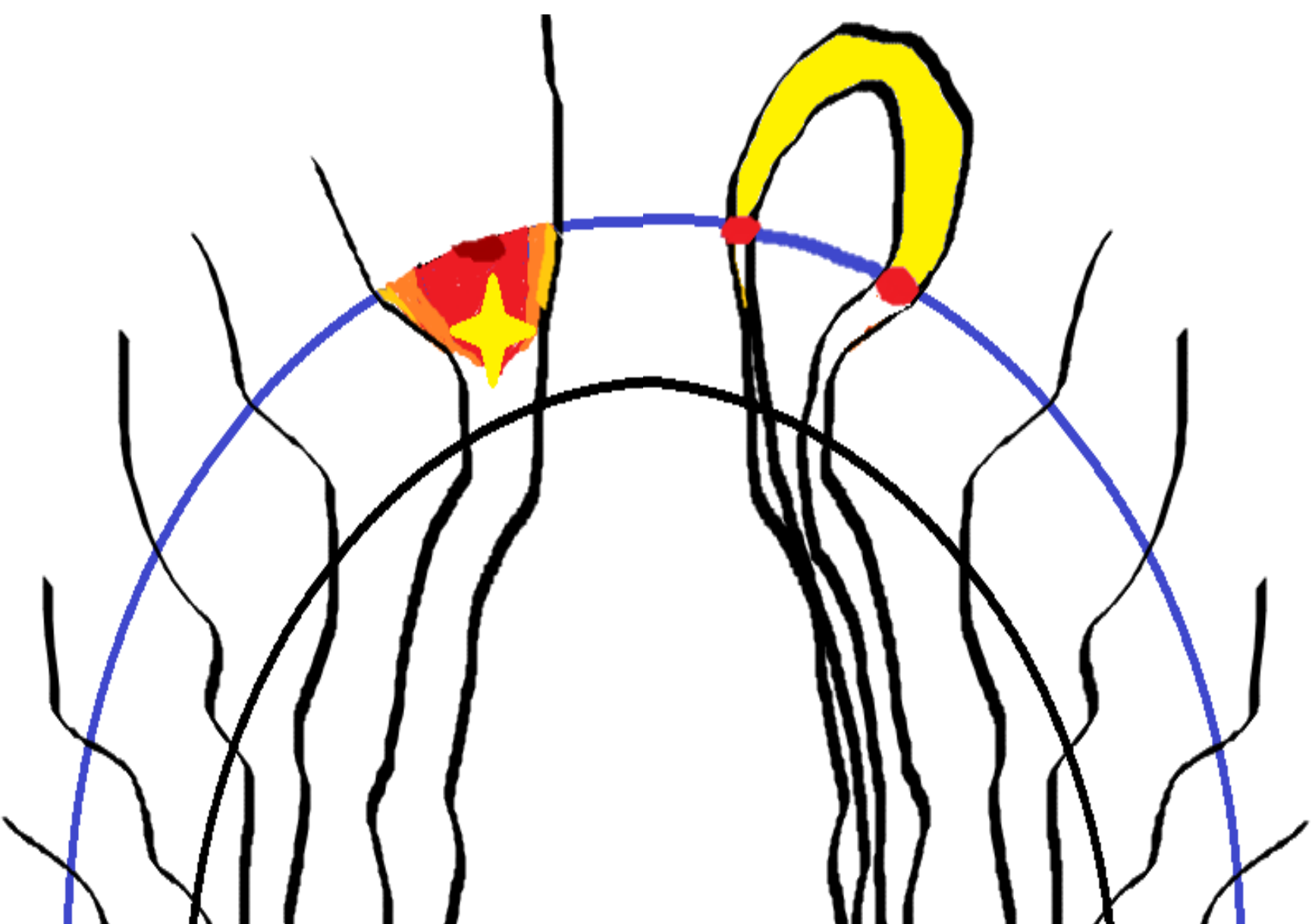


[Anzuini, Melatos, Dehman et al. 2021, MNRAS] } GM1A — with hyperons in
[Anzuini, Melatos, Dehman et al. 2022, MNRAS] } the core

[Marino, Dehman, Kowlakas, Rea et al. 2024, Nature Astronomy]



Magnetic Field Evolution

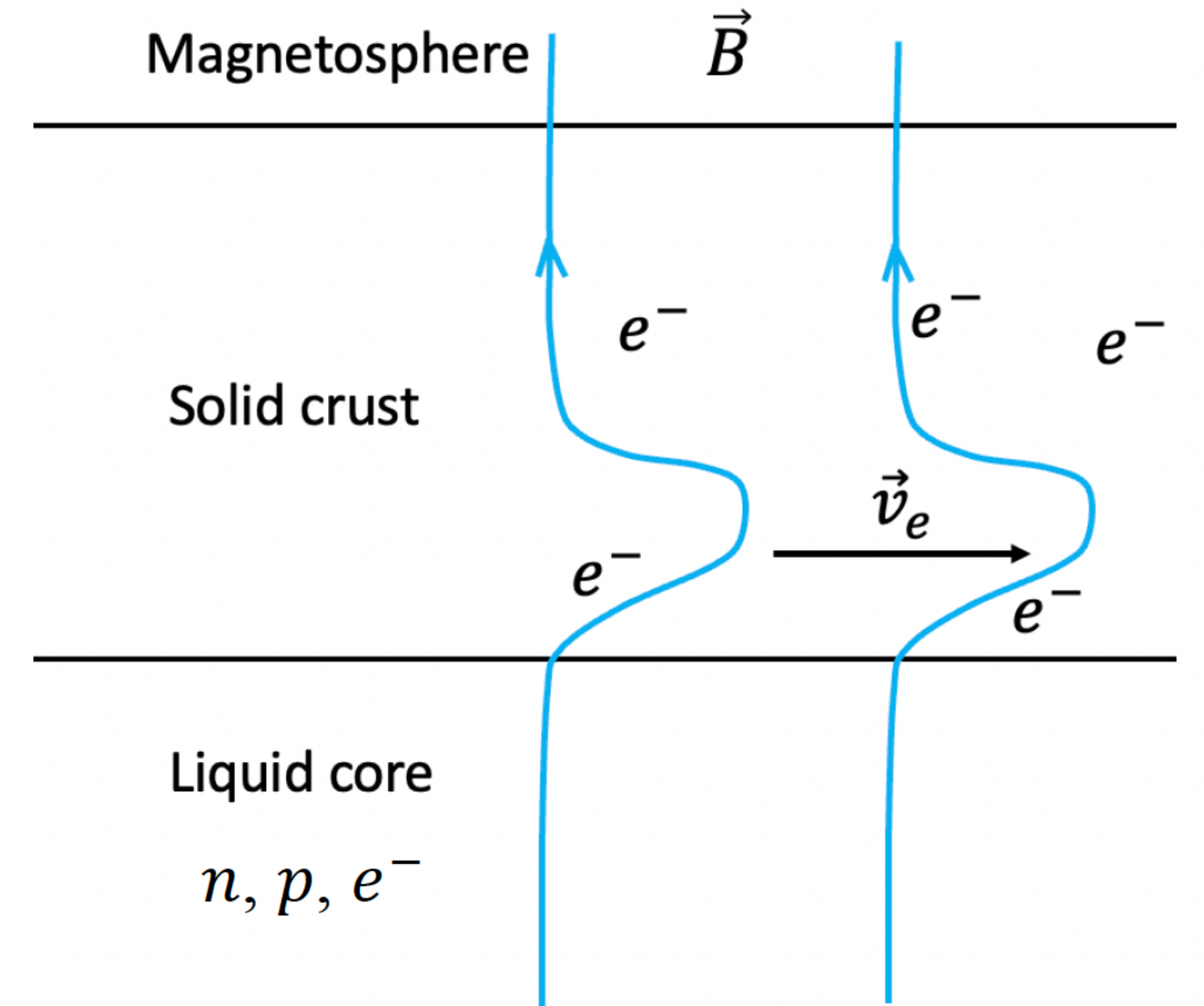


Magnetic field evolution in the crust

- **Neutron star interior:** complex multi-fluid system.
- **Crust:** solidifies early; nuclei mobility is limited; conductivity governed by electrons.
- **Core:** remains a full multi-fluid on long timescales.
- **Limit:** eMHD used in the **crust of a neutron star**

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\eta \nabla \times \mathbf{B} - \eta k_5 \mathbf{B} + f_h (\nabla \times \mathbf{B}) \times \mathbf{B} \right]$$

$$\left(\eta = \frac{c^2}{4\pi\sigma_e}; \quad k_5 = \frac{4\alpha\mu_5}{\hbar c}; \quad f_h = \frac{c}{4\pi en_e} \right)$$



Ohmic term $\propto (\eta \nabla \times \mathbf{B})$: the magnetic diffusivity is sensitive to temperature evolution and electron density (strong radial gradients).

Chiral term $\propto (\eta k_5 \mathbf{B})$: It redistributes magnetic energy among different magnetic spatial scales.

Hall term $\propto (f_h (\nabla \times \mathbf{B}) \times \mathbf{B})$: It naturally creates magnetic discontinuity and transfers energy between different scales.

$$k_5 = \frac{(\nabla \times \mathbf{B}) \cdot \mathbf{B}}{\frac{\mu_e^2}{8\pi\alpha^2\eta(\hbar c)}\Gamma_f + B^2} = \frac{(\nabla \times \mathbf{B}) \cdot \mathbf{B}}{\left(\frac{2\mu_e^2}{m_e^2 c^4} \right) \frac{B_{\text{QED}}^2}{3\pi} + B^2}$$

Perfect conductor B.C. at the crust-core interface (Meissner/type-I) and potential B.C. at the surface.

MATINS the brand new 3D code

– Open Access: <https://github.com/ice-csic-astroexotic/MATINS>–

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \left[\eta \nabla \times (e^\nu \mathbf{B}) - \eta k_5 \mathbf{B} + f_h \nabla \times (e^\nu \mathbf{B}) \times \mathbf{B} \right] c_V(T) \frac{\partial (T e^\nu)}{\partial t} = \vec{\nabla} \cdot (e^\nu \hat{\mathbf{k}} \cdot \vec{\nabla} (e^\nu T)) + e^{2\nu} (Q_J - Q_\nu)$$

[Dehman, Viganò, Pons & Rea 2023, MNRAS \(DOI: \[10.1093/mnras/stac2761\]\(https://doi.org/10.1093/mnras/stac2761\)\)](#): Cubed-sphere grid + Magnetic formalism

[Dehman, Viganò, Ascenzi, Pons & Rea 2023, MNRAS \(DOI: \[10.1093/mnras/stad1773\]\(https://doi.org/10.1093/mnras/stad1773\)\)](#): First 3D magneto-thermal simulation

[Ascenzi, Viganò, Dehman, Pons & Rea, Perna 2024, MNRAS \(DOI: \[10.1093/mnras/stae1749\]\(https://doi.org/10.1093/mnras/stae1749\)\)](#): Thermal formalism

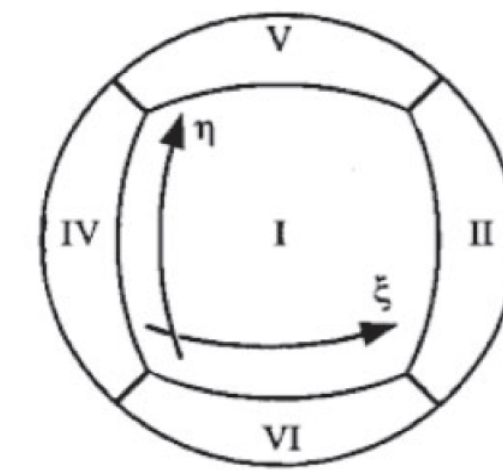
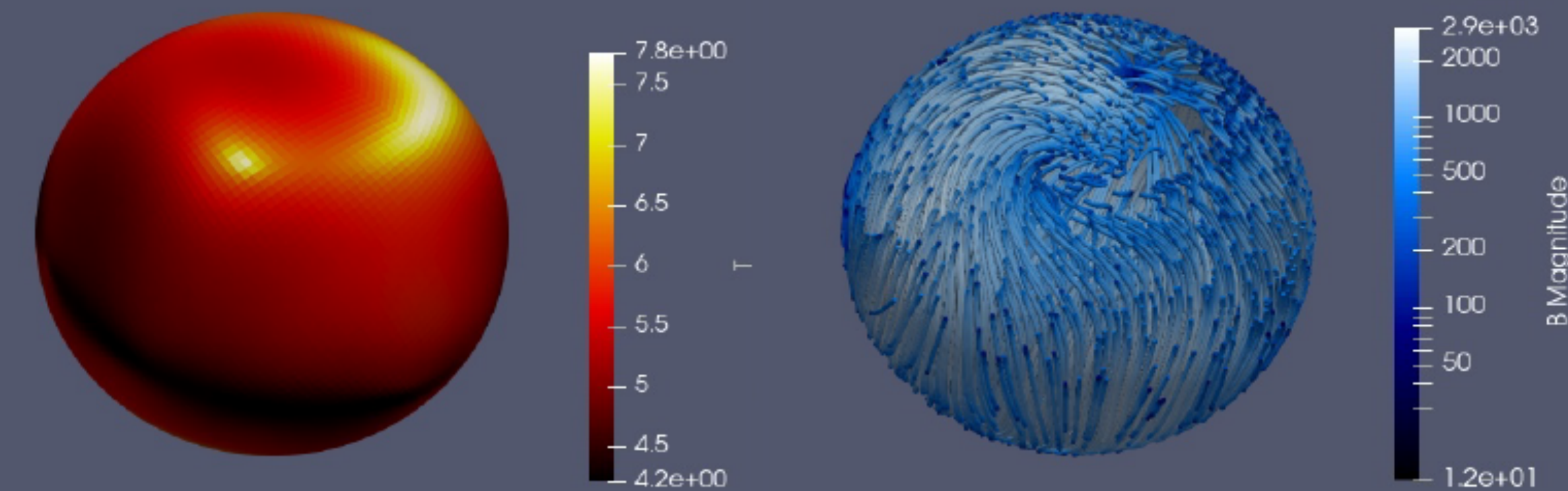
[Dehman & Pons 2025, PRR \(DOI: \[10.1103/rhv5-nd4v\]\(https://doi.org/10.1103/rhv5-nd4v\)\)](#): Chiral magnetic effect

