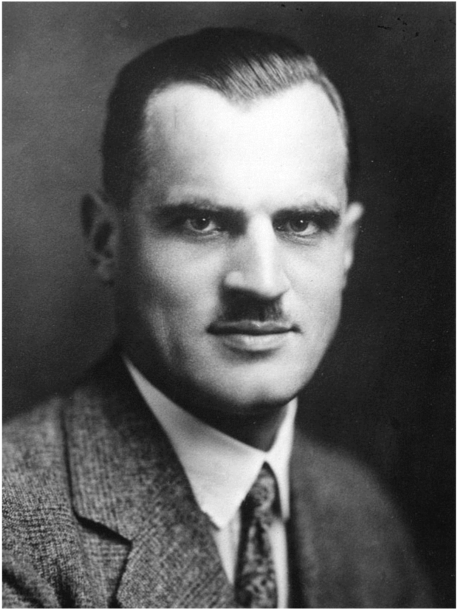
Compton scattering

R.A. Ionescu

September 16, 2021



- Phys. Rev. **21**(1923)483

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THE

PHYSICAL REVIEW

A QUANTUM THEORY OF THE SCATTERING OF X–RAYS BY LIGHT ELEMENTS

By Arthur H. Compton

Abstract

A quantum theory of the scattering of X-rays and γ-rays by light elements.—The hypothesis is suggested that when an X-ray quantum is scattered it spends all of its energy and momentum upon some particular electron. This electron in turn scatters the ray in some definite direction. The change in momentum of the X-ray quantum due to the change in its direction of propagation results in a recoil of the scattering electron. The energy in the scattered

- Phys. Rev. **81**(1951)15

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On Bound States and Scattering in Positron Theory

W. H. Furry
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(Received July 18, 1950)

The use of bound-state wave functions in calculations in positron theory is justified by the introduction of a new representation, in a certain sense intermediate between the Heisenberg and interaction representations. In the bound-state representation the definition of a stable vacuum state is possible only for a restricted class of external fields. Some attention is given to the problem of vacuum polarization, and it is shown that a very simple procedure accomplishes the charge renormalization with sufficient accuracy to be of use in certain scattering problems. The application to the scattering of radiation is discussed in some detail, in order to show the relation between the different points of view that may be adopted in problems of the coherent scattering by a bound electron and the "Delbrück scattering" by virtual electron pairs.

► Z. Phys. **94**(1935)250

Über eine Klasse von Lösungen der Diracschen Gleichung.

Von **D. M. Wolkow** in Leningrad.

(Eingegangen am 12. Februar 1935.)

1. Der Fall eines sinusoidalen Feldes. — 2. Lösung für den Fall, daß das äußere Feld aus polarisierten, in einer bestimmten Richtung fortschreitenden Wellen besteht, die ein *abzählbares* Spektrum nach Frequenz und Anfangsphasen haben.

1. Der Fall des sinusoidalen Feldes.

Es sei das skalare Potential des auf das relativistische Quantenelektron wirkenden äußeren Feldes gleich Null, das Vektorpotential sei

$$A = a \cos 2\pi\nu\left[t - \frac{nx}{c} + \alpha\right] = a\cos\varphi \quad \text{mit} \quad \varphi = 2\pi\nu\left[t - \frac{nx}{c} + \alpha\right],$$

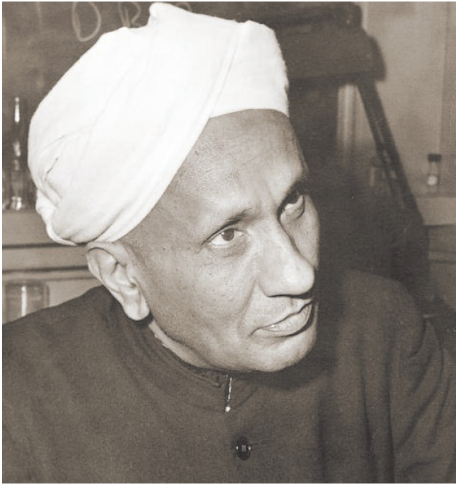
- C.V. Raman, K.S. Krishnan - Nature **121**(1928)501

