Vacuum-polarization effects in intense laser beams

Antonino Di Piazza

ELI-NP Summer School 2019
Sinaia, 12 September 2019
Outline

• Short introduction on QED
• Quantum vacuum
  – Low-energy vacuum-polarization effects
• Strong-field QED in an intense laser field
  – High-energy vacuum-polarization effects
• Conclusions
• For more information see the books/reviews:

## Typical scales of QED

<table>
<thead>
<tr>
<th>Strength:</th>
<th>Energy:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = \frac{e^2}{4\pi\hbar c} = 7.3 \times 10^{-3}$</td>
<td>$mc^2 = 0.511$ MeV</td>
</tr>
<tr>
<td>(Fine-structure constant)</td>
<td>(Electron rest energy)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Length:</th>
<th>Field:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_C = \frac{\hbar}{mc} = 3.9 \times 10^{-11}$ cm</td>
<td>$E_{cr} = \frac{m^2c^3}{\hbar</td>
</tr>
<tr>
<td>(Compton wavelength)</td>
<td>$B_{cr} = \frac{m^2c^3}{\hbar</td>
</tr>
<tr>
<td></td>
<td>(Critical fields of QED)</td>
</tr>
</tbody>
</table>

## Intensity scale

$$E_{cr} = \frac{m^2c^3}{\hbar|e|} = 1.3 \times 10^{16} \text{ V/cm}$$

$$B_{cr} = \frac{m^2c^3}{\hbar|e|} = 4.4 \times 10^{13} \text{ G}$$

$$I_{cr} = cE_{cr}^2 = 4.6 \times 10^{29} \text{ W/cm}^2$$
Critical fields of QED and vacuum physics

- The vacuum state is the lowest-energy state of the theory where no particles are present.
- In quantum field theory,
  - “Fluctuations” of particles-antiparticles are present in the vacuum.
  - They cover a very short distance and annihilate again after a very short time (for electrons and positrons $\lambda_c=\hbar/mc\sim10^{-11}\text{ cm}$ and $\tau=\lambda_c/c\sim10^{-21}\text{ s}$, respectively).

**Diagram:**

- Quantum Mechanics
  - Time-Energy Uncertainty Principle $\Delta\varepsilon\Delta t \geq \hbar$
  - Quantum Field Theory
  - Quantum Vacuum Fluctuations
  - Special Relativity
  - Mass-Energy Equivalence $\varepsilon = mc^2$
• Physical meaning of the critical fields:

\[ |e|E_{cr} \times \frac{\hbar}{mc} = mc^2 \]

• Vacuum instability and electromagnetic cascades (Bell et al., PRL 2008, Bulanov et al., PRL 2010, Fedotov et al., PRL 2010)
• The interaction energy of a Bohr magneton with a magnetic field of the order of \( B_{cr} \) is of the order of the electron rest energy
• In the presence of background electromagnetic fields of the order of the critical ones a new regime of QED, the strong-field QED regime, opens:

1. where the properties of the vacuum are substantially altered by the fields
2. where a tight interplay unavoidably exists between collective (plasma-like) and quantum effects
3. which is unaccessible to conventional accelerators because it requires coherent fields
Probing the quantum vacuum: effective Lagrangian approach

• At a quantum level photons interact with each other and with external electromagnetic fields:
  – The interaction occurs through the virtual electrons and positrons
  – The interaction is non local, with the typical interaction distance being the Compton wavelength $\lambda_C = \frac{\hbar}{mc}$
  – If the interacting photon fields and the external fields do not vary much on a Compton wavelength, the local approximation can be used (effective Lagrangian approach)

• Approach valid if $\hbar \omega \ll mc^2$, with $\omega$ being the typical angular frequency of the (photon and external) fields
Methods to obtain the effective Lagrangian $L_{\text{eff}}$

1. The Lagrangian density $L_{\text{eff}}$ is such that the Maxwell equations in vacuum are given by (Schwinger 1951)

$$\partial_\mu F^{\mu \nu} = \langle 0(A) | j^\nu(x; A) | 0(A) \rangle$$

$$= -i\epsilon \langle 0(A) | \text{Tr}[\gamma^\nu G(x, x'; A)]_{x' \rightarrow x} | 0(A) \rangle$$

with $|0(A)\rangle$ and $G(x, x'; A)$ being the vacuum state and the electron propagator, respectively, in the presence of the field electromagnetic $F^{\mu \nu}(x) = \partial^\mu A^\nu(x) - \partial^\nu A^\mu(x)$