What's better than 2D (Viganò et al. 2021):

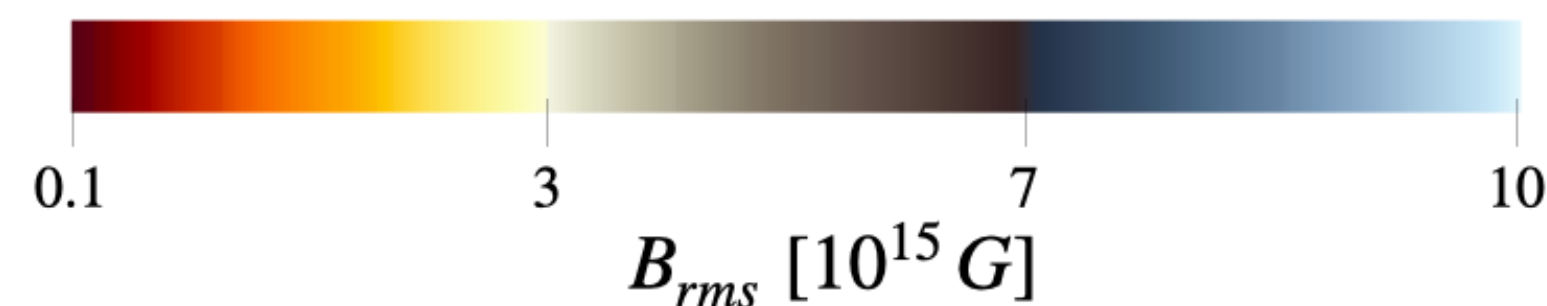
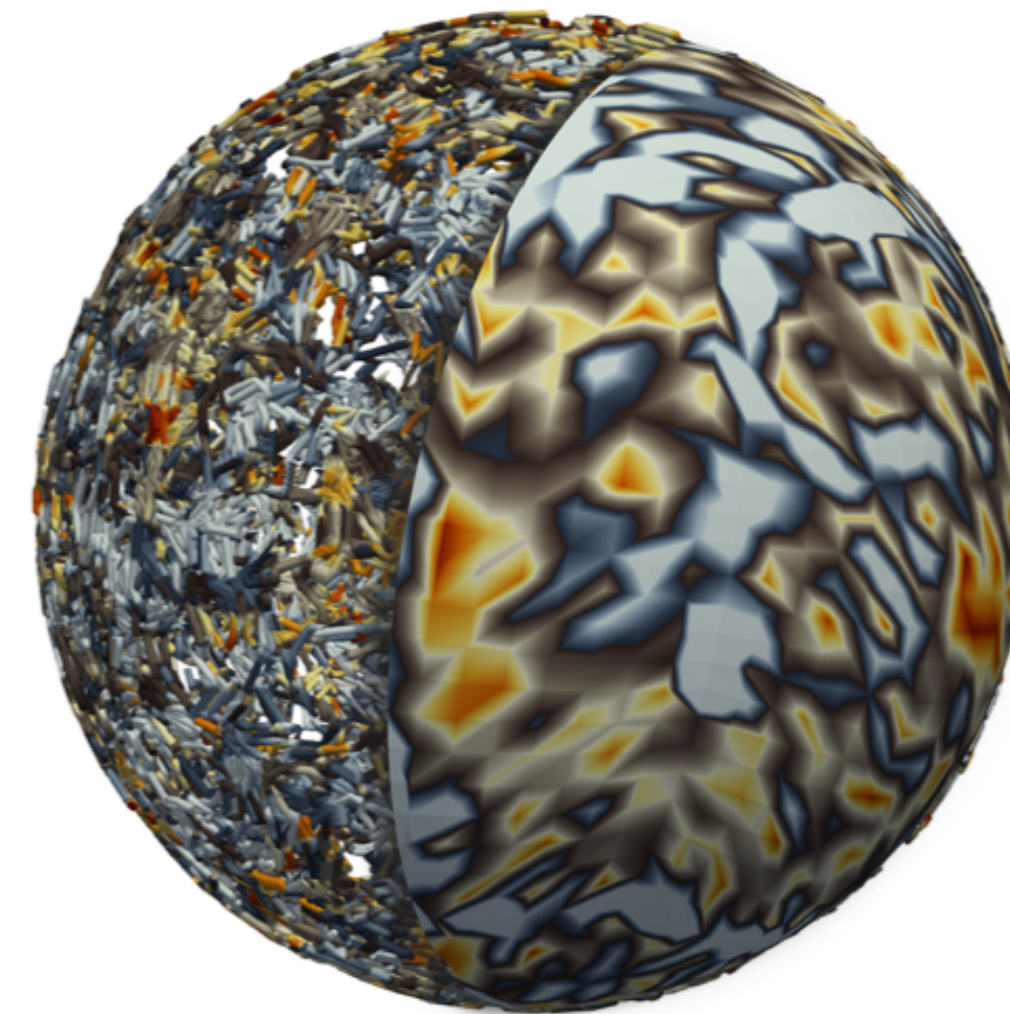
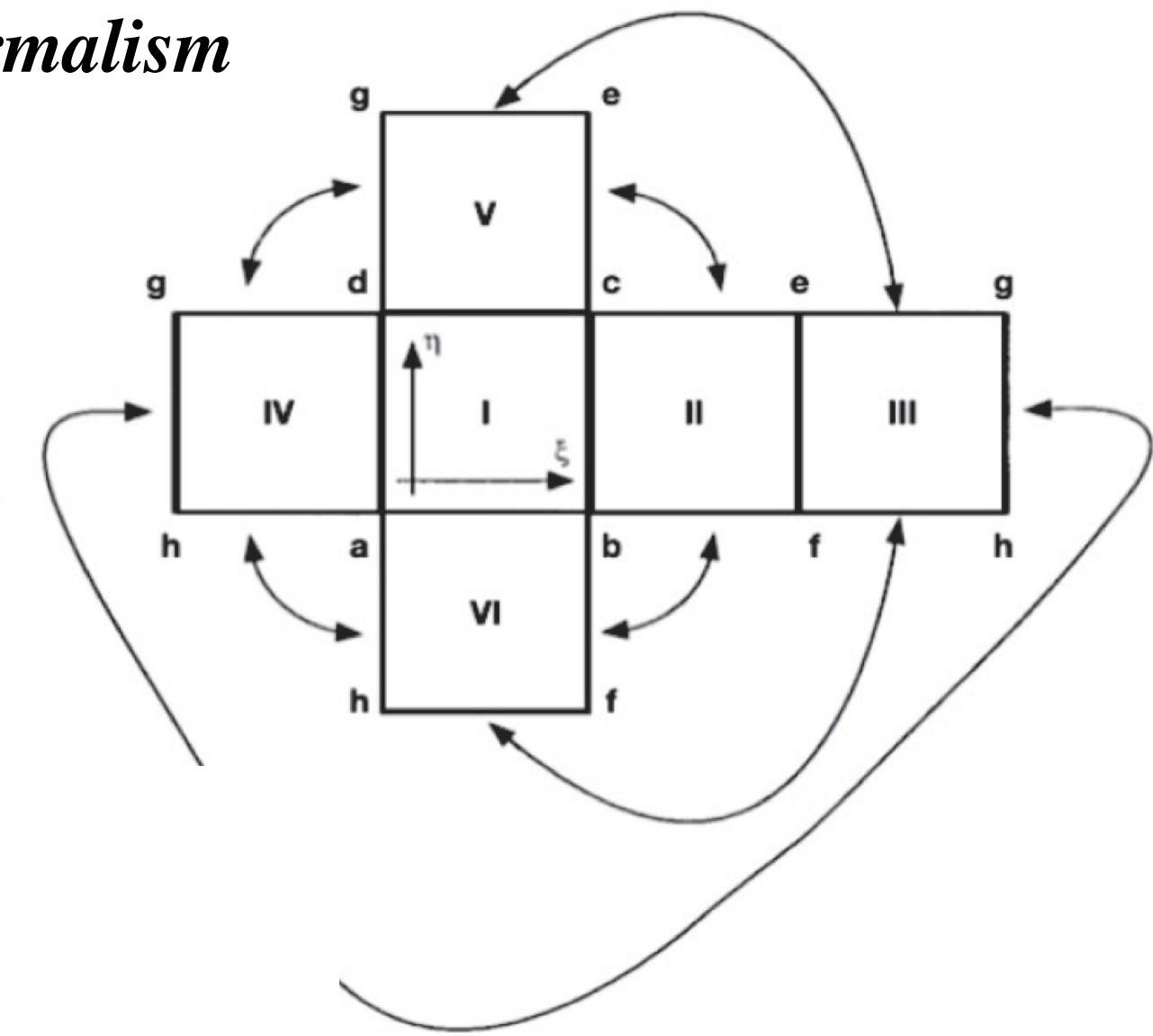
- Simulation of 3D magnetic modes, hotspots, and light curves
- Better documentation, **use of novel coordinates (cubed-sphere)**
- Optimization and use of OpenMP

Advance obtained:

- **Realistic 3D evolution and topology, appearance of hotspots**
- **State-of-the-art microphysics, various masses and EOSs, with realistic structure obtained by solving the TOV equation.**
- Numerical scheme to better capture non-linear dynamics
- **General relativistic correction**
- State of art envelope model
- Flexibility in implementing new physics