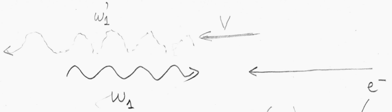
Nature (*London*) **121 501–502 (1928)**

A new type of secondary radiation

If we assume that the X-ray scattering of the 'unmodified' type observed by Prof. Compton corresponds to the normal or average state of the atoms and molecules, while the 'modified' scattering of altered wavelength corresponds to their fluctuations from that state, it would follow that we should expect also in the case of ordinary light two types of scattering, one determined by the normal optical properties of the atoms or molecules, and another representing the effect of their fluctuations from their normal state. It accordingly becomes necessary to test whether this is actually the case. The experiments we have made have

- **Chandrasekhara Venkata Raman**





$$\begin{pmatrix}1\\1\end{pmatrix}\frac{\hbar\omega_1}{c^2} \qquad \begin{pmatrix}p_0\\p_\parallel\end{pmatrix} = \begin{pmatrix}1\\-\sqrt{1-\left(\frac{m_0c^2}{E}\right)^2}\end{pmatrix}\frac{E}{c^2}$$

$$p_0^{tot} = \frac{\hbar\omega_1}{c^2}+\frac{E}{c^2}$$

$$p_\parallel^{tot} = \frac{\hbar\omega_1}{c^2}-\sqrt{1-\left(\frac{m_0c^2}{E}\right)^2}\,\frac{E}{c^2}$$

$$p_\parallel^{tot'} = \gamma\left(p_\parallel^{tot}+\frac{V}{c}p_0^{tot}\right) = 0$$

$$\Rightarrow \frac{V}{c} = -\frac{p_\parallel^{tot}}{p_0^{tot}} = -\frac{\frac{\hbar\omega_1}{c^2}-\sqrt{1-\left(\frac{m_0c^2}{E}\right)^2}\,\frac{E}{c^2}}{\frac{\hbar\omega_1}{c^2}+\frac{E}{c^2}}$$

$$\omega_1^{cm} = \omega_1\sqrt{\frac{1+\frac{V}{c}}{1-\frac{V}{c}}} \qquad \omega_1' = \omega_1'^{cm}\sqrt{\frac{1+\frac{V}{c}}{1-\frac{V}{c}}} \qquad \left(\omega_1'^{cm}=\omega_1^{cm}\right)$$

$$\omega_1' = \omega_1\frac{1+\frac{V}{c}}{1-\frac{V}{c}} = \omega_1\frac{\frac{E}{c^2}+\sqrt{\left(\frac{E}{c^2}\right)^2-m_0^2}}{2\frac{\hbar\omega_1}{c^2}+\frac{E}{c^2}-\sqrt{\left(\frac{E}{c^2}\right)^2-m_0^2}}$$

- e^- at rest $\qquad E = m_0c^2$

$$\omega_1' = \omega_1 \frac{1}{1 + 2\frac{\hbar\omega_1}{m_0c^2}}$$

- relativistic e^- $\qquad E \gg m_0c^2$

$$\omega_1' \simeq \omega_1 \frac{2\left(\frac{E}{m_0c^2}\right)}{\cancel{2\frac{\hbar\omega_1}{m_0c^2}} + \frac{1}{2}\frac{1}{\left(\frac{E}{m_0c^2}\right)}} \longrightarrow 4\left(\frac{E}{m_0c^2}\right)^2 \omega_1$$

$$\mathcal{L}(\vec{X},\vec{V};t) = -m\sqrt{1-\vec{V}^2} + e\vec{V}\vec{A}(\vec{X};t) \qquad \vec{V} = \frac{d\vec{X}}{dt} \qquad c=1$$