2. The Lagrangian density $L_{\text{eff}}$ is such that

$$\Delta L = L_{\text{eff}} - L_M = -[\mathcal{E}_{\text{vac}}(A) - \mathcal{E}_{\text{vac}}(0)]$$

with $L_M = (E^2 - B^2)/2$ being the classical Maxwell Lagrangian density and $\mathcal{E}_{\text{vac}}(0/A)$ the vacuum energy in absence (in the presence) of the external field

3. More formal methods through Feynman path integral (see, e.g., Peskin and Schroeder or any modern book on QFT)
• Euler and Heisenberg in 1936 obtained the following expression of the effective Lagrangian for a constant and uniform electromagnetic field \((E, B)\) described by the two invariants \(\mathcal{F} = - \frac{(E^2 - B^2)}{2}, \mathcal{G} = -E \cdot B\)

\[
L_{\text{eff}} = - \mathcal{F}
- \frac{1}{2\pi} \int_0^\infty \frac{ds}{s^3} e^{-ms} \left\{ (es)^2 \mathcal{G} \frac{\text{Re} \cosh \left[ |e|s \sqrt{2(\mathcal{F} + i\mathcal{G})} \right]}{\text{Im} \cosh \left[ |e|s \sqrt{2(\mathcal{F} + i\mathcal{G})} \right]} - 1 - \frac{2}{3} (es)^2 \mathcal{F} \right\}
\]

• The quantum part of the Lagrangian density depends on the ratios \(\mathcal{F}/F_{cr}^2\) and \(|\mathcal{G}|/F_{cr}^2\)

• The imaginary part of \(L_{\text{eff}}\) is connected with the pair production probability per unit volume and unit time

• If the external field is purely magnetic \(L_{\text{eff}}\) is real then no pair production, while for purely electric fields \(L_{\text{eff}}\) contains an imaginary part (Dittrich et al. 1985)

• A plane-wave field with \(\mathcal{F} = \mathcal{G} = 0\) cannot give rise to any nonlinear vacuum-polarization effect
Light-by-light diffraction

Probe polarization before the interaction

Probe polarization after the interaction

The scheme also works with a single strong traveling wave (electromagnetic cascade generation in a standing wave in the presence of residual electrons: A. Bell and J. Kirk, Phys. Rev. Lett. 101, 200403 (2008))
Effects of diffraction on the probe polarization

Strong beam:
\[ I_L = 10^{23} \text{ W/cm}^2, \]
\[ w_L = \lambda_L = 0.745 \text{ \mu m} \]

Probe beam:
\[ \lambda_p = 0.4 \text{ nm, } w_p = 8 \text{ \mu m} \]

- The Euler-Heisenberg effective Lagrangian at the lowest-order in \( \mathcal{F}/F_{cr}^2 \) and \( |\mathcal{G}|/F_{cr}^2 \) reads

\[
L_{\text{eff}} = \frac{1}{2} (E^2 - B^2) + \frac{2\alpha^2}{45m^4} \left[ (E^2 - B^2)^2 + 7(E \cdot B)^2 \right]
\]

- \( \psi \) and \( \varepsilon \) are proportional to \( I_L \)
- \( \psi \) and \( \varepsilon \sim 10^{-9} \text{ rad} \) are envisaged to be soon measurable in the soft x-ray regime (Marx et al. 2011)

A matterless double-slit

- The double slit experiment has played a fundamental role in our understanding of quantum mechanics, in particular the so-called wave-particle duality of particles.
- All double-slit schemes proposed so far have always involved matter (either the particles employed like electrons, neutrons and so on or the wall where the double slit is). By exploiting the quantum interaction among laser beams in the vacuum, we have put forward a matterless double slit setup (King et al. 2010, ibid. 2012)
• Strong field: 150 PW (ELI), 800 nm, 30 fs, focused to one wavelength
• Weak field: 200 TW, 527 nm, 100 fs focused to 290 µm
• The × are at: \((n+1/2)\lambda_p=D \sin \vartheta\)
• With the above parameters one obtains about 4 diffracted photons per shot
• Elastic photon-photon scattering

• Multi PW-class laser systems may open the possibility of observing for the first time either direct photon-photon scattering in vacuum (Lundstroem et al. Phys. Rev. Lett. 2006)
High-energy vacuum-polarization effects

- At photon energies much lower than the electron rest energy, the effective-Lagrangian approach can be successfully applied.
- If two photons with four-momenta $k_1^\mu$ and $k_2^\mu$ collide, the Lorentz-invariant condition to apply the effective-Lagrangian approach reads: $(k_1k_2) \ll m^2$.
- However, the cross-section of photon-photon scattering scales as $[(k_1k_2)/m^2]^6$ for $(k_1k_2) \ll m^2$ (Berestetskii et al. 1982).
- It is more convenient to work at $(k_1k_2) \sim m^2$. Units with $\hbar=c=1$. 