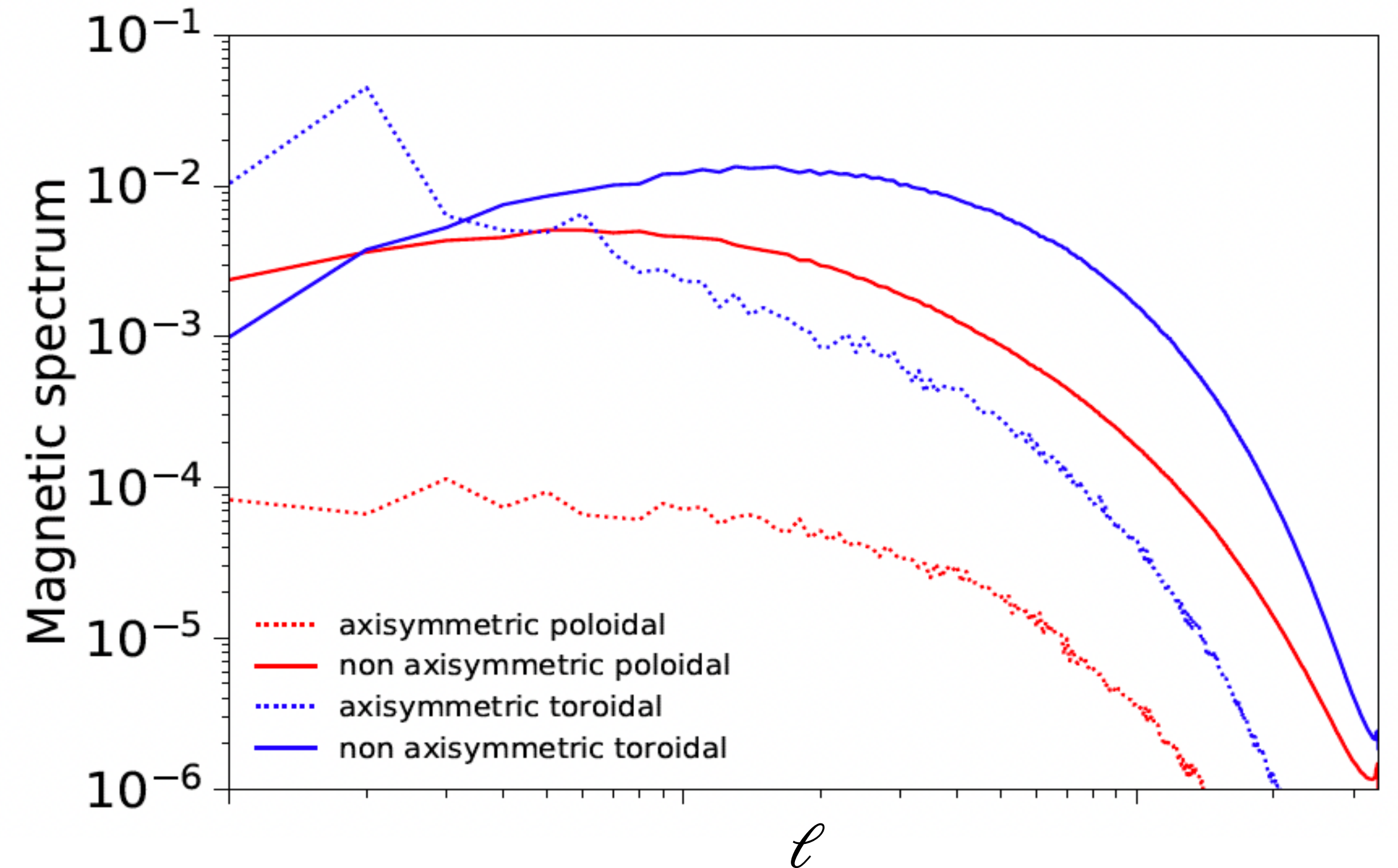
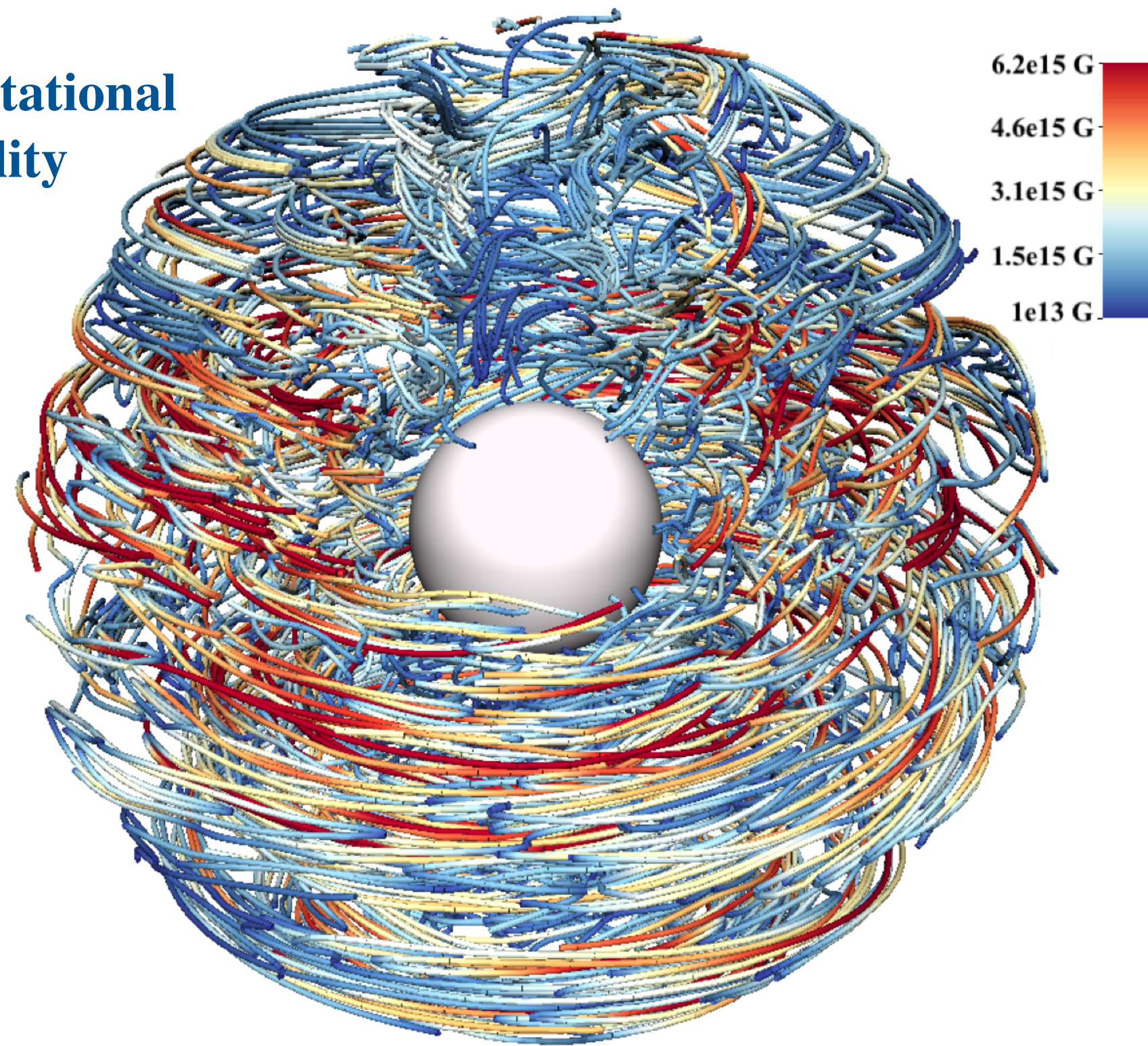


**cubed
sphere**



Magnetic field at birth: Magnetic spectrum after core-collapse

Magneto-rotational
instability



Magnetic energy is distributed across a broad range of scales.

Toroidal axisymmetric quadrupole and non-axisymmetric components dominate.

Dipolar field accounts for less than 5% of the total magnetic energy ($\approx 10^{12}$ G).

From post-collapse to neutron star phase, plenty of MHD timescales to approach an equilibrium, with dynamo still going on and at the same time dissipating the smallest scales.

But how does a strong dipole of 10^{14} G form in magnetars, leading to their long spin periods?

[Reboul-Salze et al. 2022]

[Dehman et al. 2023c]

[Barrère et al. 2025]

[Igoshev et al. 2025]

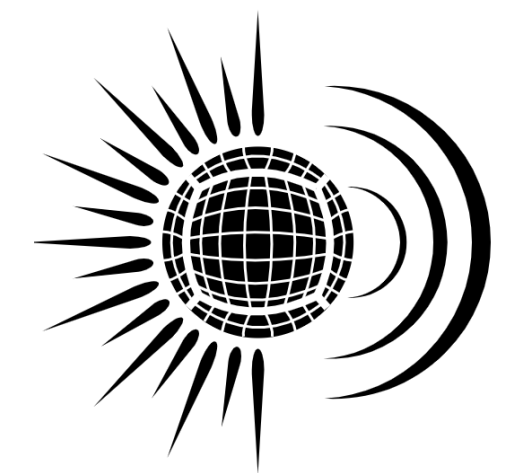
Magneto-rotational
instability

Taylor-Spruit
dynamo

Candidates to
explain CCOs, low
field magnetars



Weak dipolar
component (order
of 10^{12} G)



MATINS
MAgneto-Thermal evolution
of Isolated Neutron Stars

Hall-driven inverse cascade in neutron star crusts

Initial field:

Helical magnetic field.
Random initial field peaking at $\ell_0 \sim 100$.
Causal spectrum as used in the cosmological context.
Correct aspect ratio of the NS crust.

Inverse Cascade occurs!

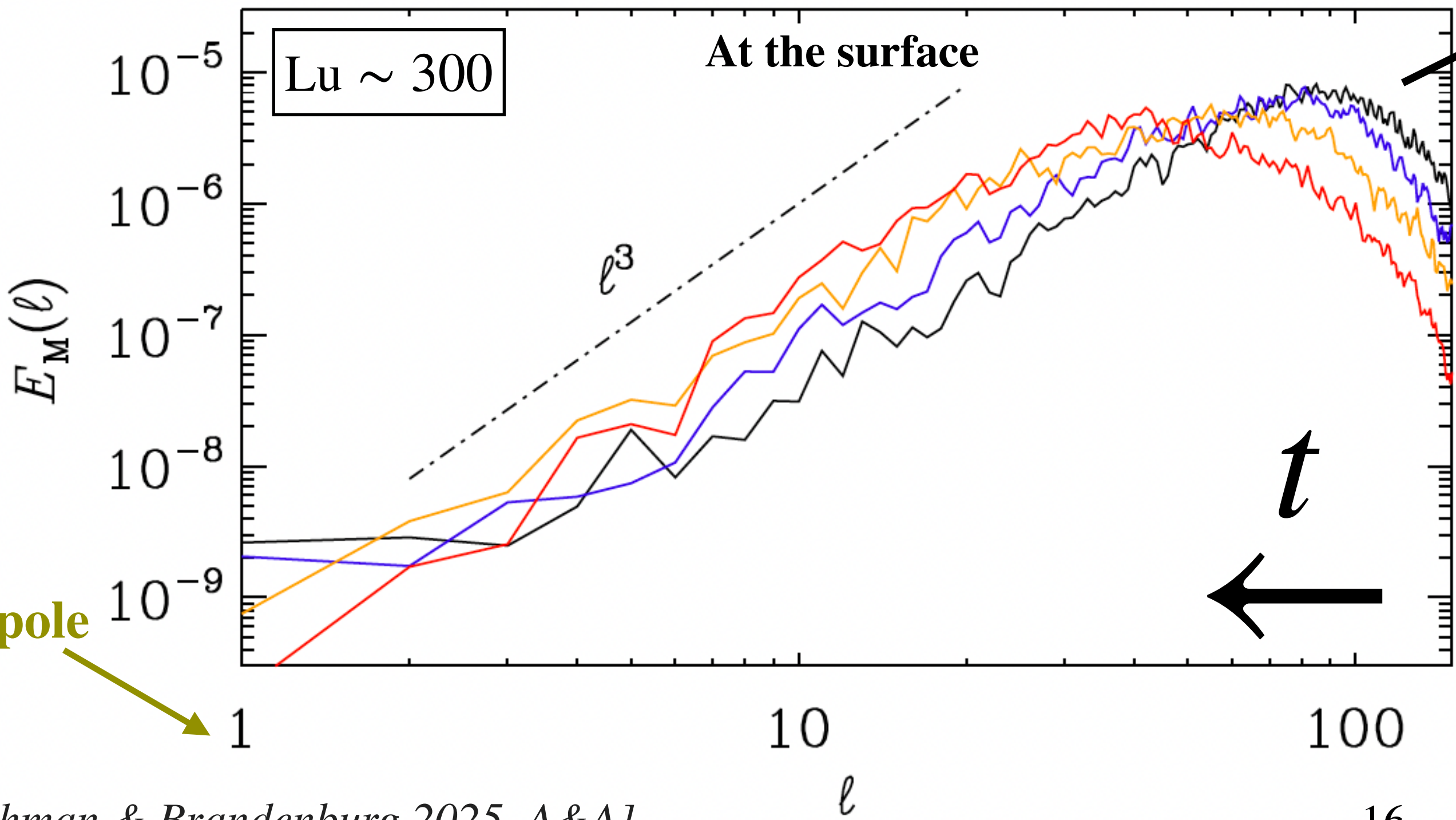
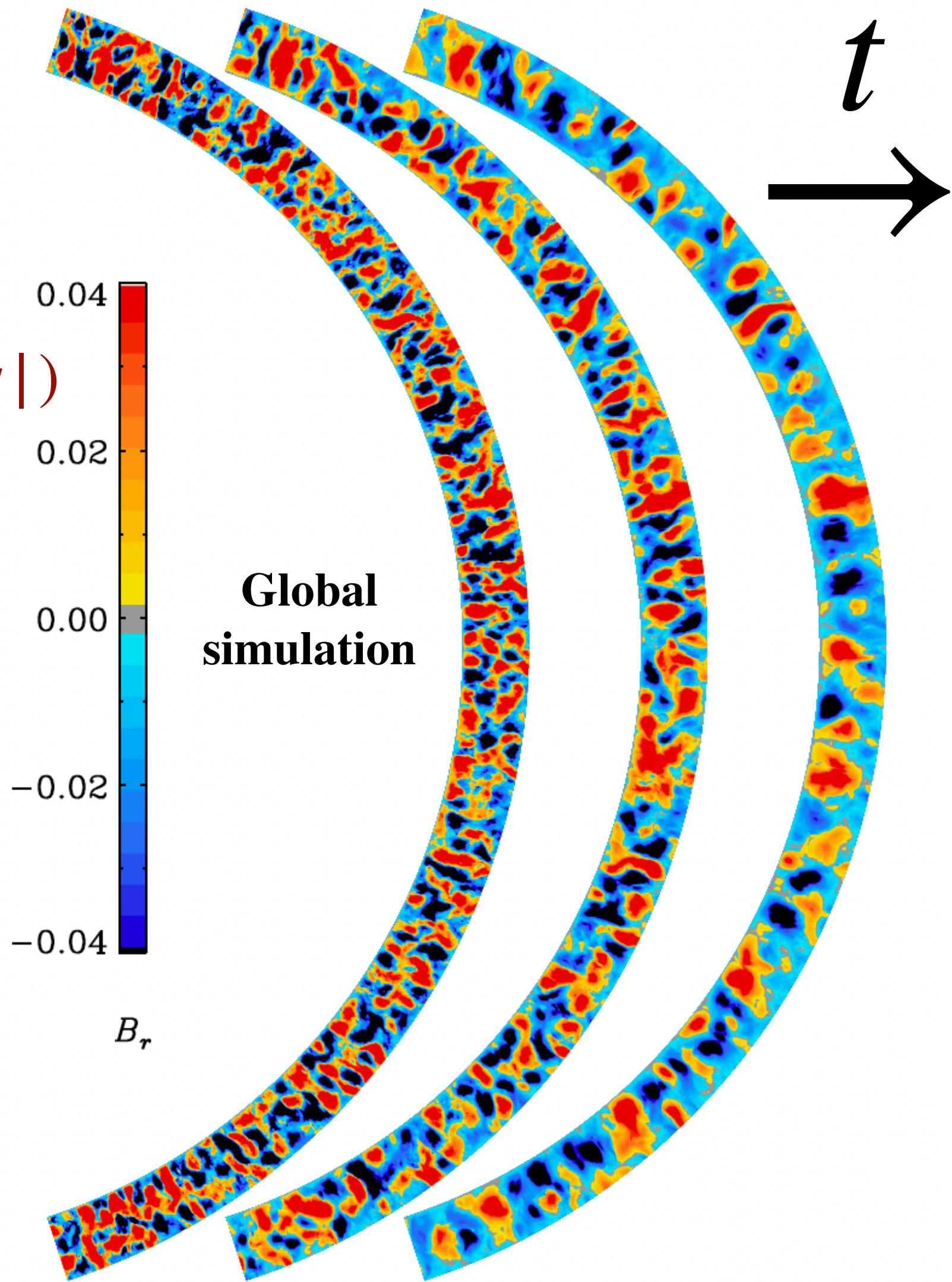
Energy transferred from small to large-scale multipoles.
Not observed in previous neutron star simulation studies.
Extreme aspect ratio (1:30)—thin crust— limits the inverse cascade.

