Procese QED în câmpuri laser intense
tema 38

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.

”Verbum sapienti sat est”

Cuprins

1.1 Cuantificarea câmpului Dirac

- Cuantificarea oricăror câmpuri se face prin exprimarea amplitudinilor din dezvoltarea generală Fourier a câmpului, ca superpoziţie de câmpuri libere.

  Dacă în teoria câmpurilor clasice, amplitudinile dezvoltării după câmpuri libere se determină ca *transformata Fourier* a acestor dezvoltări, în teoria câmpurilor cuantice, amplitudinile dezvoltării după câmpurile cuantice libere, ca vectori de bază ai unui spaţiu Hilbert, se determină prin proiectarea pe fiecare din aceste câmpuri cuantice libere.

  Să exprimăm întâi Lagrangian-ul şi Hamiltonian-ul de câmp Dirac cuantic.

- Lagrangian-ul, de *câmpuri Dirac* cuantice exprimat prin $\hat{\varphi}$ şi $\hat{\overline{\varphi}}$:

$$\boxed{\mathcal{L} = i\hbar\hat{\overline{\varphi}}\gamma^\mu\partial_\mu\hat{\varphi} - mc\hat{\overline{\varphi}}\hat{\varphi}} \tag{1.1}$$

- Câmpul Dirac de impuls conjugat canonic este dat de operatorul:

$$\hat{\pi} = \frac{\partial\mathcal{L}}{\partial\hat{\dot{\varphi}}} = i\hbar\hat{\overline{\varphi}}\gamma^0 = i\hbar\hat{\varphi}^\dagger \tag{1.2}$$

- Folosind ecuaţia Dirac $\boxed{i\hbar\gamma^\mu\partial_\mu\varphi - mc\varphi = 0}$ [1]

  Hamiltonian-ul de *câmp Dirac* cuantic,

$$\mathcal{H}_D = \hat{\pi}\hat{\dot{\varphi}} - \mathcal{L} = \underbrace{\overbrace{i\hbar\hat{\overline{\varphi}}\gamma^0}^{\hat{\pi}}\partial_0\hat{\varphi} - i\hbar\hat{\overline{\varphi}}\gamma^\mu\partial_\mu\hat{\varphi}}_{-i\hbar\hat{\overline{\varphi}}\gamma^i\partial_i\hat{\varphi}} + mc\hat{\overline{\varphi}}\hat{\varphi} = \hat{\overline{\varphi}}\underbrace{\left(-i\hbar\gamma^i\partial_i + mc\right)}_{-i\hbar\gamma^0\partial_0 \text{ din ec.Dirac}}\hat{\varphi} \tag{1.3}$$
$$= \hat{\overline{\varphi}}\left(i\hbar\gamma^0\partial_0\hat{\varphi}\right) = i\hbar\hat{\varphi}^\dagger\partial_0\hat{\varphi}$$

- Hamiltonian-ul de câmp Dirac cuantic, folosind (1.3), este:

$$\hat{H}_D = \int\mathcal{H}_D\, d^3\vec{x} \tag{1.4}$$

  deoarece $i\hbar\frac{d\hat{H}}{dt} = \left[\hat{H}, \hat{H}\right] \equiv 0 \Rightarrow \hat{H}_D$ e independent de t

- Ecuaţia Heisenberg de mişcare pentru operatorul de câmp Dirac cuantic $\hat{\varphi}(\vec{x}, t)$ este:

$$i\hbar\frac{\partial\hat{\varphi}}{\partial t} = \left[\hat{\varphi}, \hat{H}_D\right] \tag{1.5}$$

- Soluţiile de undă plană de câmp liber Dirac $\varphi(\vec{x}, t)$ şi $\overline{\varphi}(\vec{x}, t)$, se folosesc acum ca operatori de câmpuri cuantice $\hat{\varphi}$ şi $\hat{\overline{\varphi}}$, exprimate cu ajutorul spinorilor Dirac $u_s(\vec{p})$ şi $v_s(\vec{p})$, sunt:

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[1] Atenţie ! γ^μ nu este 4-vector, iar ∂_μ este 4-vector covariant cu metrica $(+, +, +, +)$.
Apoi, deşi trecerea la câmpuri cuantice se face prin $\dot{q} \to \partial_\mu\varphi$, pentru densităţi de Hamiltonian şi de câmpuri se împarte cu $d^3\vec{x}$, atunci rămâne valabilă trecerea $\dot{q} \to \partial_0\varphi$.

