Procese QED în câmpuri laser intense
tema 51

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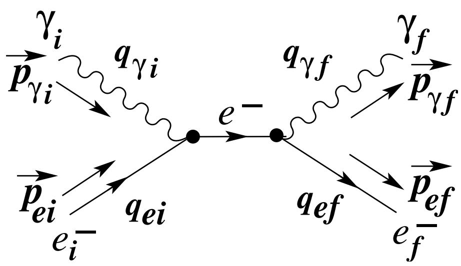
”Verbum sapienti sat est”

Cuprins

1.1 Cinematica împrăştierii $e_i\,\gamma_i \to e_f\,\gamma_f$



$$E_\gamma = h\nu \qquad \vec{p}_\gamma = \frac{E_\gamma}{c}\vec{n}_\gamma$$

$$E_e = \gamma_e\, mc^2 \qquad \vec{p}_e = \gamma_e\, m\vec{v}_e = \frac{E_e}{c^2}\vec{v}_e = \frac{E_e}{c}\vec{\beta}_e$$

$$q \equiv \left(\frac{E}{c}\,,\,\vec{p}\right) \text{ cu metrica: } g_{\mu\nu} = (1,-1,-1,-1)$$

$$q^2 = \frac{E^2}{c^2} - \vec{p}^{\,2} = m^2c^2$$

$$q_{\gamma i} = \frac{E_{\gamma i}}{c}\Big(1, \vec{n}_{\gamma i}\Big) \qquad q_{\gamma f} = \frac{E_{\gamma f}}{c}\Big(1, \vec{n}_{\gamma f}\Big) \qquad \beta_e = \frac{p_e c}{E_e} = \frac{v_e}{c}$$

$$q_{ei} = \frac{E_{ei}}{c}\Big(1, \vec{\beta}_{ei}\Big) \qquad q_{ef} = \frac{E_{ef}}{c}\Big(1, \vec{\beta}_{ef}\Big) \qquad \gamma_e = \frac{E_e}{mc^2} = \frac{1}{\sqrt{1-\beta_e^2}}$$

Figura 1.1: Variabilele şi relaţiile cinematice ale împrăştierii $e_i\,\gamma_i \to e_f\,\gamma_f$

- Conservarea 4-impulsului: $\boxed{q_{\gamma i} + q_{ei} = q_{\gamma f} + q_{ef}}$ (1.1)
- Prin ridicarea la pătrat:

$$\Big(q_{\gamma i} + q_{ei}\Big)^2 = \underbrace{q_{\gamma i}^2}_{=0} + 2\, q_{\gamma i}\cdot q_{ei} + \underbrace{q_{ei}^2}_{=m^2c^2} = 2\, q_{\gamma i}\cdot q_{ei} + m^2c^2 \tag{1.2}$$

$$\Big(q_{\gamma f} + q_{ef}\Big)^2 = \underbrace{q_{\gamma f}^2}_{=0} + 2\, q_{\gamma f}\cdot q_{ef} + \underbrace{q_{ef}^2}_{=m^2c^2} = 2\, q_{\gamma f}\cdot q_{ef} + m^2c^2 \tag{1.3}$$

Din egalarea celor două relaţii rezultă: $\boxed{q_{\gamma i}\cdot q_{ei} = q_{\gamma f}\cdot q_{ef}}$ (1.4)

1.1.1 Energia fotonului împrăştiat (Compton) funcţie de energia fotonului iniţial

- Inmulţim relaţia (1.1) cu $q_{\gamma f}$, adică o proiectăm pe direcţia fotonului final.

$$q_{\gamma f}\cdot\Big[q_{\gamma i} + q_{ei} = q_{\gamma f} + q_{ef}\Big] \quad \longrightarrow \quad \text{folosind (1.4)}$$

$$q_{\gamma f}\cdot q_{\gamma i} + q_{\gamma f}\cdot q_{ei} = \underbrace{q_{\gamma f}\cdot q_{\gamma f}}_{=0} + \underbrace{q_{\gamma f}\cdot q_{ef}}_{=q_{\gamma i}\cdot q_{ei}}$$