Maximally helical
(e.g., positive helicity):

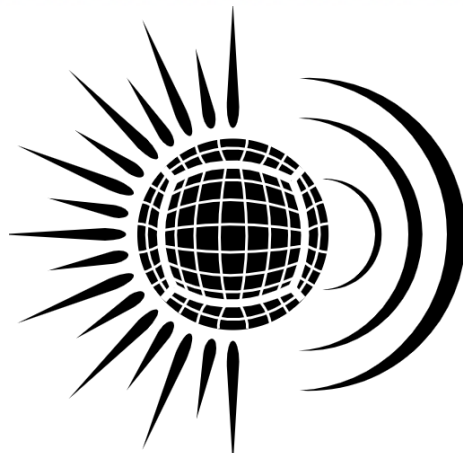
$$k = p + q$$

$$k |H_M(k)| / 2E_M(k) \leq 1$$
$$H_M = 2E_M(k)/k$$

$$|k| \leq \max(|p|, |q|)$$



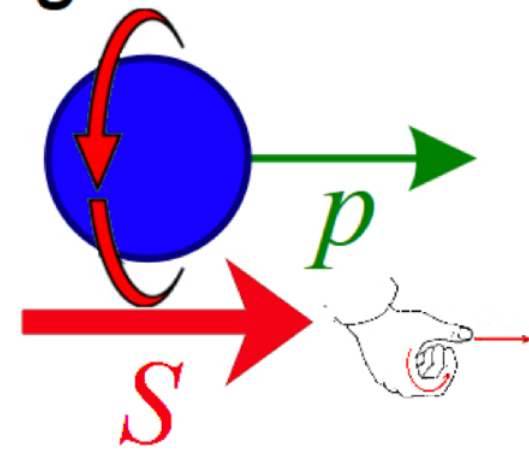
$t = 0$



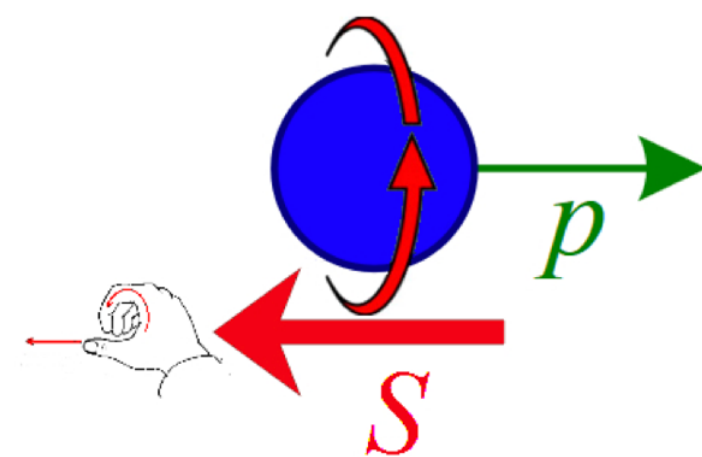
MATINS
MAgneto-Thermal evolution
of Isolated Neutron Stars

Chiral Anomaly

Right-handed



Left-handed



A particle is moving with momentum p represented by the green arrow.

— Particle Chirality —

The chirality of a particle is the projection of the spin along the direction of movement:

- right-handed if it is parallel to the movement;
- left-handed otherwise.

— Magnetic Helicity —

- Pure poloidal & toroidal components are unstable
- Both components are necessary for the stability (linked structures)
- Magnetic helicity quantify the topological stability

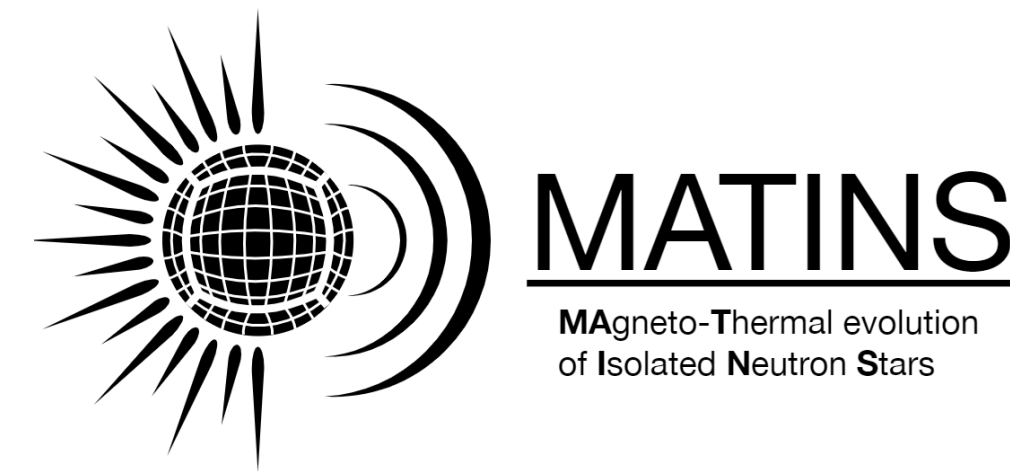
$$\frac{\partial (A \cdot B)}{\partial t} = -2cE \cdot B - c\nabla \cdot (E \times A)$$