$$\hat{\varphi}(\vec{x},t)=u_s(\vec{p})\,e^{-i\,p\cdot x/\hbar} \qquad \hat{\overline{\varphi}}(\vec{x},t)=v_s(\vec{p})\,e^{+i\,p\cdot x/\hbar} \tag{1.6}$$

$$u_{(1,2)}=\begin{pmatrix}1\\0\\ \frac{p_3c}{E+mc^2}\\ \frac{(p_1+ip_2)c}{E+mc^2}\end{pmatrix},\ \begin{pmatrix}0\\1\\ \frac{(p_1-ip_2)c}{E+mc^2}\\ \frac{-p_3c}{E+mc^2}\end{pmatrix} \quad ; \quad v_{(1,2)}=\begin{pmatrix}\frac{p_3c}{|E|+mc^2}\\ \frac{(p_1+ip_2)c}{|E|+mc^2}\\1\\0\end{pmatrix},\ \begin{pmatrix}\frac{(p_1-ip_2)c}{|E|+mc^2}\\ \frac{-p_3c}{|E|+mc^2}\\0\\1\end{pmatrix}$$

- Soluţia generală de câmp Dirac o scriem ca o superpoziţie de unde plane, de data asta ca operatori luaţi la acelaşi timp, ca în cazul oricărui câmp cuantic,

$$\begin{cases}\hat{\varphi}(\vec{x},t)=\sum_s\int\frac{d^3\vec{p}}{\sqrt{(2\pi)^3\,2E_{\vec{p}}}}\Big(\hat{a}_s(\vec{p})\,u_s(\vec{p})\,e^{-i\,p\,\cdot\,x/\hbar}+\hat{b}_s^\dagger(\vec{p})\,v_s(\vec{p})\,e^{i\,p\,\cdot\,x/\hbar}\Big)\\ \hat{\varphi}^\dagger(\vec{x},t)=\sum_s\int\frac{d^3\vec{p}}{\sqrt{(2\pi)^3\,2E_{\vec{p}}}}\Big(\hat{b}_s(\vec{p})\,v_s^\dagger(\vec{p})\,e^{-i\,p\,\cdot\,x/\hbar}+\hat{a}_s^\dagger(\vec{p})\,u_s^\dagger(\vec{p})\,e^{i\,p\,\cdot\,x/\hbar}\Big)\\ \hat{\overline{\varphi}}(\vec{x},t)=\sum_s\int\frac{d^3\vec{p}}{\sqrt{(2\pi)^3\,2E_{\vec{p}}}}\Big(\hat{b}_s(\vec{p})\,\overline{v}_s(\vec{p})\,e^{-i\,p\,\cdot\,x/\hbar}+\hat{a}_s^\dagger(\vec{p})\,\overline{u}_s(\vec{p})\,e^{i\,p\,\cdot\,x/\hbar}\Big)\end{cases} \tag{1.7}$$

- Coeficienţii $\hat{a}_s(\vec{p})$ şi $\hat{b}_s(\vec{p})$ au structură de operatori, pe când $u_s(\vec{p})$ şi $v_s(\vec{p})$ sunt cunoscuţii spinori Dirac.

- Va trebui să ţinem cont de relaţiile de ortogonalitate pentru spinori:

$$\begin{array}{ll}\overline{u}_r(\vec{p})u_s(\vec{p})=\delta_{rs} & \overline{v}_r(\vec{p})u_s(\vec{p})=0\\ \overline{v}_r(\vec{p})v_s(\vec{p})=-\delta_{rs} & \overline{u}_r(\vec{p})v_s(\vec{p})=0\end{array} \tag{1.8}$$

- De asemenea, avem nevoie de identităţile:

$$\overline{u}_r(\vec{p})\,\gamma^0u_s(\vec{p})=u_r^\dagger(\vec{p})\,u_s(\vec{p})=\frac{\omega}{m}\delta_{rs} \tag{1.9}$$

$$\overline{v}_r(\vec{p})\,\gamma^0v_s(\vec{p})=v_r^\dagger(\vec{p})\,v_s(\vec{p})=\frac{\omega}{m}\delta_{rs} \tag{1.10}$$

Aceste identităţi pot fi demonstrate după cum urmează:

Pentru I-a identitate (1.9), plecăm de la: $\boxed{u_r^\dagger(\vec{p})\,u_s(\vec{p})=\overline{u}_r(\vec{p})\,\gamma^0u_s(\vec{p})}$

Folosind ecuaţiile Dirac pentru spinori:

$$(\gamma^\mu p_\mu-m)\,u_s(\vec{p})=0 \quad\longrightarrow\quad \overline{u}_s(\vec{p})\,(\gamma^\mu p_\mu-m)=0$$

$$\downarrow \qquad\qquad\qquad\qquad \downarrow$$

$$u_s(\vec{p})=\frac{1}{m}\gamma^\mu p_\mu u_s(\vec{p}) \qquad\qquad \overline{u}_s(\vec{p})=\frac{1}{m}\overline{u}_s(\vec{p})\,\gamma^\mu p_\mu$$

$$
\begin{aligned}
u_r^\dagger(\vec p)u_s(\vec p) &= \frac{1}{2}\Big(\overline{u}_r(\vec p)\gamma^0 u_s(\vec p) + \overline{u}_r(\vec p)\gamma^0 u_s(\vec p)\Big)\\
&= \frac{1}{2m}\Big(\overline{u}_r(\vec p)\gamma^0 \overbrace{\gamma^\mu p_\mu u_s(\vec p)}^{u_s(\vec p)} + \overbrace{\overline{u}_r(\vec p)\gamma^\mu p_\mu}^{\overline{u}_r(\vec p)}\gamma^0 u_s(\vec p)\Big)\\
&= \frac{1}{2m}\overline{u}_r(\vec p)\underbrace{\left\{\gamma^0,\gamma^\mu\right\}}_{2g^{0\mu}} p_\mu u_s(\vec p)\\
&\qquad \{\gamma^0,\gamma^\mu\} = 2g^{0\mu}\\
u_r^\dagger(\vec p)u_s(\vec p) &= \frac{\omega}{2m}\overline{u}_r(\vec p)u_s(\vec p) = \frac{\omega}{m}\delta_{rs}
\end{aligned}
$$

1.1.1 Determinarea amplitudinilor Fourier

- Să trecem la exprimarea amplitudinii $a_r(\vec p)$:

$$
\varphi(\vec x,t) = \sum_s \int \frac{d^3\vec p}{(2\pi)^3}\frac{m}{\omega}\Big[a_s(\vec p)u_s(\vec p)e^{-i\,p\cdot x} + b_s^\dagger(\vec p)v_s(\vec p)e^{i\,p\cdot x}\Big] \tag{1.11}
$$

Proiectăm (1.11) pe componenta armonică $e^{i\,p'\cdot x}$ de funcţii ortogonale:

$$
\begin{aligned}
\boxed{\int d^3\vec x\,\varphi(\vec x,t)e^{i\,p'\cdot x}} &= \sum_s \int d^3\vec x \frac{d^3\vec p}{(2\pi)^3}\frac{m}{\omega}\Big[a_s(\vec p)u_s(\vec p)e^{-i(p-p')\cdot x} + b_s^\dagger(\vec p)v_s(\vec p)e^{i\,(p+p')\cdot x}\Big]\\
&= \sum_s \int d^3\vec x \frac{d^3\vec p}{(2\pi)^3}\frac{m}{\omega}\Big[a_s(\vec p)u_s(\vec p)\,e^{-i\,(\omega-\omega')t}\,e^{i\,(\vec p-\vec p')\cdot\vec x} +\\
&\qquad + b_s^\dagger(\vec p)v_s(\vec p)\,e^{i\,(\omega'+\omega)t}\,e^{-i\,(\vec k'+\vec k)\cdot\vec x}\Big]
\end{aligned}
$$

Pentru integrala după x, folosim funcţia δ:

$$
\int \frac{d^3\vec x}{(2\pi)^3}e^{-i\,(\vec k-\vec k')\cdot\vec x} = \delta^3(\vec k-\vec k') \quad ; \quad \int \frac{d^3\vec x}{(2\pi)^3}e^{i\,(\vec k+\vec k')\cdot\vec x} = \delta^3(\vec k'+\vec k) \tag{1.12}
$$