Adică $\boxed{q_{\gamma f}\cdot\Big(q_{\gamma i} + q_{ei}\Big) = q_{\gamma i}\cdot q_{ei}}$ (1.5)

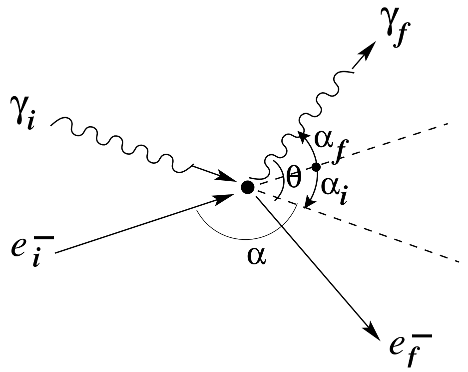


Figura 1.2: Cinematica împrăştierii $\gamma_i\, e_i^- \to \gamma_f\, e_f^-$

Relaţia (1.5) exprimă împrăştierea Compton $\gamma - e$. Aceasta, scrisă pe componente, devine:

$$\overbrace{\frac{E_{\gamma f}}{c}\frac{E_{\gamma i}}{c} - \frac{E_{\gamma f}}{c}\frac{E_{\gamma i}}{c}\,\vec{n}_{\gamma f}\cdot\vec{n}_{\gamma i}}^{q_{\gamma f}\cdot q_{\gamma i}} + \overbrace{\frac{E_{\gamma f}}{c}\gamma_{ei}\,mc - \frac{E_{\gamma f}}{c}\,\vec{n}_{\gamma f}\cdot\gamma_{ei}\,m\vec{v}_{ei}}^{q_{\gamma f}\cdot q_{ei}} =$$

$$= \overbrace{\frac{E_{\gamma i}}{c}\,\gamma_{ei}\,mc - \frac{E_{\gamma i}}{c}\,\vec{n}_{\gamma i}\cdot\gamma_{ei}\,m\,\vec{v}_{ei}}^{q_{\gamma i}\cdot q_{ei}}$$

sau

$$\left.\begin{aligned}\frac{E_{\gamma f}}{c}\frac{E_{\gamma i}}{c}\left(1 - \underbrace{\vec{n}_{\gamma f}\cdot\vec{n}_{\gamma i}}_{\cos\theta}\right) + \frac{E_{\gamma f}}{c}\,\gamma_{ei}\,m\left(c - \underbrace{\vec{n}_{\gamma f}\cdot\vec{v}_{ei}}_{v_{ei}\cos\alpha_f}\right) &= \\ = \frac{E_{\gamma i}}{c}\,\gamma_{ei}\,m\left(c - \underbrace{\vec{n}_{\gamma i}\cdot\vec{v}_{ei}}_{v_{ei}\cos\alpha_i}\right) &\end{aligned}\right| \times c^2$$

$$E_{\gamma f}E_{\gamma i}\Big(1-\cos\theta\Big) + E_{\gamma f}\,\underbrace{\gamma_{ei}\,m\,c^2}_{E_{ei}}\left(1-\underbrace{\frac{v_{ei}}{c}}_{\beta_{ei}}\cos\alpha_f\right) = E_{\gamma i}\,\underbrace{\gamma_{ei}\,m\,c^2}_{E_{ei}}\left(1-\underbrace{\frac{v_{ei}}{c}}_{\beta_{ei}}\cos\alpha_i\right)$$

de unde energia fotonului împrăştiat funcţie de cea a fotonului iniţial, este:

$$\boxed{E_{\gamma f} = \frac{E_{\gamma i}\,E_{ei}\Big(1 - \beta_{ei}\cos\alpha_i\Big)}{E_{\gamma i}\Big(1-\cos\theta\Big) + E_{ei}\Big(1 - \beta_{ei}\cos\alpha_f\Big)}} \tag{1.6}$$