$$A_x = a \sin\omega(t-z)$$
$$A_y = -a\cos\omega(t-z)$$

circular polarization

exact solution

$$x_0' = \frac{1}{\omega'}\frac{ea}{\sqrt{m^2+e^2a^2}}\cos\omega' t'$$

$$y_0' = \frac{1}{\omega'}\frac{ea}{\sqrt{m^2+e^2a^2}}\sin\omega' t'$$

$$z_0' = 0$$

$$t' = \gamma(t - v_z z) \qquad \omega' = \frac{\omega}{\gamma(1+v_z)}$$
$$z' = \gamma(z - v_z t)$$

$$\Rightarrow \quad \begin{aligned} x_0 &= x_0' = \frac{\gamma(1+v_z)}{\omega}\frac{ea}{\sqrt{m^2+e^2a^2}}\cos\frac{\omega}{1+v_z}(t-v_z z_0) \\ y_0 &= y_0' = \frac{\gamma(1+v_z)}{\omega}\frac{ea}{\sqrt{m^2+e^2a^2}}\sin\frac{\omega}{1+v_z}(t-v_z z_0) \\ z_0 &= v_z t \end{aligned}$$

$$v_x = \dot{x}_0 = -\frac{1}{\gamma} \frac{ea}{\sqrt{m^2+e^2a^2}} \sin \frac{\omega}{1+v_z}\left(t - v_z z\right)$$

$$v_y = \dot{y}_0 = \frac{1}{\gamma} \frac{ea}{\sqrt{m^2+e^2a^2}} \cos \frac{\omega}{1+v_z}\left(t - v_z z\right)$$

$$\Rightarrow v_x^2 + v_y^2 = \frac{e^2a^2}{m^2+e^2a^2}\left(1 - v_z^2\right)$$

$$e\vec{V}\cdot\vec{A} = -\frac{1}{\gamma} \frac{e^2a^2}{\sqrt{m^2+e^2a^2}} \underbrace{\left[\cos\omega(t-z)\cos\omega\frac{t-v_z z}{1+v_z} + \sin\omega(t-z)\sin\omega\frac{t-v_z z}{1+v_z}\right]}_{\cos\omega\frac{z-v_z t}{1+v_z}}$$

$\uparrow$ $\sqrt{1-v_z^2}$

$$\mathcal{L} = -m\sqrt{1 - \frac{e^2a^2}{m^2+e^2a^2}\left(1-v_z^2\right) - v_z^2} - \frac{e^2a^2}{\sqrt{m^2+e^2a^2}} \cos\omega\frac{z-v_z t}{1+v_z}\sqrt{1-v_z^2}$$

$$\mathcal{L} = -m\sqrt{\frac{m^2}{m^2+e^2\varrho^2}\left(1-v_z^2\right)} - \frac{e^2\varrho^2}{\sqrt{m^2+e^2\varrho^2}}\cos\omega\frac{z-v_z t}{1+v_z}\sqrt{1-v_z^2}$$

$$\downarrow$$

$$1.$$

$$\mathcal{L} = -\underbrace{\sqrt{m^2+e^2\varrho^2}}_{m_{eff}}\sqrt{1-v_z^2}$$

$$p_z = \frac{\partial\mathcal{L}}{\partial v_z} = \frac{m_{eff}\,v_z}{\sqrt{1-v_z^2}} \Rightarrow v_z = \frac{p_z}{\sqrt{m_{eff}^2+p_z^2}}$$

$$H = p_z v_z(p_z) - \mathcal{L}\left(z, v_z(p_z)\right) = \frac{p_z^2}{\sqrt{m_{eff}^2+p_z^2}} + m_{eff}\sqrt{1-\frac{p_z^2}{m_{eff}^2+p_z^2}} =$$

$$= \frac{m_{eff}^2+p_z^2}{\sqrt{m_{eff}^2+p_z^2}} = \sqrt{m_{eff}^2+p_z^2}$$

$$H = \sqrt{m^2 + e^2 a^2 + p_z^2}$$

envelope $\quad a(z-t)$

$$\left(v_z = \frac{p_z}{\sqrt{m^2 + e^2 a^2 + p_z^2}} \right)$$

to be continued ?

- Multumesc!