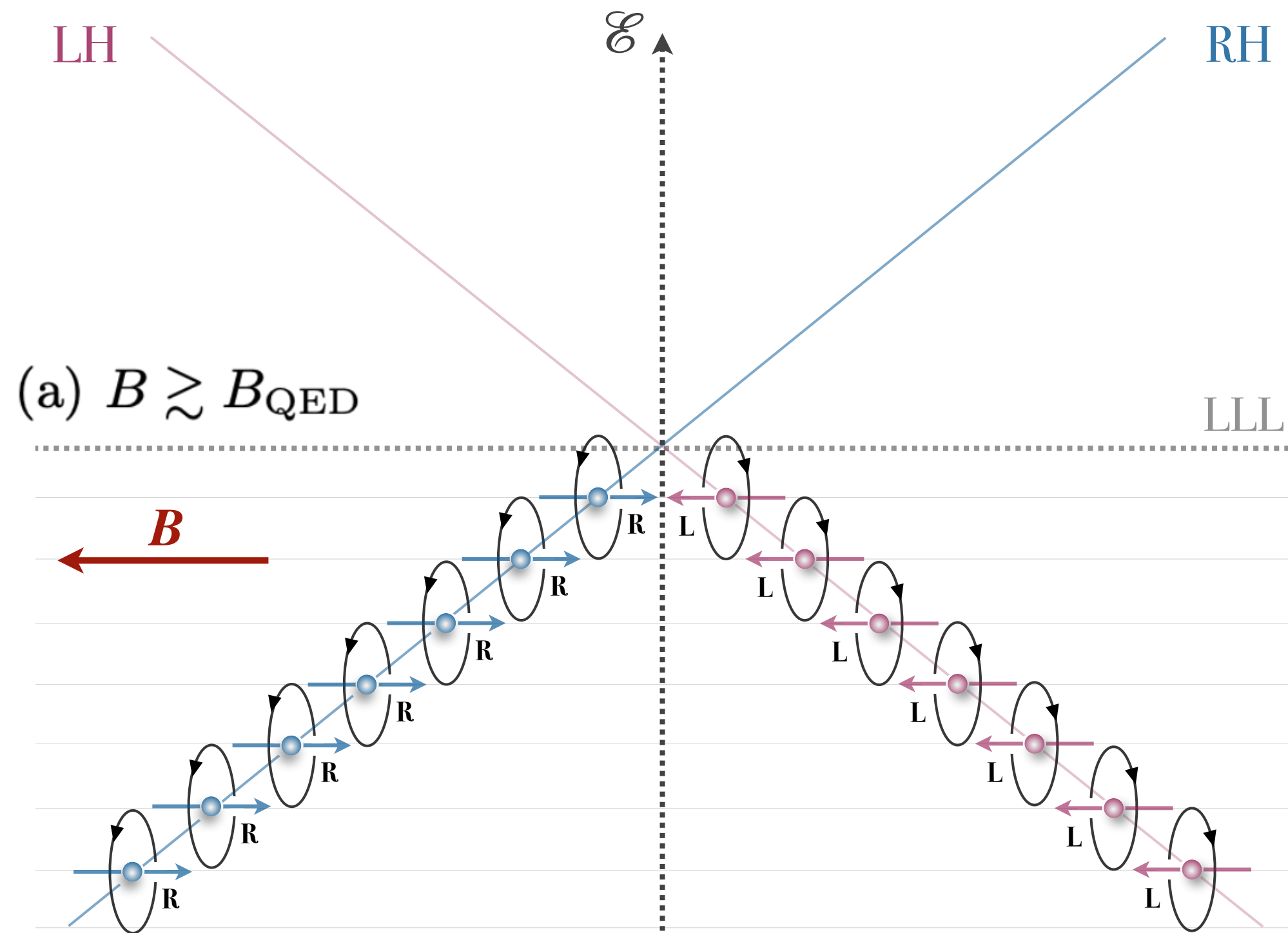
— Chiral Anomaly —

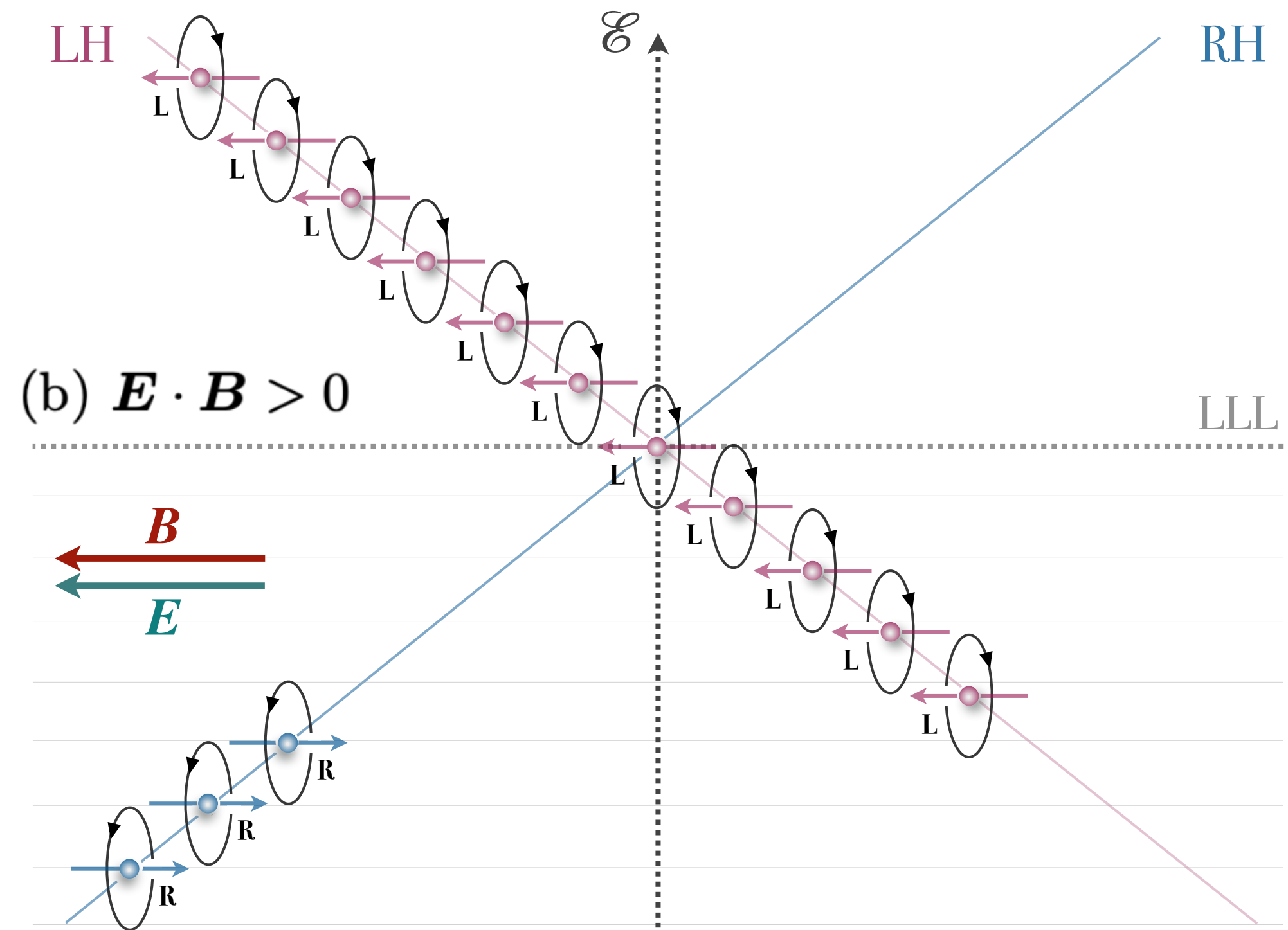
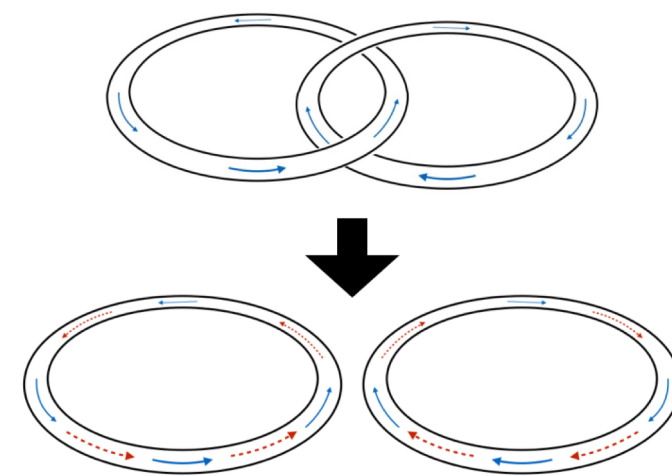
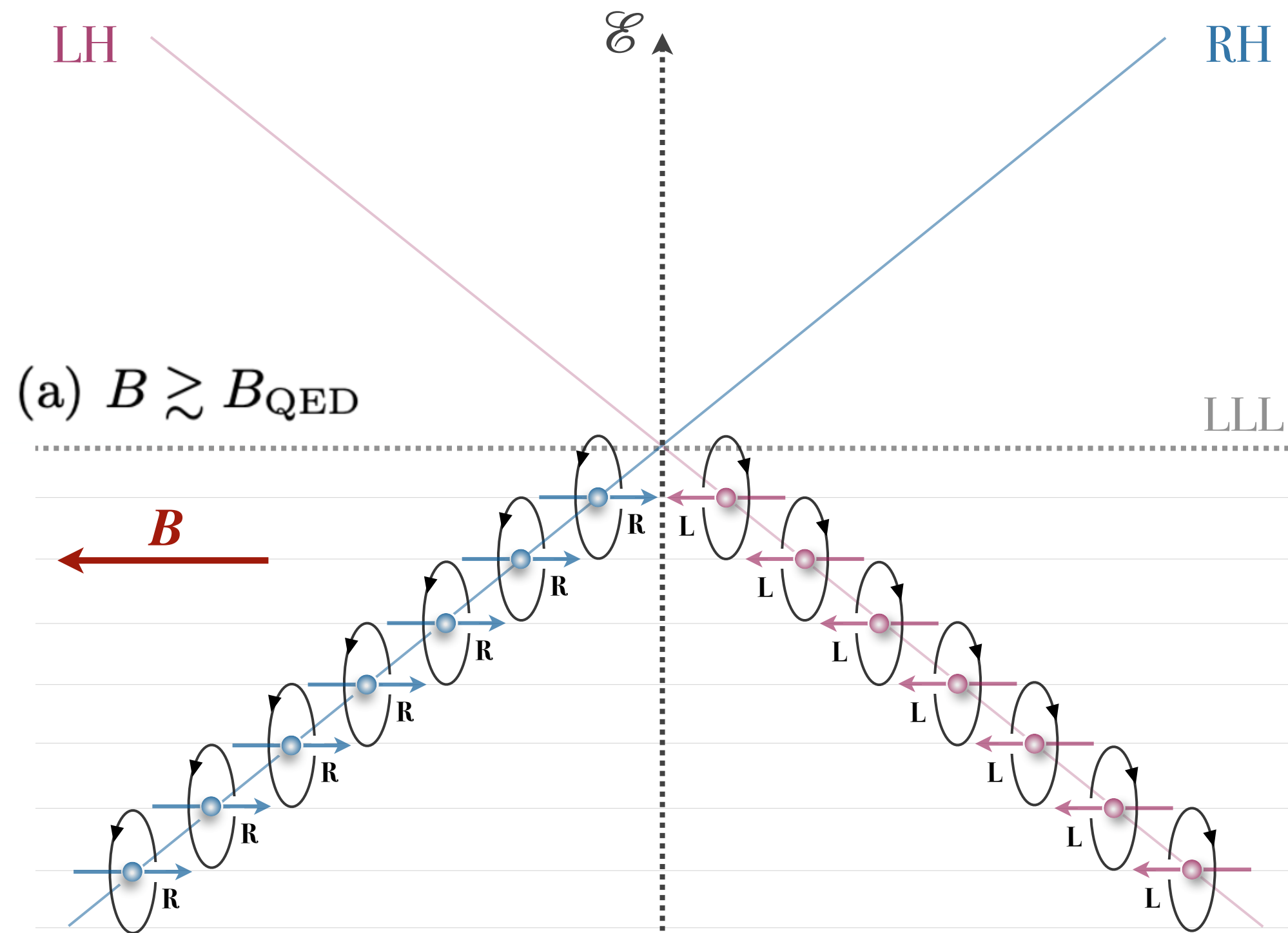
- Changes in magnetic helicity create or destroy chiral asymmetry (vice versa).
- Chiral electric current induced along field lines

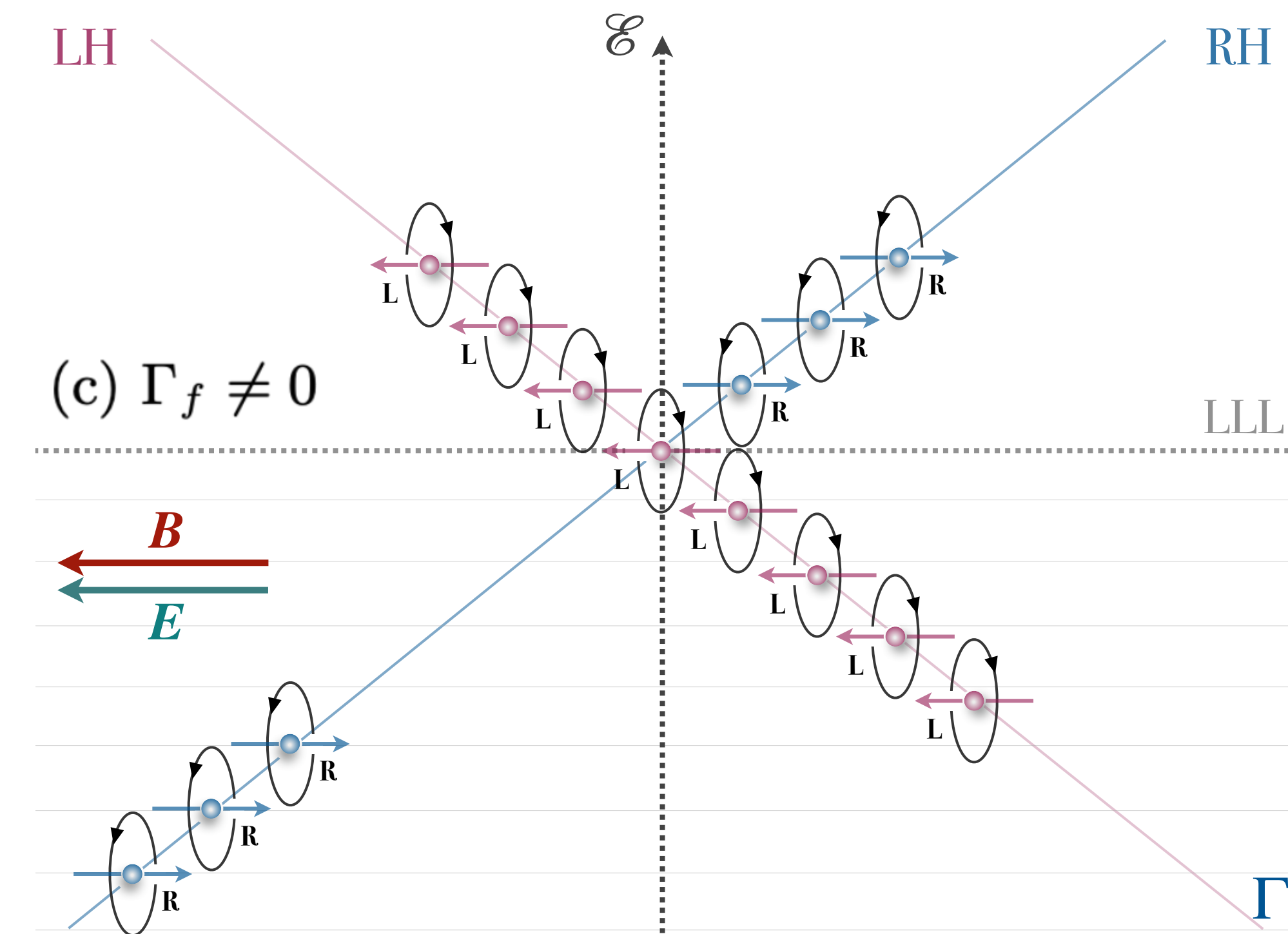
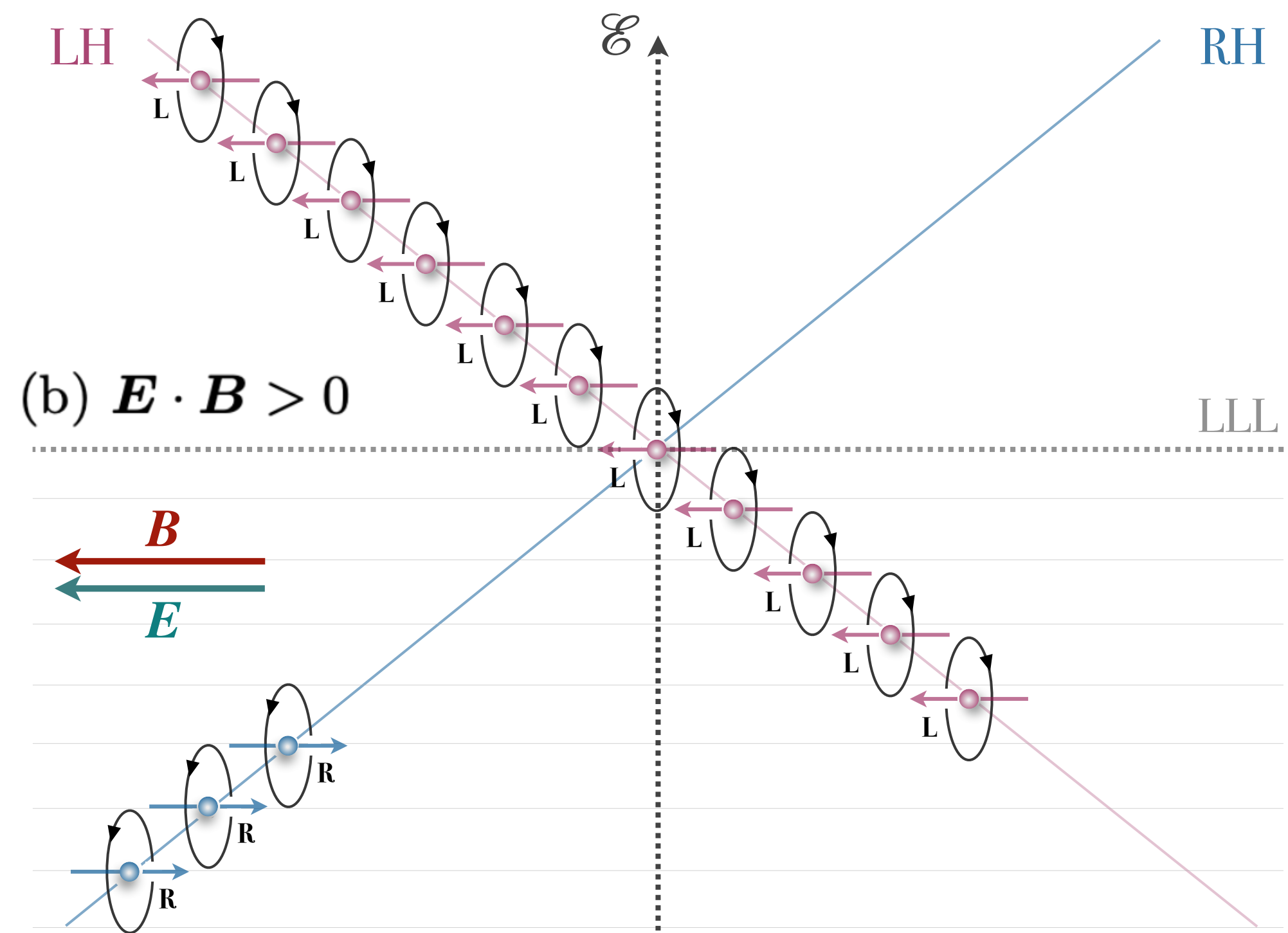
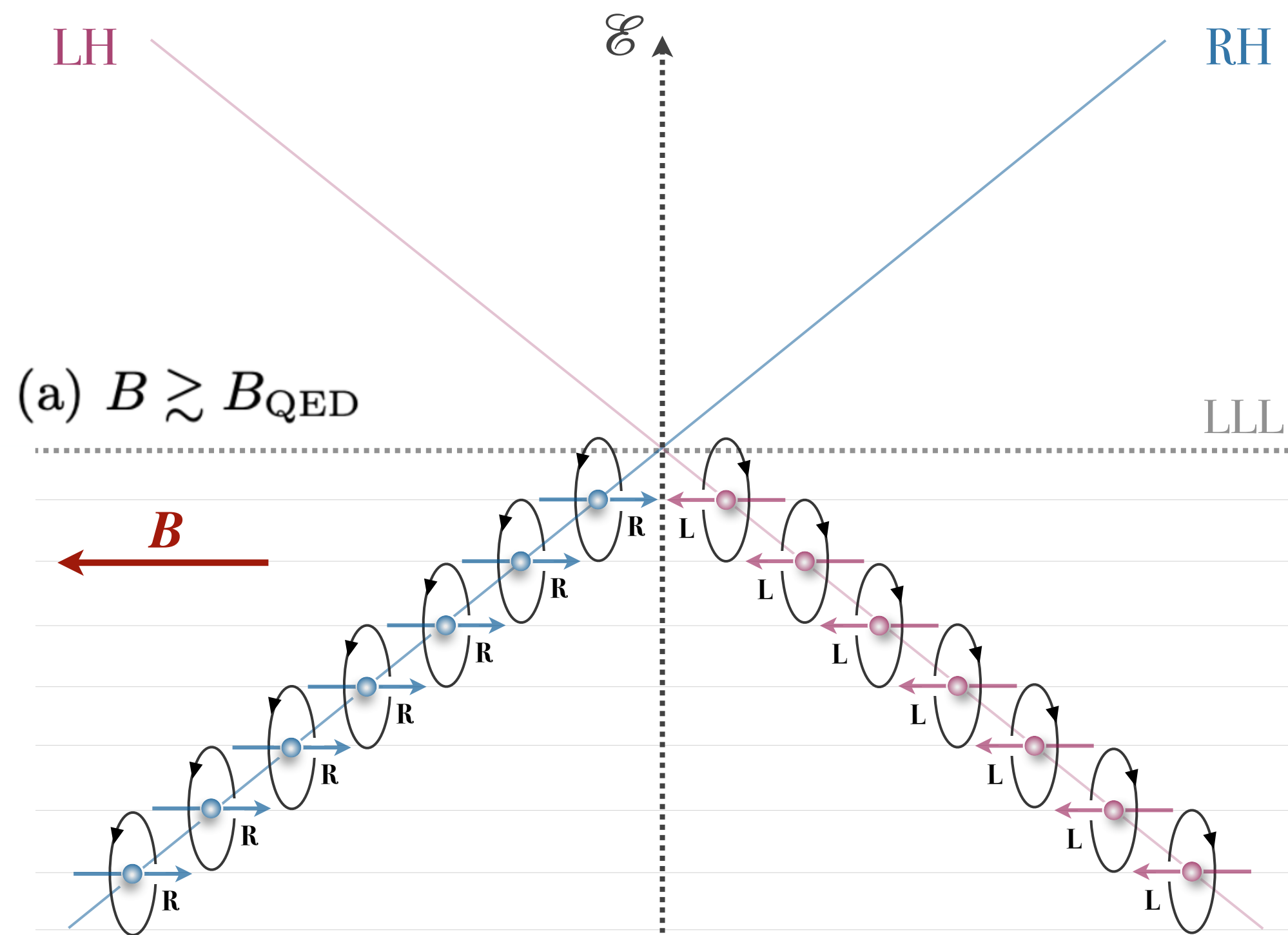
$$\frac{\partial n_5}{\partial t} = \frac{2\alpha}{\pi\hbar} E \cdot B - n_5 \Gamma_f$$