![Graph showing the cross-section of photon-photon scattering vs. center-of-momentum energy.](image)
• In the regime \((k_1 k_2) \sim m^2\) the propagation of a photon through a laser beam can be studied by means of the polarization operator

\[
\text{Note: no odd number of photon lines: Furry theorem}
\]

- The propagation of a photon through a laser field can be studied by means of the (nonlocal) Dyson equation for the photon wave function

\[
-\partial^2 \Phi^\mu(x) = \int d^4 y P^{\mu\nu}(x, y) \Phi_\nu(y)
\]

by neglecting higher-order contributions to the polarization operator

\[
\text{S. Bragin et al., Phys. Rev. Lett. 119, 250403 (2017)}
\]

• The Dyson equation can be written diagrammatically as
• In the laser-photon head-on collision, the photon state can be expanded into two polarization states $\Lambda_1^\mu = (0, \Lambda_1)$ and $\Lambda_2^\mu = (0, \Lambda_2)$, with $\Lambda_1 \parallel E_L$ and $\Lambda_2 \parallel B_L$.

• By solving the Dyson equation, one finds that

$$\Phi_i^\mu(x) = e_i^\mu e^{-i(kx)} = \sum_{l=1,2} c_l \Lambda_l^\mu e^{-i(kx)} \rightarrow \Phi_f^\mu(x) = e_f^\mu e^{-i(kx)} = \sum_{l=1,2} c_l \Lambda_l^\mu e^{-i(kx)} e^{-i\phi_l} e^{-\lambda_l}$$

• The two additional phases $\phi_l$ (real exponents $\lambda_l$) indicate that the vacuum is a birefringent (dichroic) medium (the physical origin of dichroism is the possibility of pair production).

• If $\delta\phi = \phi_2 - \phi_1$ and $\delta\lambda = \lambda_2 - \lambda_1$ ($N$ is the number of laser cycles)

- $\omega = 0.1$ GeV
- $\omega = 0.5$ GeV
- $\omega = 1$ GeV

• Anomalous dispersion for $\chi > 2.5$
The experimental setup

- At such high photon energies the only detection process is pair production, which is sensitive to the polarization of the photon
- Importance of employing circularly-polarized photons for detecting vacuum birefringence
- For measuring vacuum birefringence the most suitable facility turns out to be ELI-Beamlines (11 operational hours at for $\chi=0.25$)
- For measuring vacuum dichroism the most suitable facility turns out to be ELI-NP (4 operational days at for $\chi=2.5$)
Photon splitting in a laser field

General features

• Due to charge-parity conservation (Furry theorem), photon splitting can happen in a laser field with absorption of an odd number of laser photons

• If the initial photon and the laser field are counterpropagating the initial and the final photons lie in the same plane

• The photon-splitting rate depends only on the two gauge- and Lorentz-invariants

\[ \eta = \frac{(k_0 k_1)}{2m^2} = \frac{\omega_0 \omega_1}{m^2}, \quad \chi = \frac{1}{2m^3} \sqrt{-(eF_{\mu\nu}k_1^\nu)^2} = \frac{\omega_1}{m} \frac{E}{E_{cr}} \]
Results (strong optical laser field)

- The parameter $\eta=\omega_0\omega_1/m^2$ is much smaller than unity and the final photons are almost collinear to the initial one.
- Due to angular momentum conservation, in a circularly polarized laser field only one or three laser photons can be absorbed:

<table>
<thead>
<tr>
<th>I</th>
<th>L</th>
<th>F1</th>
<th>F2</th>
</tr>
</thead>
<tbody>
<tr>
<td>+</td>
<td>+</td>
<td>-/+</td>
<td>+/−</td>
</tr>
<tr>
<td>+</td>
<td>+++</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>−</td>
<td>+</td>
<td>−</td>
<td>−</td>
</tr>
</tbody>
</table>

- Example for $\chi=1$: $\omega_1=50$ MeV, $I_L=5 \times 10^{24}$ W/cm²
- In a linearly polarized laser field any odd number of photons can be absorbed from the laser

Conclusions

• According to quantum electrodynamics the vacuum is not simply ‘nothing’ but it is an interesting and fascinating object to investigate
• Its properties can be studied with the help of ultra-intense lasers as envisaged at ELI
• Under the action of ultra-strong laser fields, the vacuum behaves like a birefringent and dichroic medium, allowing for exotic effects like
  – modification of the vacuum refractive index
  – light-by-light diffraction
  – photon splitting
  – vacuum anomalous dispersion