Atunci, integrala de mai sus după $d^3\vec x$ va fi:

$$
\begin{aligned}
\boxed{\int d^3\vec x\,\varphi(\vec x,t)\,e^{i\,p'\cdot x}} &= \sum_s \int d^3\vec p\,\frac{m}{\omega}\Big[a_s(\vec p)u_s(\vec p)\,e^{-i(\omega-\omega')t}\,\delta^3(\vec p-\vec p')\\
&\qquad + b_s^\dagger(\vec p)v_s(\vec p)e^{i\,(\omega'+\omega)t}\,\delta^3(\vec p'+\vec p)\Big]
\end{aligned}
$$

Acum, putem face şi integrala după p

$$\boxed{\int d^3\vec{x}\,\varphi(\vec{x},t)\,e^{ip'\cdot x}} = \sum_s \frac{m}{\omega}\left[a_s(\vec{p}\,')\,u_s(\vec{p}\,') + b_s^\dagger(-\vec{p}\,')\,v_s(-\vec{p}\,')\,e^{i\,2\omega' t}\right]$$

$$\boxed{\int d^3\vec{x}\,\varphi(\vec{x},t)\,e^{ip\cdot x}} = \sum_s \frac{m}{\omega}\left[a_s(\vec{p})\,u_s(\vec{p}) + b_s^\dagger(-\vec{p})\,v_s(-\vec{p})\,e^{i\,2\omega t}\right]$$

$$\boxed{\int d^3\vec{x}\,\varphi(\vec{x},t)\,e^{-ip\cdot x}} = \sum_s \frac{m}{\omega}\left[a_s(-\vec{p})\,u_s(-\vec{p}) + b_s^\dagger(\vec{p})\,v_s(\vec{p})\,e^{i\,2\omega t}\right]$$

Proiectăm mai departe pe $\overline{u}_r$

$$\boxed{\int d^3\vec{x}\,\overline{u}_r(\vec{p})\,\varphi(\vec{x},t)\,e^{-ip\cdot x}} = \sum_s \frac{m}{\omega}\left[a_s(-\vec{p})\,\overline{u}_r(\vec{p})\,u_s(-\vec{p}) + b_s^\dagger(\vec{p})\,\underbrace{\overline{u}_r(\vec{p})\,v_s(\vec{p})}_{=0\,(1.8)}\,e^{i\,2\omega t}\right]$$

$$\boxed{\int d^3\vec{x}\,\underbrace{\overline{u}_r(\vec{p})}\,\varphi(\vec{x},t)\,e^{-ip\cdot x} = \sum_s \frac{m}{\omega}a_s(-\vec{p})\,\overline{u}_r(\vec{p})\,\underbrace{u_s(-\vec{p})}}$$

$$\overline{u}_r(\vec{p}) = \overline{u}_r(-\vec{p})\gamma^0 \quad ; \quad u_s(-\vec{p}) = \gamma^0 u_s(\vec{p})$$

$$\boxed{\int d^3\vec{x}\,\overbrace{\overline{u}_r(-\vec{p})\,\gamma^0}\,\varphi(\vec{x},t)\,e^{-ip\cdot x}} = \sum_s \frac{m}{\omega}a_s(-\vec{p})\,\underbrace{\overline{u}_r(\vec{p})\,\overbrace{\gamma^0 u_s(\vec{p})}}_{=\frac{\omega}{m}\,\delta_{rs}}$$

din (1.9) avem $\overline{u}_r(\vec{p})\,\gamma^0 u_s(\vec{p}) = \dfrac{\omega}{m}\delta_{rs}$

$$\boxed{\int d^3\vec{x}\,\overline{u}_r(-\vec{p})\,\gamma^0\varphi(\vec{x},t)\,e^{-ip\cdot x}} = \sum_s a_s(-\vec{p})\,\delta_{rs} = a_r(-\vec{p})$$

Operatorul căutat, ca amplitudinea $a_r(\vec{p})$ a dezvoltării Fourier (1.11), este:

$$\boxed{a_r(\vec{p}) = \int d^3\vec{x}\,\overline{u}_r(\vec{p})\,\gamma^0\varphi(\vec{x},t)\,e^{ip\cdot x}} \tag{1.13}$$

sau $$a_r(\vec{p}) = \int d^3\vec{x}\,u_r^\dagger(\vec{p})\,\varphi(\vec{x},t)\,e^{ip\cdot x}$$