Image credit: Y. Hirono

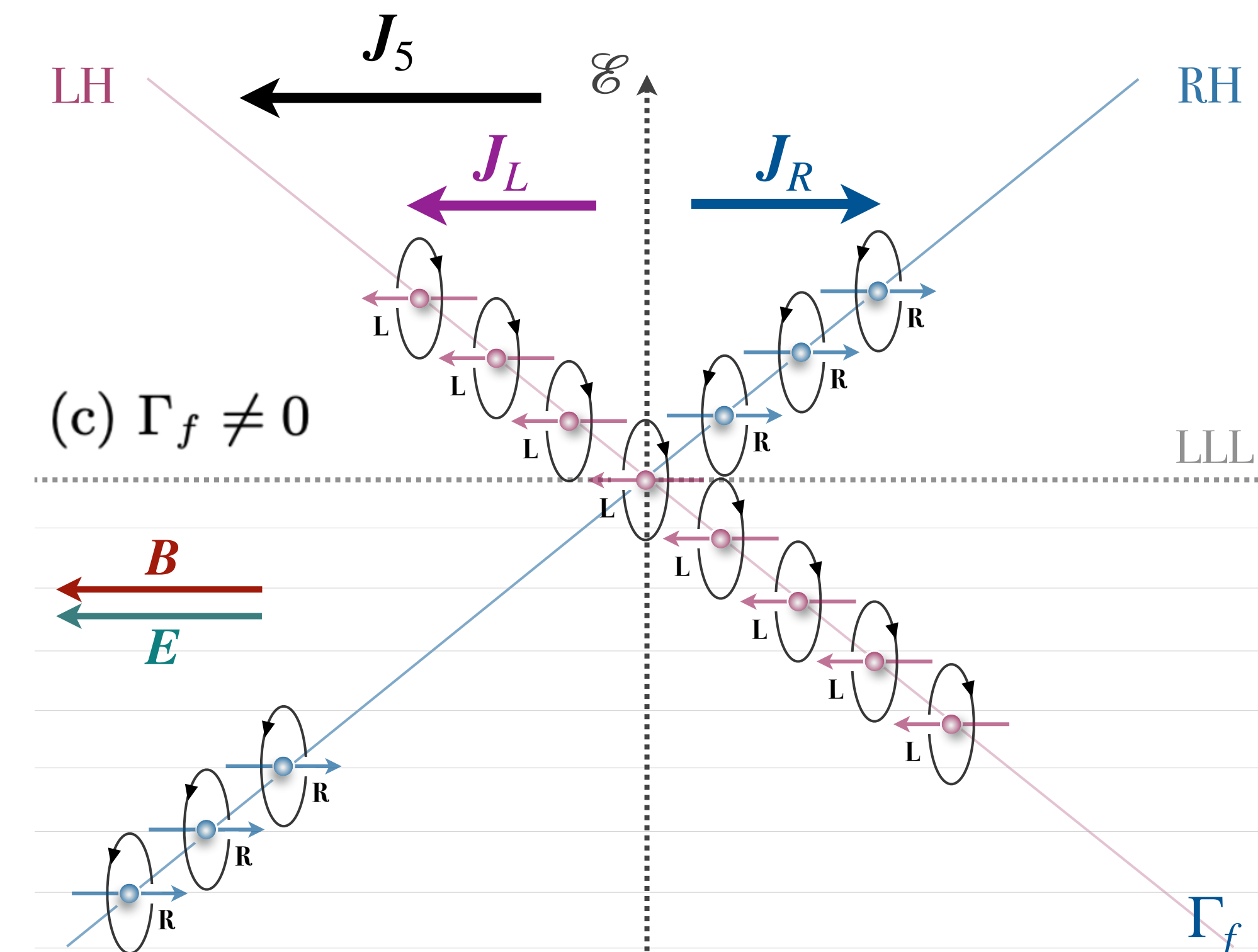
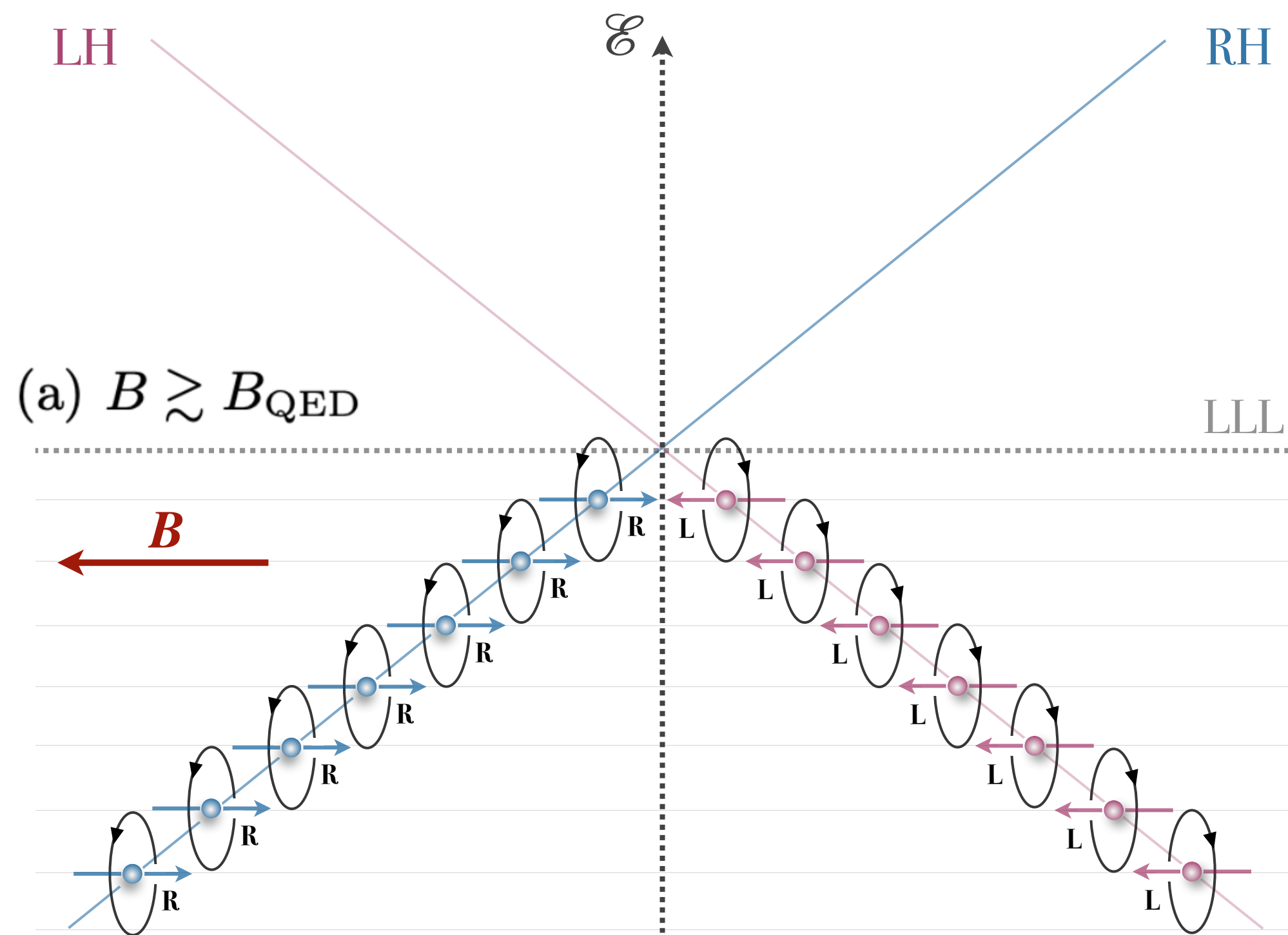






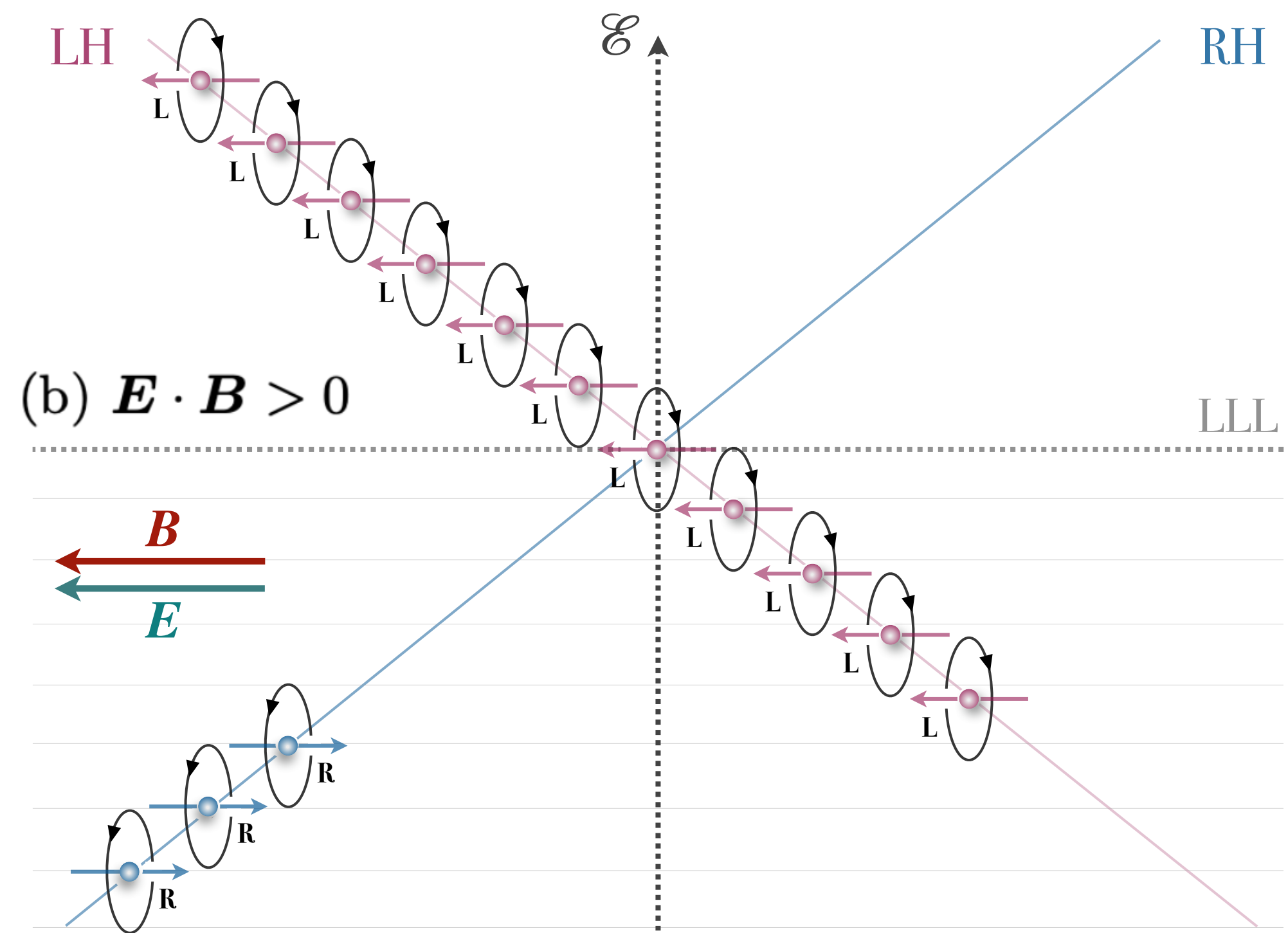


$$\Gamma_f = \left(\frac{m_e}{\mu_e} \right)^2 \nu_{\text{coll}} = \frac{4\alpha}{3\pi\sigma_e} \frac{m_e^2 c^4}{\hbar^2}$$