Modificarea energiei fotonului:

$$\frac{E_{\gamma f}}{E_{\gamma i}} = \frac{1 - \beta_{ei}\cos\alpha_i}{\dfrac{E_{\gamma i}}{E_{ei}}\Big(1-\cos\theta\Big) + \Big(1 - \beta_{ei}\cos\alpha_f\Big)} \tag{1.7}$$

In SR al electronului iniţial:

$\beta_{ei}=0$; $\gamma_{ei}=1$; $E_{ei}=mc^2$.

$$\boxed{\frac{E_{\gamma f}}{E_{\gamma i}} = \frac{1}{1 + \dfrac{E_{\gamma i}}{mc^2}\Big(1-\cos\theta\Big)}} \tag{1.8}$$

- Să exprimăm (1.8) prin variaţia lungimii de undă λ a fotonului. O scriem sub forma:

$$E_{\gamma f} = \frac{1}{\dfrac{1}{E_{\gamma i}} + \dfrac{1-\cos\theta}{mc^2}} \quad \Longrightarrow \quad \frac{1}{E_{\gamma i}} + \frac{1-\cos\theta}{mc^2} = \frac{1}{E_{\gamma f}} \tag{1.9}$$

trecem la lungimea de undă: $E_{\gamma i} = h\nu_i = \dfrac{hc}{\lambda_i} \quad \Longrightarrow \quad \dfrac{1}{E_{\gamma i}} = \dfrac{\lambda_i}{hc}$

(1.9) devine: $\dfrac{\lambda_i}{hc} + \dfrac{1-\cos\theta}{mc^2} = \dfrac{\lambda_f}{hc}$ sau $\boxed{\lambda_f - \lambda_i = \dfrac{h}{mc}\Big(1-\cos\theta\Big)}$ (1.10)

1.1.2 Energie foton împrăştiat (invers Compton) funcţie de energie electron iniţial

- Inmulţim (1.5) cu q_{ei}

$$q_{\gamma f}\cdot q_{\gamma i}\cdot q_{ei} + q_{\gamma f}\cdot \underbrace{q_{ei}\cdot q_{ei}}_{m^2c^2} = q_{\gamma i}\cdot q_{ei}\cdot q_{ei}$$

$$q_{\gamma f}\cdot\Big(q_{\gamma i}\cdot q_{ei} + m^2c^2\Big) = \Big(q_{\gamma i}\cdot q_{ei}\Big)\cdot q_{ei}$$

- De unde 4-impulsul fotonului final este,

$$q_{\gamma f} = \frac{\Big(q_{\gamma i}\cdot q_{ei}\Big)}{q_{\gamma i}\cdot q_{ei} + m^2c^2}\, q_{ei}$$

- Componenta temporală exprimă energia fotonului final

$$\boxed{E_{\gamma f} = \frac{\Big(q_{\gamma i}\cdot q_{ei}\Big)}{\Big(q_{\gamma i}\cdot q_{ei}\Big) + m^2c^2}\, E_{ei}} \tag{1.11}$$

unde

$$q_{\gamma i}\cdot q_{ei} = \frac{E_{\gamma i}}{c}\frac{E_{ei}}{c} - \frac{E_{\gamma i}}{c}\frac{E_{ei}}{c}\,\vec{n}_{\gamma i}\cdot\vec{\beta}_{ei} = \frac{E_{\gamma i}E_{ei}}{c^2}\Big(1-\beta_{ei}\cos(\overbrace{\pi-\alpha}^{\alpha_i})\Big) \tag{1.12}$$

adică

$$\boxed{q_{\gamma i}\cdot q_{ei} = \frac{E_{\gamma i}E_{ei}}{c^2}\Big(1+\beta_{ei}\cos\alpha\Big)} \tag{1.13}$$

