# Photon-photon scattering

## Zero momentum frame

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two photons collision

$\omega_1$ $\omega_2$

$$\begin{pmatrix} p_0^{(1)} \\ p_{\parallel}^{(1)} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \frac{\hbar\omega_1}{c^2} \qquad \begin{pmatrix} p_0^{(2)} \\ p_{\parallel}^{(2)} \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \frac{\hbar\omega_2}{c^2}$$

$$\begin{pmatrix} p_0 \\ p_{\parallel} \end{pmatrix} = \begin{pmatrix} 1 \\ \sqrt{1-\left(\frac{m_0c^2}{E}\right)^2} \end{pmatrix} \frac{E}{c^2}$$

$$\xleftarrow{\quad V \quad}$$

$$\gamma = \frac{1}{\sqrt{1-\left(\frac{V}{c}\right)^2}}$$

$\omega_1'$ $\omega_2'$

$$\begin{pmatrix} p_0' \\ p_{\parallel}' \end{pmatrix} = \begin{pmatrix} \gamma & \gamma\frac{V}{c} \\ \gamma\frac{V}{c} & \gamma \end{pmatrix} \begin{pmatrix} p_0 \\ p_{\parallel} \end{pmatrix} \Rightarrow \omega_1' = \omega_1 \frac{1}{\sqrt{1-\left(\frac{V}{c}\right)^2}}\left(1+\frac{V}{c}\right) \qquad \omega_2' = \omega_2 \frac{1}{\sqrt{1-\left(\frac{V}{c}\right)^2}}\left(1-\frac{V}{c}\right)$$

relativistic longitudinal Doppler eff. $= \omega_1 \sqrt{\frac{1+\frac{V}{c}}{1-\frac{V}{c}}}$ $\qquad = \omega_2 \sqrt{\frac{1-\frac{V}{c}}{1+\frac{V}{c}}}$

zero total momentum in the new frame $\left(p_{\parallel}^{(1)'} + p_{\parallel}^{(2)'}\right)$

$$\Longrightarrow \quad \omega_1' = \omega_2'$$

$$\omega_1 \left(1 + \frac{V}{c}\right) = \omega_2 \left(1 - \frac{V}{c}\right)$$

$$\frac{V}{c} = \frac{\omega_2 - \omega_1}{\omega_2 + \omega_1}$$

$$\omega_1' = \sqrt{\frac{1 + \frac{V}{c}}{1 - \frac{V}{c}}}\,\omega_1 = \sqrt{\frac{1 + \frac{\omega_2 - \omega_1}{\omega_2 + \omega_1}}{1 - \frac{\omega_2 - \omega_1}{\omega_2 + \omega_1}}}\,\omega_1 = \sqrt{\frac{2\omega_2}{2\omega_1}}\,\omega_1$$

$$= \sqrt{\omega_1 \omega_2} = \omega_2'$$

total momentum zero reference frame – center of mass frame

ELI – NP

$\omega_1$ : 1,5 eV (820 nm)

$\omega_2$ : 20 MeV - $\gamma$ - source

$\omega_1' = \omega_2' = \sqrt{\omega_1 \omega_2}$ : $\sqrt{1.5 \cdot 20.10^6}$ eV $\simeq 5.5$ keV

N photons $\omega_1$ $\equiv \omega_1 \to N\omega_1$

equivalent kinematics $5.5\sqrt{N}$ keV

$e^+ + e^-$ threshold $\Rightarrow 5.5\sqrt{N}$ keV $= 511$ keV

$\Rightarrow \boxed{N \simeq 10^4}$ (8632)

but $e^+$ and $e^-$ appear in a strong em field

$\to$ effective mass!

SLAC E-144 PRL 79 (1997) 1626

46.6 GeV $e^-$ $\Rightarrow$ Compton 29.1 GeV

$\omega_1$ : 2.35 eV (527 nm)

$\omega_2$ : 29.1 GeV

$\omega_1' = \omega_2' = \sqrt{\omega_1 \omega_2}$ ; $\sqrt{2.35 \cdot 29.1 \cdot 10^9}$ eV $\simeq$ 262 keV

$e^+ + e^-$ thr. $\Rightarrow (N\omega_1)$ $262\sqrt{N} \geq 511$ $\Rightarrow$ $\underline{N \geq 4}$

H.R. Reiss – Eur. Phys. J D75 (2021) 158 $N \geq 5$

$\Rightarrow$ effective threshold $m \to m\sqrt{1+\eta^2} \simeq m + \frac{1}{2}\frac{(ea)^2}{mc^2}$