[Dehman 2026, PRD]

$$\Gamma_f = \left(\frac{m_e}{\mu_e} \right)^2 \nu_{\text{coll}} = \frac{4\alpha}{3\pi\sigma_e} \frac{m_e^2 c^4}{\hbar^2}$$



$$J_5 = J_R + J_L = \frac{\alpha}{\pi\hbar} \mathbf{B}(\mu_R - \mu_L) = \frac{\alpha\mu_5}{\pi\hbar} \mathbf{B}$$

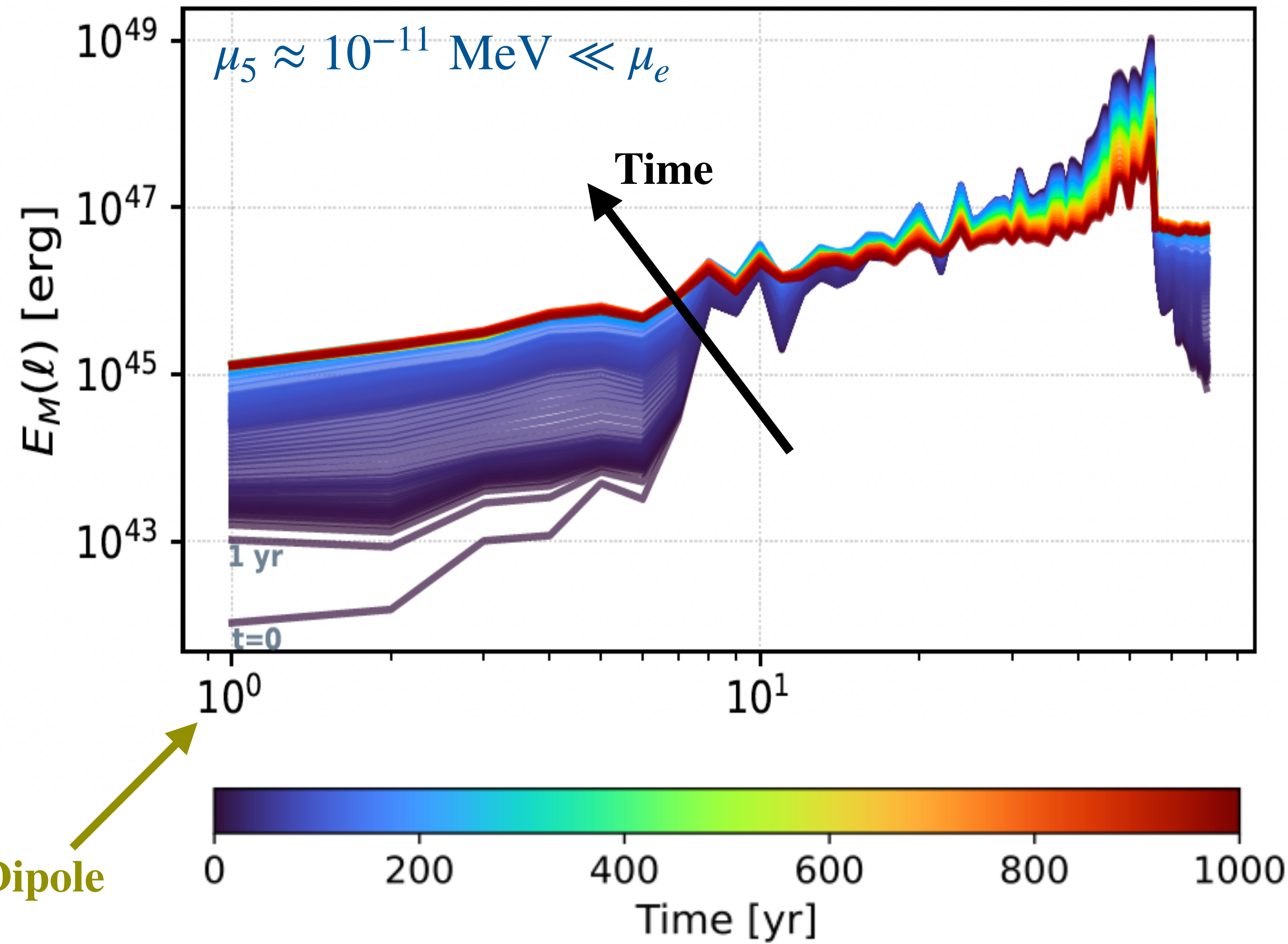
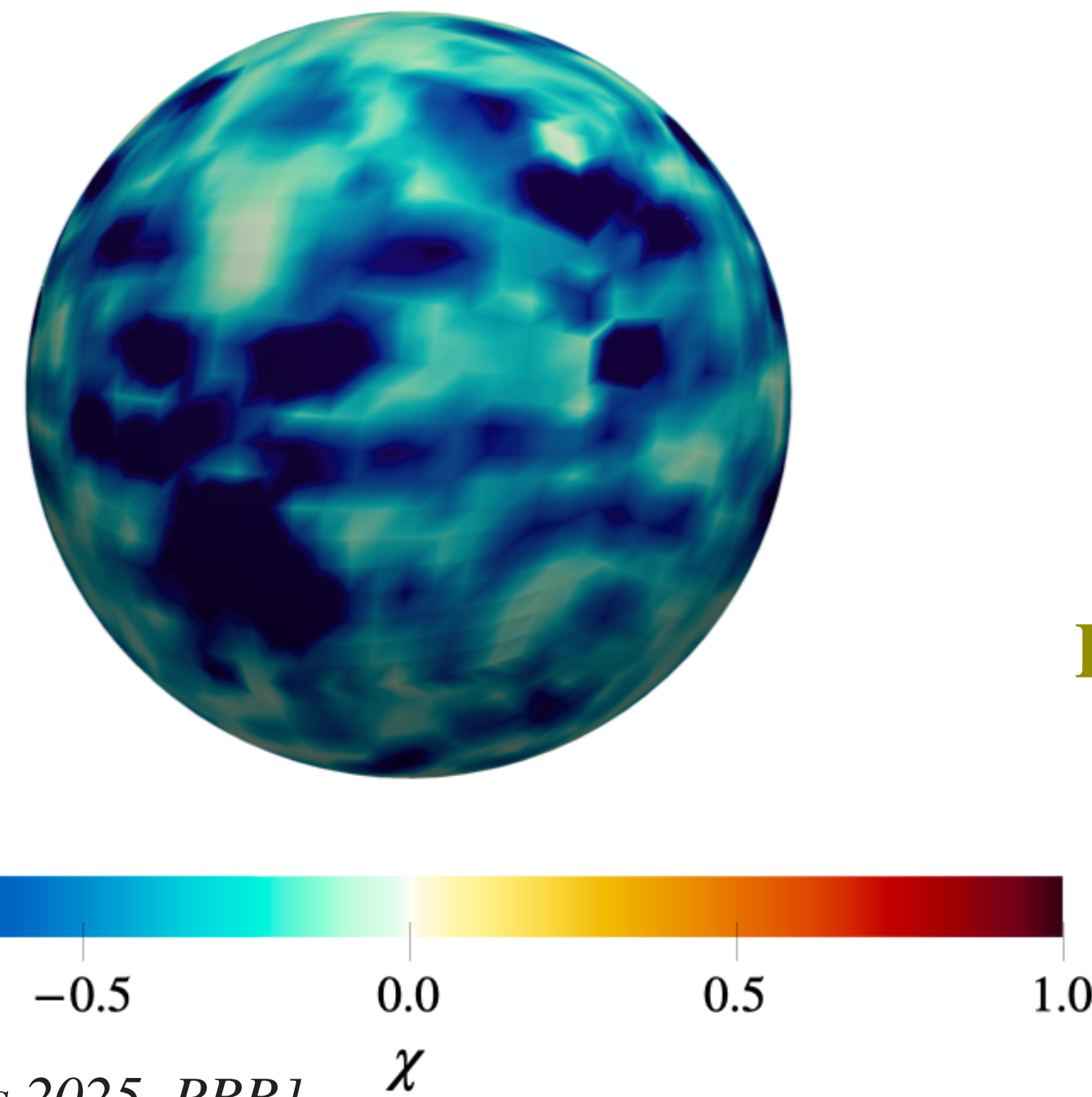
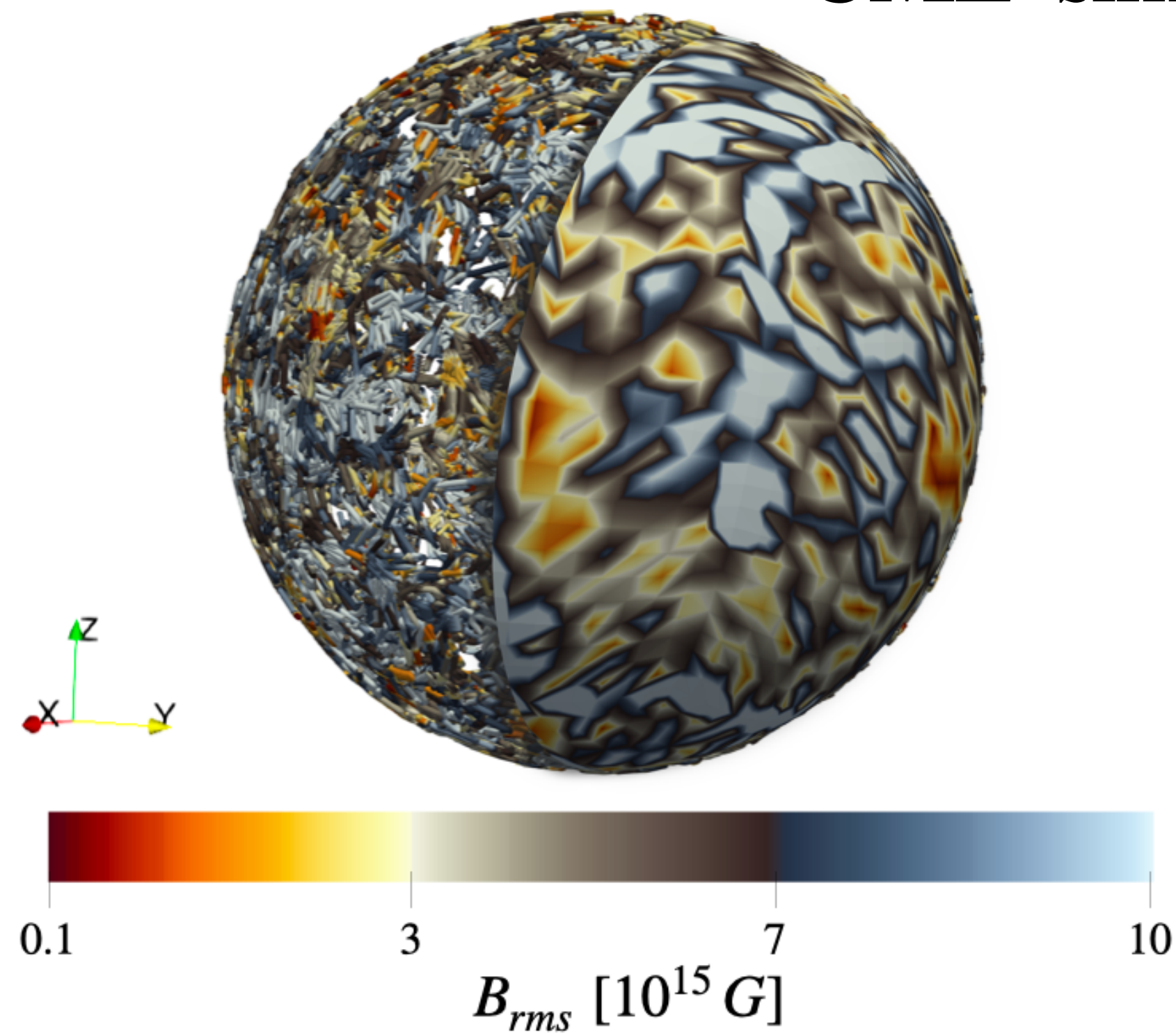
$$|\mu_5| \approx 10^{-12} \dots 10^{-11} \text{ MeV} \ll \mu_e = 10 \dots 100 \text{ MeV}$$

$$\mathbf{J} = \frac{c}{4\pi} (\nabla \times \mathbf{B}) = \sigma_e \mathbf{E} + \mathbf{J}_5$$