Operatorul hermitic conjugat $a_r^\dagger(\vec{p})$ este:

$$a_r^\dagger(\vec{p}) = \int d^3\vec{x}\,\varphi^\dagger(\vec{x},t)\,u_r(\vec{p})\,e^{-ip\cdot x} \tag{1.14}$$

$$\boxed{a_r^\dagger(\vec{p}) = \int d^3\vec{x}\,\overline{\varphi}(\vec{x},t)\,\gamma^0 u_r(\vec{p})\,e^{-ip\cdot x}} \tag{1.15}$$

- Să trecem acum să exprimăm şi amplitudinea $b_r(\vec{p})$ din dezvoltarea adjunctă:

$$\overline{\varphi}(\vec{x},t) = \sum_s \int \frac{d^3\vec{p}}{(2\pi)^3}\frac{m}{\omega}\left[b_s(\vec{p})\overline{v}_s(\vec{p})e^{-i\,p\cdot x} + a_s^\dagger(\vec{p})\overline{u}_s(\vec{p})e^{i\,\vec{p}\cdot x}\right] \qquad (1.16)$$

Proiectăm (1.16) pe componenta armonică $e^{i\,p'\cdot x}$ de funcţii ortogonale:

$$\boxed{\int d^3\vec{x}\,\overline{\varphi}(\vec{x},t)e^{i\,p'\cdot x}} = \sum_s \int d^3\vec{x}\frac{d^3\vec{p}}{(2\pi)^3}\frac{m}{\omega}\left[b_s(\vec{p})\overline{v}_s(\vec{p})e^{-i(p-p')\cdot x} + a_s^\dagger(\vec{p})\overline{u}_s(\vec{p})e^{i\,(p+p')\cdot x}\right]$$
$$= \sum_s \int d^3\vec{x}\frac{d^3\vec{p}}{(2\pi)^3}\frac{m}{\omega}\Big[b_s(\vec{p})\overline{v}_s(\vec{p})\,e^{-i\,(\omega-\omega')t}\,e^{i\,(\vec{p}-\vec{p}')\cdot\vec{x}} +$$
$$+\, a_s^\dagger(\vec{p})\overline{u}_s(\vec{p})\,e^{i\,(\omega'+\omega)t}\,e^{-i\,(\vec{k}'+\vec{k})\cdot\vec{x}}\Big]$$

Pentru integrarea după x, folosim expresiile (1.12), cu funcţia δ

$$\int \frac{d^3\vec{x}}{(2\pi)^3}e^{-i(\vec{k}-\vec{k}')\cdot\vec{x}} = \delta^3(\vec{k}-\vec{k}') \quad ; \quad \int \frac{d^3\vec{x}}{(2\pi)^3}e^{i(\vec{k}+\vec{k}')\cdot\vec{x}} = \delta^3(\vec{k}'+\vec{k}) \qquad (1.12)$$

Atunci, integrala de mai sus după $d^3\vec{x}$ va fi:

$$\boxed{\int d^3\vec{x}\,\overline{\varphi}(\vec{x},t)e^{i\,p'\cdot x}} = \sum_s \int d^3\vec{p}\,\frac{m}{\omega}\Big[b_s(\vec{p})\overline{v}_s(\vec{p})\,e^{-i\,(\omega-\omega')t}\,\delta^3(\vec{p}-\vec{p}') +$$
$$+ a_s^\dagger(\vec{p})\overline{u}_s(\vec{p})\,e^{i\,(\omega'+\omega)t}\,\delta^3(\vec{p}'+\vec{p})\Big]$$

Acum, facem şi integrala după p

$$\boxed{\int d^3\vec{x}\,\overline{\varphi}(\vec{x},t)\,e^{i\,p'\cdot x}} = \sum_s \frac{m}{\omega}\left[b_s(\vec{p}')\,\overline{v}_s(\vec{p}') + a_s^\dagger(-\vec{p}')\,\overline{u}_s(-\vec{p}')\,e^{i\,2\omega' t}\right]$$
$$\boxed{\int d^3\vec{x}\,\overline{\varphi}(\vec{x},t)\,e^{i\,p\cdot x}} = \sum_s \frac{m}{\omega}\left[b_s(\vec{p})\,\overline{v}_s(\vec{p}) + a_s^\dagger(-\vec{p})\,\overline{u}_s(-\vec{p})\,e^{i\,2\omega t}\right]$$
$$\boxed{\int d^3\vec{x}\,\overline{\varphi}(\vec{x},t)\,e^{-i\,p\cdot x}} = \sum_s \frac{m}{\omega}\left[b_s(-\vec{p})\,\overline{v}_s(-\vec{p}) + a_s^\dagger(\vec{p})\,\overline{u}_s(\vec{p})\,e^{i\,2\omega t}\right]$$