$\eta = \frac{ea}{mc^2}$

ponderomotive potential

K.T. McDonald (Princeton U), 1986 – proposal


$$\begin{pmatrix} p_0 \\ p_{\parallel} \end{pmatrix} \simeq \begin{pmatrix} 1 \\ -\sqrt{1-\left(\frac{m_0c^2}{E}\right)^2} \end{pmatrix} \frac{E}{c^2}$$

Compton scattering

$$p'_{\parallel} = \gamma p_{\parallel} + \gamma \frac{V}{c} p_0$$

$$= \gamma \left( -\sqrt{1-\left(\frac{m_0c^2}{E}\right)^2} + \frac{V}{c} \right) \frac{E}{c^2}$$

rest frame of the electron

$$\frac{V}{c} = \sqrt{1-\left(\frac{m_0c^2}{E}\right)^2} \simeq 1 - \frac{1}{2}\left(\frac{m_0c^2}{E}\right)^2$$

$$\omega'_1 = \omega_1 \sqrt{\frac{1+\frac{V}{c}}{1-\frac{V}{c}}} \simeq \omega_1 \sqrt{\frac{2}{\frac{1}{2}\left(\frac{m_0c^2}{E}\right)^2}} = \omega_1 \frac{2E}{m_0c^2}$$

ELI - NP

$w_\gamma$ : 1.5eV (820 nm)

$e^-$ : 1 GeV

$$\Rightarrow \frac{2E}{m_0c^2} = \frac{2\cdot 10^6}{511} \simeq 3914$$

1.5eV $\longrightarrow$ $e^-$ rest frame $= \underline{\underline{5.87\ keV}}$

Thomson scatt (negligible recoil)

$$E_{\gamma\, scatt} = \frac{E_\gamma}{1+2\frac{E_\gamma}{m_0c^2}}$$

$(1-\cos\theta)$

$\sim 5.75$ keV

$\downarrow$

22.5 MeV

SLAC E-144

$w_\gamma$ : 2.35eV (527nm) $\Rightarrow \frac{2E}{m_0c^2} \simeq 182\cdot 10^3 \Rightarrow$ laser in $e^-$ rest frame $\sim 428$ keV

$e^-$ : 46.6 GeV

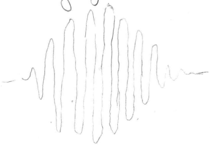
$E_{\gamma\,scatt} \simeq 160\ keV$ → in lab frame ⇒ 29.1 GeV

$182.10^3$ factor

SLAC. E-320

13 GeV $e^-$ + 10 TW $(\lambda = ?)$

strong field + $e^-$



$e^-$ at rest

$\omega \longrightarrow$

what is the drift velocity of $e^-$ in the wave?

Kirk McDonald argument:

$p_\mu$ – 4-momentum of the electron $(m, 0, 0, 0)$

$w_\mu$ – 4-vector of the wave photon $(\omega, 0, 0, \omega)$

$\bar{p}_\mu$ – the effective 4-momentum of the electron in the wave $\bar{p}_\mu \bar{p}^\mu = m^2(1+\eta^2)$

$\bar{p}_\mu$ "must have the form" $\bar{p}_\mu = p_\mu + \varepsilon w_\mu \Rightarrow \varepsilon = \frac{\eta^2 m^2}{2 p_\mu w^\mu} = \frac{\eta^2 m}{2\omega}$

!!!

$$\overline{p}_\mu = \left(m\left(1+\frac{\eta^2}{2}\right), 0, 0, \frac{\eta^2 m}{2}\right)$$

$$\frac{\frac{\eta^2 m}{2}}{m\left(1+\frac{\eta^2}{2}\right)} = \frac{V_{\parallel}^{drift}}{c} \Rightarrow \frac{V_{\parallel}^{drift}}{c} = \frac{\eta^2}{2+\eta^2}$$

alternative $\overline{p}_\mu = \alpha p_\mu + \varepsilon \omega_\mu$ !

$\alpha \neq 1$.

$$\omega' = \omega \sqrt{\frac{1-\frac{\eta^2}{2+\eta^2}}{1+\frac{\eta^2}{2+\eta^2}}} = \omega\sqrt{\frac{2}{2+2\eta^2}} = \omega\frac{1}{\sqrt{1+\eta^2}}$$

SLAC E-144 $\Rightarrow \eta \simeq 0{,}2$

ELI-NP $\Rightarrow \eta \geq 200 \Rightarrow \omega = 5.878\,\text{eV} \Rightarrow \omega' \simeq 30\,\text{eV}$

Conclusions!

- relevant physics may be understood using relativistic Doppler shift
- photon + multiphoton threshold for pair production

Breit - Wheeler $n\,\omega_1 + \omega_2 \rightarrow e^+ + e^-$

- Multumesc!