(1.11) devine

$$E_{\gamma f} = \frac{E_{ei}}{1+\dfrac{m^2c^2}{q_{\gamma i}\cdot q_{ei}}} = \frac{E_{ei}}{1+\dfrac{m^2c^4}{E_{\gamma i}E_{ei}}\,\dfrac{1}{\Big(1+\beta_{ei}\cos\alpha\Big)}} \tag{1.14}$$

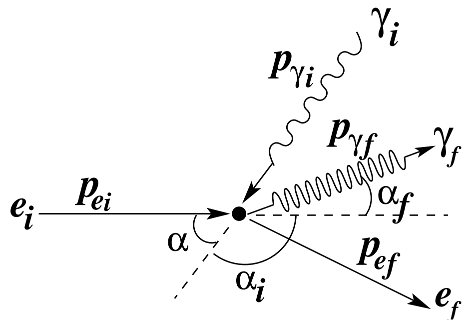


Figura 1.3: Geometria SLAC E-144 a împrăştierii Compton inverse

Experimentul SLAC E-144
In geometria experimentului SLAC E-144 (vezi Fig. 1.3) avem: $\beta_{ei} \approx 1 \quad ; \quad \alpha \approx 0$

(1.13) devine
$$q_{\gamma i} \cdot q_{ei} = \frac{2E_{\gamma i}E_{ei}}{c^2} \tag{1.15}$$

pentru interacţia multifoton $E_{\gamma i} \rightarrow nE_{\gamma i}$ energia fotonului final (1.11) devine

$$E_{\gamma f} \approx \frac{2n\,E_{\gamma i}E_{ei}}{2nE_{\gamma i}E_{ei} + m^2c^4} E_{ei} \tag{1.16}$$

(similar cu (23) din preprint SLAC 1999)

1.1.3 Energie electron împrăştiat (invers Compton) funcţie de energie electron iniţial

- Din relaţia (1.4) $\quad q_{\gamma i} \cdot q_{ei} = q_{\gamma f} \cdot q_{ef}$

  folosind dezvoltarea produsului de 4-impulsuri (1.12), avem:

$$q_{\gamma i} \cdot q_{ei} = \frac{E_{\gamma i}E_{ei}}{c^2}\Big(1 - \beta_{ei}\cos\alpha_{\gamma iei}\Big) \tag{1.17}$$

$$q_{\gamma f} \cdot q_{ef} = \frac{E_{\gamma f}E_{ef}}{c^2}\Big(1 - \beta_{ef}\cos\alpha_{\gamma fef}\Big) \tag{1.18}$$

- Din egalarea celor două ecuaţii de mai sus, exprimăm imediat E_{ef},

  adică
$$\boxed{E_{ef} = \frac{E_{\gamma i}\big(1 - \beta_{ei}\cos\alpha_{\gamma iei}\big)}{E_{\gamma f}\big(1 - \beta_{ef}\cos\alpha_{\gamma fef}\big)} E_{ei}} \tag{1.19}$$

Experimentul SLAC E-144

In geometria experimentului SLAC E-144 (vezi Fig. 1.3) avem:

$\beta_{ei} \approx 1 \quad ; \quad \alpha_{\gamma iei} \equiv \alpha_i = \pi - \alpha, \quad \Longrightarrow \quad 1 - \beta_{ei}\cos\alpha_{\gamma iei} = 1 + \cos\alpha$

$$E_{ef} = \frac{E_{\gamma i}\big(1 + \cos\alpha\big)}{E_{\gamma f}\big(1 - \beta_{ef}\cos\alpha_{\gamma fef}\big)} E_{ei} \;\overset{\substack{(1.14)\\ E_{\gamma f}\\ \downarrow}}{=}\; \frac{E_{\gamma i}\big(1 + \cos\alpha\big) + \dfrac{m^2c^4}{E_{ei}}}{1 - \beta_{ef}\cos\alpha_{\gamma fef}} \tag{1.20}$$