Proiectăm mai departe pe $v_r(\vec{p})$

$$\boxed{\int d^3\vec{x}\;\overline{\varphi}(\vec{x},t)\,v_r(\vec{p})\,e^{-i\,p\cdot x}} = \sum_s \frac{m}{\omega}\Big[b_s(-\vec{p})\,\overline{v}_s(-\vec{p})v_r(\vec{p}) + a_s^\dagger(\vec{p})\;\underbrace{\overline{u}_s(\vec{p})\,v_r(\vec{p})}_{=0\,(1.8)}\,e^{i\,2\omega t}\Big]$$

$$\boxed{\int d^3\vec{x}\ \overline{\varphi}(\vec{x},t)\, \underbrace{v_r(\vec{p})}\, e^{-i\,p\cdot x}} = \sum_s \frac{m}{\omega} b_s(-\vec{p})\ \underbrace{\overline{v}_s(-\vec{p})}\, v_r(\vec{p})$$

$$v_r(\vec{p}) = -\gamma^0 v_r(-\vec{p}) \quad ; \quad \overline{v}_s(-\vec{p}) = -\overline{v}_s(\vec{p})\gamma^0$$

$$\boxed{-\int d^3\vec{x}\ \overline{\varphi}(\vec{x},t)\, \overbrace{\gamma^0 v_r(-\vec{p})}\, e^{-i\,p\cdot x}} = -\sum_s \frac{m}{\omega} b_s(-\vec{p})\ \underbrace{\overbrace{\overline{v}_s(\vec{p})\,\gamma^0}\, v_r(\vec{p})}_{=\frac{\omega}{m}\,\delta_{sr}}$$

din (1.10) avem $\overline{v}_r(\vec{p})\,\gamma^0 v_s(\vec{p}) = \dfrac{\omega}{m}\delta_{rs}$

$$\boxed{\int d^3\vec{x}\ \overline{\varphi}(\vec{x},t)\gamma^0\, v_r(-\vec{p})\, e^{-i\,p\cdot x}} = \sum_s b_s(-\vec{p})\,\delta_{rs} = b_r(-\vec{p})$$

$$\int d^3\vec{x}\ \overline{\varphi}(\vec{x},t)\,\gamma^0\, v_r(\vec{p})\, e^{i\,p\cdot x} = b_r(\vec{p})$$

$$\boxed{b_r(\vec{p}) = \int d^3\vec{x}\,\overline{\varphi}(\vec{x},t)\gamma^0 v_r(\vec{p})\, e^{i\,p\cdot x}} \tag{1.17}$$

sau $$b_r(\vec{p}) = \int d^3\vec{x}\,\varphi^\dagger(\vec{x},t)\, v_r(\vec{p})\, e^{i\,p\cdot x}$$

Operatorul hermitic conjugat $b_r^\dagger(\vec{p})$ este:

$$b_r^\dagger(\vec{p}) = \int d^3\vec{x}\, v_r^\dagger(\vec{p})\,\varphi(\vec{x},t)\, e^{-i\,p\cdot x}$$

$$\boxed{b_r^\dagger(\vec{p}) = \int d^3x\,\overline{v}_r^\dagger(\vec{p})\,\gamma^0\varphi(\vec{x},t)\, e^{-i\,p\cdot x}} \tag{1.18}$$

- In concluzie, setul complet de operatori căutaţi, este:

$$\boxed{\begin{aligned} a_r(\vec{p}) &= \int d^3x\,\overline{u}_r(\vec{p})\,\gamma^0\varphi(\vec{x},t)\, e^{i\,p\cdot x} \\ a_r^\dagger(\vec{p}) &= \int d^3x\,\overline{\varphi}(\vec{x},t)\,\gamma^0 u_r(\vec{p})\, e^{-i\,p\cdot x} \\ b_r(\vec{p}) &= \int d^3x\,\overline{\varphi}(\vec{x},t)\gamma^0 v_r(\vec{p})\, e^{i\,p\cdot x} \\ b_r^\dagger(\vec{p}) &= \int d^3x\,\overline{v}_r^\dagger(\vec{p})\,\gamma^0\varphi(\vec{x},t)\, e^{-i\,p\cdot x} \end{aligned}} \tag{1.19}$$