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times [\eta \nabla \times \mathbf{B} - \eta k_5 \mathbf{B} + f_h (\nabla \times \mathbf{B}) \times \mathbf{B}]$$

[Dehman & Pons 2025, PRR]

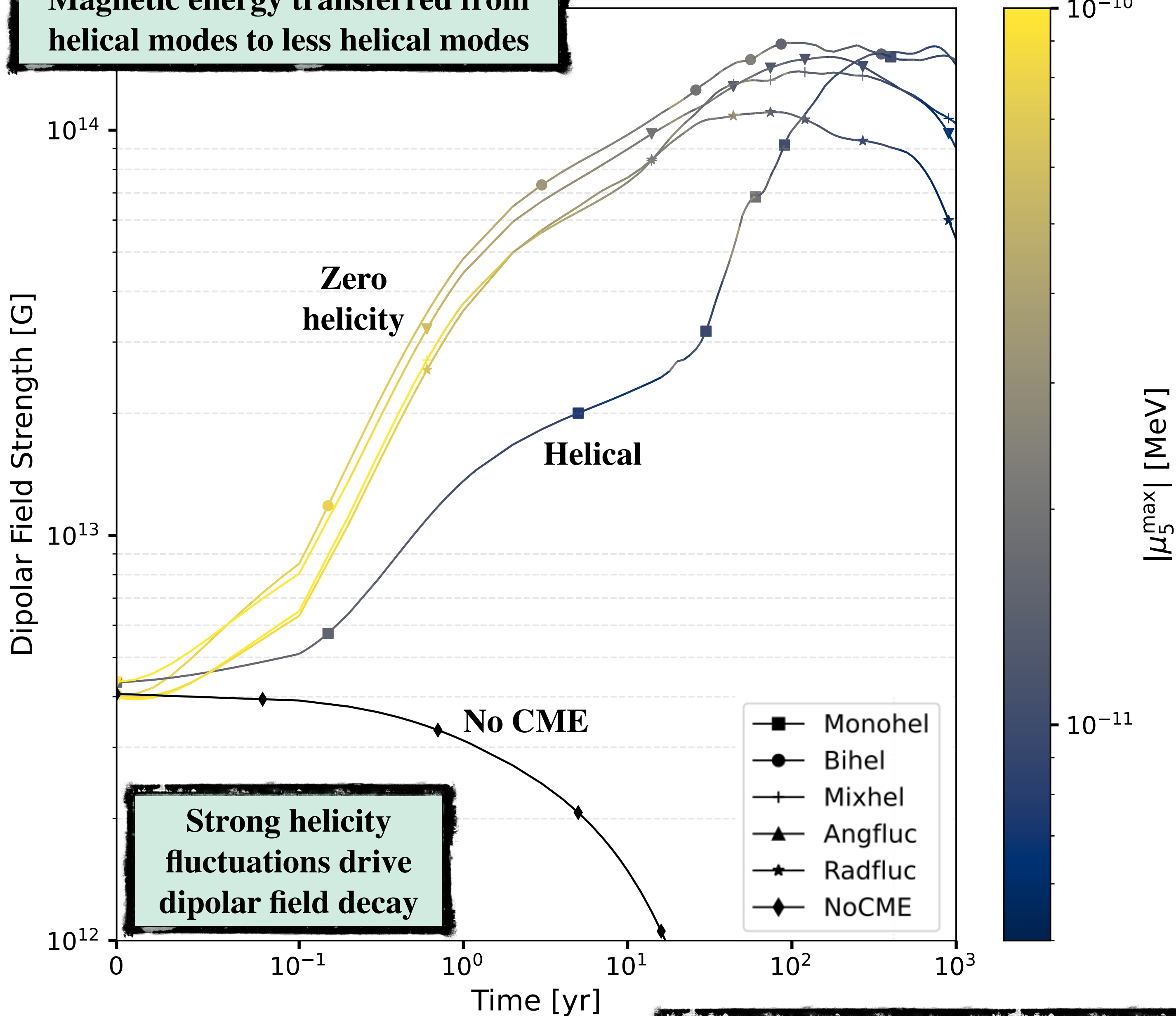
CME simulations — net magnetic helicity



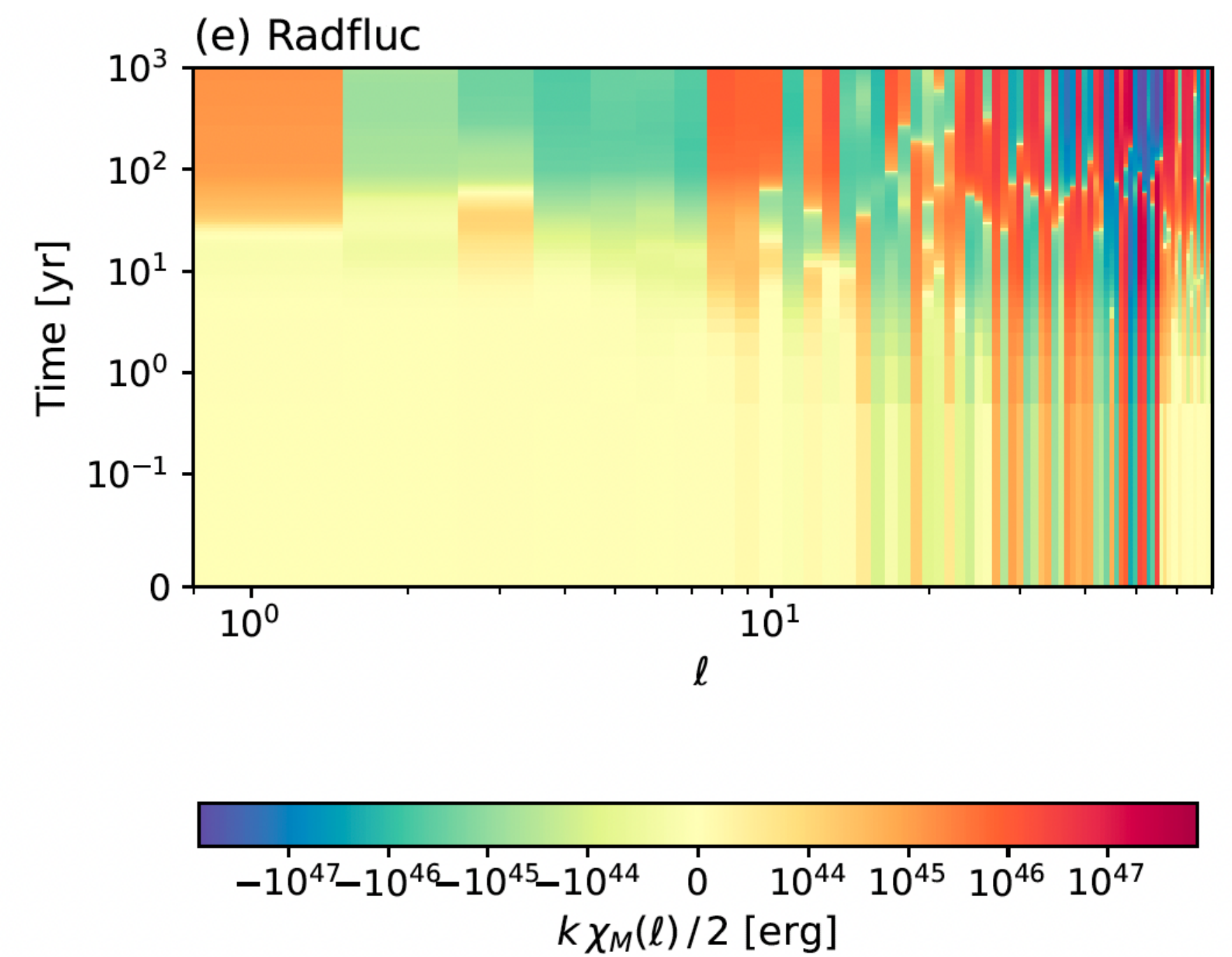
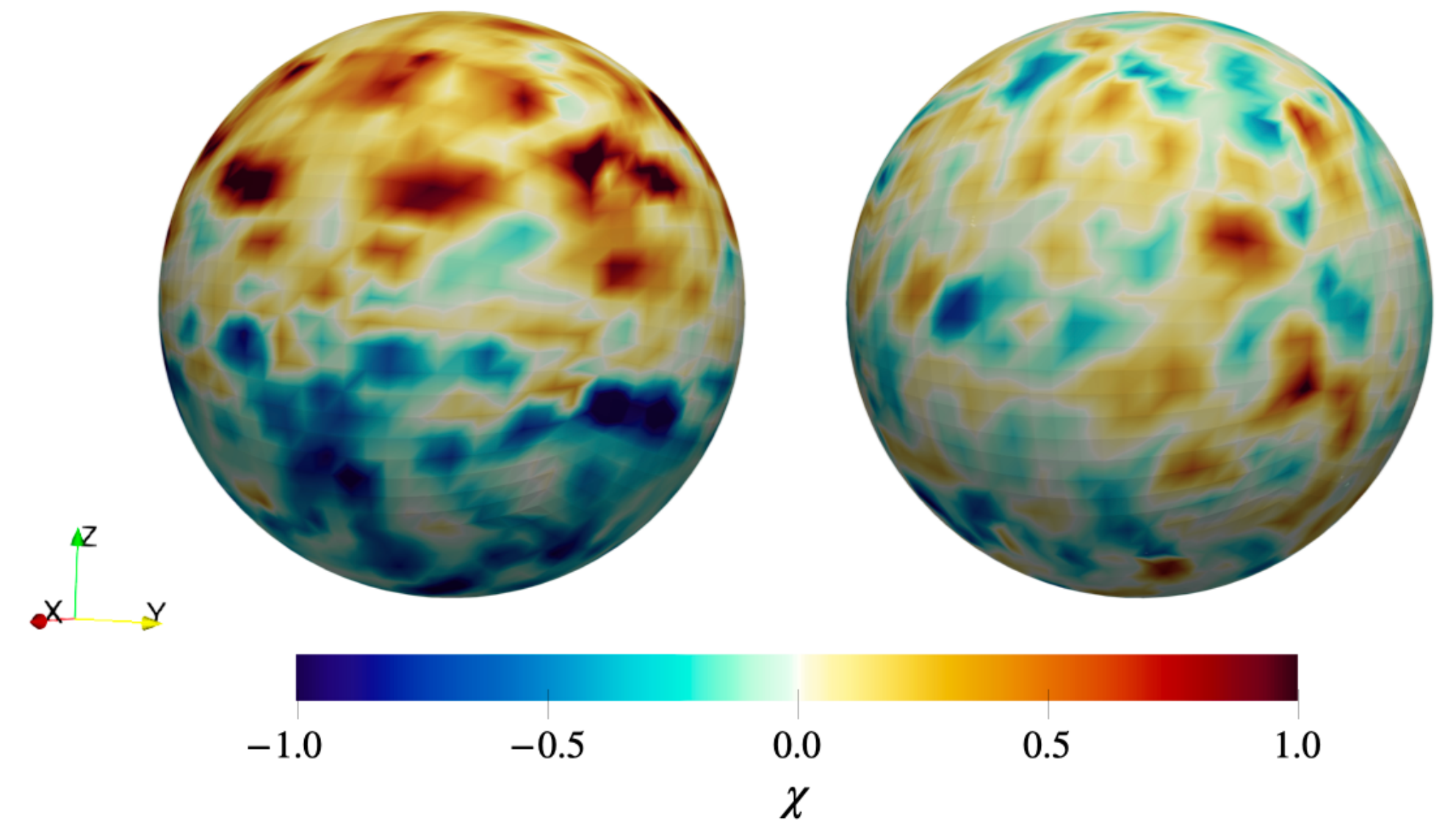
CME simulations — vanishing helicity

Onset of CMI is governed by $|\mu_5^{\max}|$

Magnetic energy transferred from helical modes to less helical modes



Strong helicity fluctuations drive dipolar field decay



CME provides a possible explanation for magnetar formation
At birth: need initial field (small scales) + helicity distribution

Summary & Conclusion:

MATINS: a new 3D code for magneto-thermal evolution of isolated neutron stars, enabling self-consistent Myr-scale evolution linking magnetic fields to X-ray emission, spin-down, and magnetar activity.



Long-term evolution (10^6 yr) with a strong magnetic field $\sim 10^{14} \dots 10^{15}$ G.

Proper treatments of microphysics, envelope models, axial singularity, field structure, temperature, etc.

Hall cascade, Inverse Hall cascade, outburst, etc.

Neutron star cooling:

- Careful microphysics treatment, including superfluid and superconducting gap effects, is essential for neutron-star cooling models.
- Nuclear EOSs must activate enhanced cooling in order to explain the observations of faint yet young neutron stars.

Magnetic field in neutron stars and magnetars:

- Dynamo simulations in core-collapse simulations and proto-neutron stars.
- Inverse cascade in neutron star
- **Chiral anomaly at the origin of magnetars large scale magnetic field.**



